Integration: Trigonometric

1. Powers of sine and cosine.
   Integrals of the form $\int \sin^m x \cos^n x\,dx$.

$\int \sin^m x \cos^n x\,dx$	*Procedure*	*Relevant Identities*
n odd	1. Split off a factor of cos(x). 2. Apply the relevant identity. 3. Make the substitution *u* = sin(x)	$\cos^2 x = 1 - \sin^2 x$
m odd	1. Split off a factor of sin(x). 2. Apply the relevant identity. 3. Make the substitution *u* = cos(x)	$\sin^2 x = 1 - \cos^2 x$
m even and *n* even	Use the relevant identities to reduce the powers on sin(x) and cos(x).	$\sin^2 x = \frac{1}{2}\ 1 - \cos(2x)$ $\cos^2 x = \frac{1}{2}\ 1 + \cos(2x)$

2. Powers of tangent and secant.
   Integrals of the form $\int \tan^m x \sec^n x\,dx$.

$\int \tan^m x \sec^n x\,dx$	*Procedure*	*Relevant Identities*
n even	1. Split off a factor of $\sec^2 x$. 2. Apply the relevant identity. (all in terms of tan) 3. Make the substitution *u* = $\tan x$	$\sec^2 x = \tan^2 x + 1$
m odd	1. Split off a factor of $\sec x \tan x$. 2. Apply the relevant identity. (all in terms of sec) 3. Make the substitution *u* = $\sec x$	$\tan^2 x = \sec^2 x - 1$
m even and *n* odd	Use the relevant identities to reduce to powers of $\sec x$ alone. Then use reduction formulas for powers of secant.	$\tan^2 x = \sec^2 x - 1$

Trigonometric Substitution

Integrals that contain expression of the form

$$\sqrt{a^2 - x^2},\ \sqrt{x^2 + a^2},\ \sqrt{x^2 - a^2}\ .$$

Expression in the integrand	*Substitution*	*Restriction on* θ
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{x^2 + a^2}$	$x = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$0 \le \theta < \frac{\pi}{2} \cup \pi \le \theta < \frac{3\pi}{2}$