Chapter 4

Systems of Equations in Two Variables

Exercise Set 4.1

1. True

3. True

5. True

7. False

9. We use alphabetical order for the variables. Substitute 1 for x and 2 for y.

$$\begin{array}{r|l} \multicolumn{2}{l}{4x-y=2} \\ \hline 4(1)-(2) & 2 \\ 4-2 & 2 \\ 2 & 2 \quad \text{TRUE} \end{array}$$

$$\begin{array}{r|l} \multicolumn{2}{l}{10x-3y=4} \\ \hline 10(1)-3(2) & 4 \\ 10-6 & 4 \\ 4 & 4 \quad \text{TRUE} \end{array}$$

The ordered pair $(1,2)$ is a solution of the system of equations.

11. We use alphabetical order for the variables. Substitute -5 for x and 1 for y.

$$\begin{array}{r|l} \multicolumn{2}{l}{x+5y=0} \\ \hline (-5)+5(1) & 0 \\ -5+5 & 0 \\ 0 & 0 \quad \text{TRUE} \end{array}$$

$$\begin{array}{l|r} \multicolumn{2}{l}{y=2x+9} \\ \hline (1) & 2(-5)+9 \\ 1 & -10+9 \\ 1 & -1 \quad \text{FALSE} \end{array}$$

The ordered pair $(-5,1)$ is not a solution of the system of equations.

13. We use alphabetical order for the variables. Substitute 0 for x and -5 for y.

$$\begin{array}{r|l} \multicolumn{2}{l}{x-y=5} \\ \hline (0)-(-5) & 5 \\ 0-(-5) & 5 \\ 5 & 5 \quad \text{TRUE} \end{array} \qquad \begin{array}{r|r} \multicolumn{2}{l}{y=3x-5} \\ \hline (-5) & 3(0)-5 \\ -5 & 0-5 \\ -5 & -5 \quad \text{TRUE} \end{array}$$

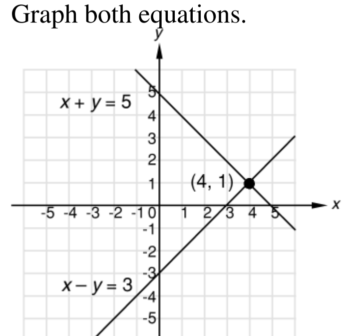
The ordered pair $(0,-5)$ is a solution of the system of equations.

15. Observe that if we multiply both sides of the first equation by 2, we get the second equation. Thus, if we find that the given point makes the one equation true, we will know that it makes the other equation true also. Substitute 3 for x and 1 for y.

$$\begin{array}{r|l} \multicolumn{2}{l}{3x+4y=13} \\ \hline 3(3)+4(1) & 13 \\ 9+4 & 13 \\ 13 & 13 \quad \text{TRUE} \end{array}$$

The ordered pair $(3,1)$ is a solution of the system of equations.

17. Graph both equations.



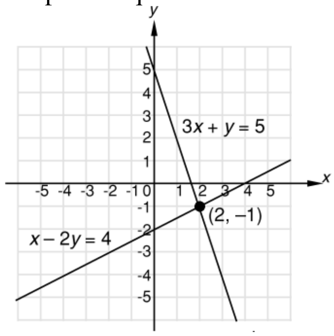
The solution (point of intersection) is apparently $(4,1)$.

Check:

$$\begin{array}{r|l} \multicolumn{2}{l}{x-y=3} \\ \hline 4-1 & 3 \\ 3 & 3 \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{l}{x+y=5} \\ \hline 4+1 & 5 \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The solution is $(4,1)$.

19. Graph the equations.



The solution (point of intersection) is apparently $(2,-1)$.

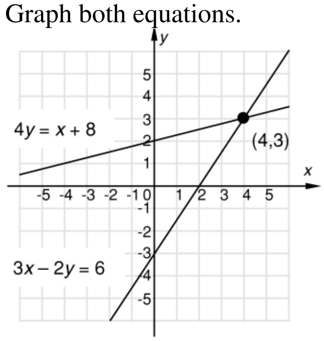
Check:

$3x+y=5$	
$3\cdot 2+(-1)$	5
$6-1$	5
5	5 TRUE

$x-2y=4$	
$2-2(-1)$	4
$2+2$	
4	4 TRUE

The solution is $(2,-1)$.

21. Graph both equations.



The solution point of intersection) is apparently $(4,3)$.

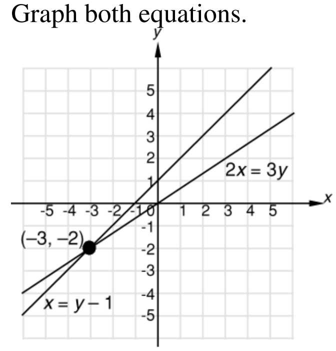
Check:

$4y=x+8$	
$4\cdot 3$	$4+8$
12	12 TRUE

$3x-2y=6$	
$3\cdot 4-2\cdot 3$	6
$12-6$	
6	6 TRUE

The solution is $(4,3)$.

23. Graph both equations.



The solution (point of intersection) is apparently $(-3,-2)$.

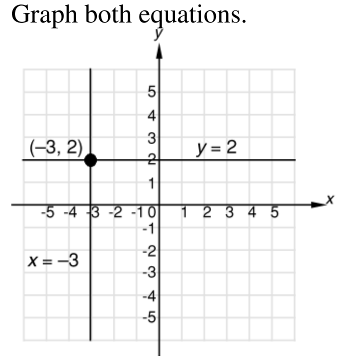
Check:

$x=y-1$	
-3	$-2-1$
-3	-3 TRUE

$2x=3y$	
$2\cdot(-3)$	$3\cdot(-2)$
-6	-6 TRUE

The solution is $(-3,-2)$.

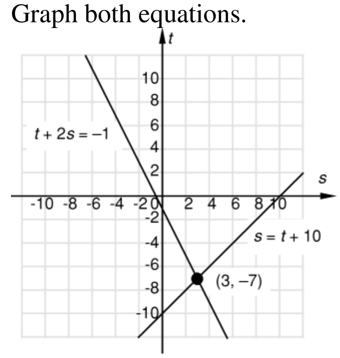
25. Graph both equations.



The solution (point of intersection) is apparently $(-3,2)$.

The ordered pair $(-3,2)$ checks in both equations. It is the solution.

27. Graph both equations.



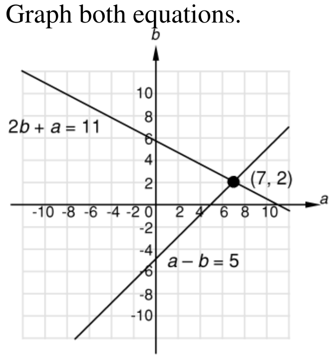
The solution (point of intersection) is apparently $(3,-7)$.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{t+2s=-1} \\ \hline -7+2\cdot 3 & -1 \\ -7+6 & \\ -1 & -1 \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{s=t+10} \\ \hline 3 & -7+10 \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The solution is $(3,-7)$.

29. Graph both equations.



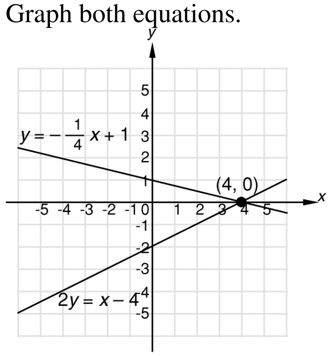
The solution (point of intersection) is apparently $(7,2)$.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{2b+a=11} \\ \hline 2\cdot 2+7 & 11 \\ 4+7 & \\ 11 & 11 \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{a-b=5} \\ \hline 7-2 & 5 \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The solution is $(7,2)$.

31. Graph both equations.



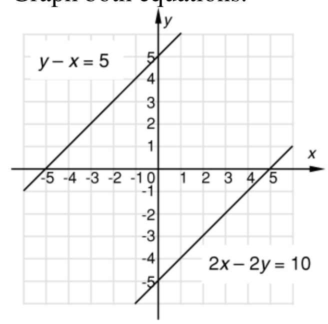
The solution (point of intersection) is apparently $(4,0)$.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{y=-\frac{1}{4}x+1} \\ \hline 0 & -\frac{1}{4}\cdot(4)+1 \\ & -1+1 \\ 0 & 0 \quad \text{TRUE} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{2y=x-4} \\ \hline 2\cdot 0 & 4-4 \\ 0 & 0 \quad \text{TRUE} \end{array}$$

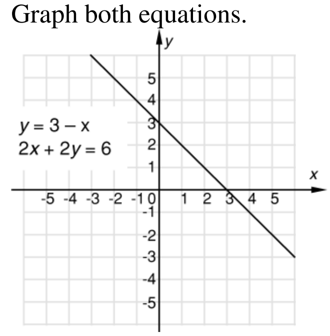
The solution is $(4,0)$.

33. Graph both equations.



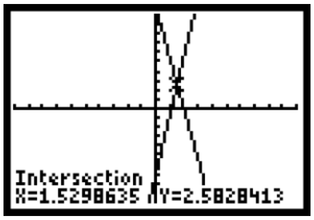
The lines are parallel. The system has no solution.

35. Graph both equations.



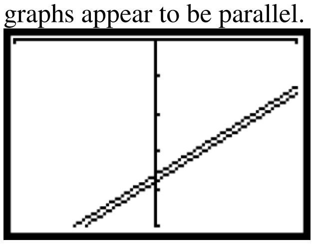
The graphs are the same. Any solution of one equation is a solution of the other. Each equation has infinitely many solutions. The solution set is the set of all pairs (x, y) for which $y = 3-x$, or $\{(x, y) \mid y = 3-x\}$. (In place of $y = 3-x$, we could have used $2x+2y=6$ since the two equations are equivalent.)

37. Enter $y_1 = -5.43x + 10.89$ and $y_2 = 6.29x - 7.04$ on a graphing calculator and use the INTERSECT feature.



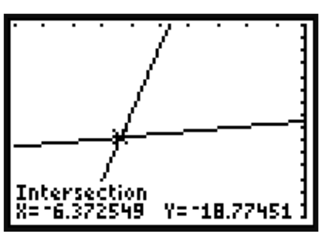
The solution is about $(1.53, 2.58)$.

39. Solve each equation for *y*. We get $y = \dfrac{-2.6x + 4}{-1.1}$ and $y = \dfrac{3.12x - 5.04}{1.32}$. Graph these equations on a graphing calculator, using a window that shows both graphs clearly. One choice is [–1, 1, –5, 0]. The graphs appear to be parallel.



This is confirmed by the error message "NO SIGN CHNG" that is returned when we use the INTERSECT feature. The system of equations has no solution.

41. Solve each equation for *y*. We get $y = 0.2x - 17.5$ and $y = \dfrac{10.6x + 30}{2}$. Graph these equations on a graphing calculator and use the INTERSECT feature.



The solution is about $(-6.37, -18.77)$.

43. A system of equations is consistent if it has at least one solution. Of the systems under consideration, only the ones in Exercises 33 and 39 have no solution. Therefore, all except the systems in Exercise 33 and 39 are consistent.

45. A system of two equations in two variables is dependent if it has infinitely many solutions. Only the system in Exercise 35 is dependent.

47. (a) Let *x* represent the number of years since 1980 and *y* represent the number of full-time faculty, in thousands, for the first regression line, and the number of part-time faculty, in thousands, for the second regression line. To determine the regression equation for each type of employment, enter the following data in lists L1, L2, and L3:

*L*1	*L*2	*L*3
0	450	236
5	459	256
11	536	291
15	551	381
19	591	437
25	676	615

Press **STAT** and move the cursor to **CALC.** Under **CALC** select **LinReg(a**x **+ b)** and select lists L1 and L2 to display the equation: $y \approx 9.0524x + 430.6778$.

(b) Press **STAT** again and move the cursor to **CALC.** **S**elect **LinReg(a**x **+ b)** and select lists L1 and L3 to display the equation: $y \approx 14.7175x + 185.3643$.

(c) Graph these equations on your calculator and use the INTERSECT feature. According to these equations, the number of full-time and part-time faculty will be the same in 2023 $(x \approx 43.30)$.

49. (a) Let *x* represent the number of years since 2004 and *y* represent the independent financial advisers, in thousands, for the first regression line, and the number of financial advisors with national firms, in thousands, for the second regression line. To determine the regression equation for each type of adviser, enter the following data in lists L1, L2, and L3:

L1	L2	L3
0	21	60
1	23	62
2	25	59
3	28	57
4	32	55

Press **STAT** and move the cursor to **CALC.** Under **CALC** select **LinReg(a**x **+ b)** and select lists L1 and L2 to display the equation: $y = 2.7x + 20.4$.

(b) Press **STAT** again and move the cursor to **CALC. S**elect **LinReg(a**x **+ b)** and select lists L1 and L3 to display the equation: $y = -1.5x + 61.6$.

(c) Graph these equations on your calculator and use the INTERSECT feature. According to these equations, the number of independent advisers will equal the number of financial advisors with national firms in about 2014 $(x \approx 9.8)$.

51. *Thinking and Writing Exercise.*

53. $2(4x-3)-7x = 9$

$8x-6-7x = 9$ Removing parentheses

$x-6 = 9$ Collecting like terms

$x = 15$ Adding 6 to both sides

55. $4x-5x = 8x-9+11x$

$-x = 19x-9$ Collecting like terms

$-20x = -9$ Adding $-19x$ to both sides

$x = \frac{9}{20}$ Mult. both sides by $-\frac{1}{20}$

57. $3x+4y = 7$

$4y = -3x+7$ Add $-3x$ both sides

$y = \frac{1}{4}(-3x+7)$ Mult. both sides by $\frac{1}{4}$

$y = -\frac{3}{4}x+\frac{7}{4}$

59. *Thinking and Writing Exercise.*

61. (a) There are many correct answers. One can be found by expressing the sum and difference of the two numbers:

$x + y = 6$

$x - y = 4$

(b) There are many correct answers. For example, write an equation in two variables. Then write a second equation by multiplying the left side of the first equation by one nonzero constant and multiplying the right side by another nonzero constant.

$x + y = 1$

$2x + 2y = 3$

(c) There are many correct answers. One can be found by writing an equation in two variables and then writing a nonzero constant multiple of that equation:

$x + y = 1$

$2x + 2y = 2$

63. Substitute 4 for *x* and –5 for *y* in the first equation:

$A(4)-6(-5) = 13$

$4A+30 = 13$

$4A = -17$

$A = -\frac{17}{4}$

Substitute 4 for *x* and –5 for *y* in the second equation:

$4-B(-5) = -8$

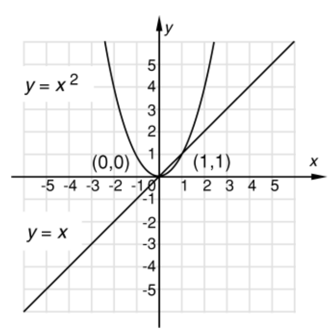
$4+5B = -8$

$5B = -12$

$B = -\frac{12}{5}$

We have $A = -\frac{17}{4}$ and $B = -\frac{12}{5}$.

65.



The solutions are apparently $(0,0)$ and $(1,1)$. Both pairs check.

67. The equations have the same slope and the same y-intercept. Thus their graphs are the same. Graph (c) matches this system.

69. The equations have the same slope and different y-intercepts. Thus their graphs are parallel lines. Graph (b) matches this system.

71. (a). Familiarize. The number of notebook PCs shipped in 2004 was 46 million and growing at a rate of $22\frac{2}{3} = \frac{68}{3}$ million per year. The number of desktop PCs shipped in 2004 was 140 million and growing at a rate of 4 million per year. Let $t =$ the number of years since 2004, and $n =$ the number of PCs shipped, in millions.
Translate. For a linear equation, the rate of growth represents the slope of the line, and the value at $t = 0$, in this case, the amount shipped in 2004 is the y-intercept. Thus for notebooks, the amount in millions shipped worldwide satisfies the equation:
$n = \frac{68}{3}t + 46.$
And for desktops the amount in millions is:
$n = 4t + 140$

(b) Graph $y_1 = \frac{68}{3}t + 46$ and $y_2 = 4t + 140$ on a graphing calculator and use the INTERSECT feature. The x value (or t value) is approximately 5. According to these equations, approximately 5 years after 2004, in 2009, the numbers of notebook PCs shipped worldwide will equal the number of desktop PCs shipped worldwide.

Exercise Set 4.2

1. False

3. True

5. $x + y = 5$ (1)
$y = x + 3$ (2)
Substitute $x + 3$ for y in the first equation and solve for x.
$$\begin{aligned} x + (x+3) &= 5 \\ x + x + 3 &= 5 \\ 2x + 3 &= 5 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$
Next substitute 1 for x in either equation of the original system and solve for y.
$y = (1) + 3 = 4$
Check the ordered pair (1, 4).

$x + y = 5$			$y = x + 3$		
$(1)+(4)$	5		(4)	$(1)+3$	
$1+4$	5		4	$1+3$	
5	5	TRUE	4	4	TRUE

The ordered pair (1, 4) is the solution.

7. $x = y + 1$ (1)
$x + 2y = 4$ (2)
Substitute $y + 1$ for x in the second equation and solve for y.
$$\begin{aligned} (y+1) + 2y &= 4 \\ y + 1 + 2y &= 4 \\ 3y + 1 &= 4 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$
Next substitute 1 for y in either equation of the original system and solve for x.
$x = y + 1 = 1 + 1 = 2$
Check the ordered pair (2, 1).

$x = y + 1$			$x + 2y = 4$		
(2)	$(1)+1$		$(2)+2(1)$	4	
2	$1+1$		$2+2$	4	
2	2	TRUE	4	4	TRUE

The ordered pair (2,1) is the solution.

9. $y = 2x - 5$ (1)
$3y - x = 5$ (2)
Substitute $2x - 5$ for y into the second equation and solve for x.
$$\begin{aligned} 3y - x &= 5 \quad (2) \\ 3(2x-5) - x &= 5 \\ 6x - 15 - x &= 5 \\ 5x - 15 &= 5 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Next substitute 4 for x in either equation of the original system and solve for y.

$y = 2x - 5 \quad (1)$

$y = 2 \cdot 4 - 5$

$y = 8 - 5$

$y = 3$

Check the ordered pair (4, 3).

$$\begin{array}{c|c} \multicolumn{2}{c}{y = 2x - 5} \\ \hline (3) & 2(4) - 5 \\ 3 & 8 - 5 \\ 3 & 3 \text{ TRUE} \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{3y - x = 5} \\ \hline 3(3) - 4 & 5 \\ 9 - 4 & 5 \\ 5 & 5 \text{ TRUE} \end{array}$$

The ordered pair (4, 3) is the solution.

11. $a = -4b \quad (1)$

$a + 5b = 5 \quad (2)$

Substitute $-4b$ for a into the second equation and solve for b.

$a + 5b = 5 \quad (2)$

$(-4b) + 5b = 5$

$-4b + 5b = 5$

$b = 5$

Next substitute 5 for b in either equation of the original system and solve for a.

$a = -4b \quad (1)$

$a = -4(5)$

$a = -20$

Check the ordered pair $(-20, 5)$.

$$\begin{array}{c|c} \multicolumn{2}{c}{a = -4b} \\ \hline (-20) & -4(5) \\ -20 & -20 \text{ TRUE} \end{array}$$

$$\begin{array}{c|c} \multicolumn{2}{c}{a + 5b = 5} \\ \hline (-20) + 5(5) & 5 \\ -20 + 25 & 5 \\ 5 & 5 \text{ TRUE} \end{array}$$

The ordered pair (-20,5) is the solution.

13. $2x + 3y = 8 \quad (1)$

$x = y - 6 \quad (2)$

Substitute $y - 6$ for x in the first equation and solve for y.

$2x + 3y = 8 \quad (1)$

$2(y - 6) + 3y = 8$

$2y - 12 + 3y = 8$

$5y - 12 = 8$

$5y = 20$

$y = 4$

Next substitute 4 for y in either equation of the original system and solve for x.

$x = y - 6 \quad (2)$

$x = 4 - 6$

$x = -2$

Check the ordered pair $(-2, 4)$.

$$\begin{array}{c|c} \multicolumn{2}{c}{2x + 3y = 8} \\ \hline 2(-2) + 3 \cdot 4 \,?\, & 8 \\ -4 + 12 & \\ 8 = & 8 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{x = y - 6} \\ \hline -2 \,?\, & 4 - 6 \\ -2 & -2 \end{array}$$

TRUE TRUE

The ordered pair $(-2, 4)$ is the solution.

15. $x = 2y + 1 \quad (1)$

$3x - 6y = 2 \quad (2)$

Substitute $2y + 1$ for x in the second equation and solve for y.

$3x - 6y = 2 \quad (2)$

$3(2y + 1) - 6y = 2$

$6y + 3 - 6y = 2$

$3 = 2$

We get a false equation, or contradiction. The system has no solution.

17. $s + t = -4 \quad (1)$

$s - t = 2 \quad (2)$

Solve the first equation for s.

$s + t = -4 \quad (1)$

$s = -t - 4 \quad (3)$

Substitute $-t - 4$ for s in the second equation and solve for t.

$s - t = 2 \quad (2)$

$(-t - 4) - t = 2$

$-2t - 4 = 2$

$-2t = 6$

$t = -3$

Next substitute –3 for t in Equation (3).

$s = -t - 4 \quad (3)$
$s = -(-3) - 4$
$s = 3 - 4$
$s = -1$
The ordered pair $(-1, -3)$ works in both equations (1) and (2). It is the solution.

19. $x - y = 5 \quad (1)$
$x + 2y = 7 \quad (2)$
Solve the first equation for *x*.
$x - y = 5 \quad (1)$
$x = y + 5 \quad (3)$
Substitute $y + 5$ for *x* in the second equation and solve for *y*.
$x + 2y = 7 \quad (2)$
$(y + 5) + 2y = 7$
$3y + 5 = 7$
$3y = 2$
$y = \frac{2}{3}$
Next substitute $\frac{2}{3}$ for *y* in Equation (3).
$x = y + 5 \quad (3)$
$x = \frac{2}{3} + \frac{15}{3}$
$x = \frac{17}{3}$
The ordered pair $\left(\frac{17}{3}, \frac{2}{3}\right)$ works in both equations (1) and (2). It is the solution.

21. $x - 2y = 7 \quad (1)$
$3x - 21 = 6y \quad (2)$
Solve the first equation for *x*.
$x - 2y = 7 \quad (1)$
$x = 2y + 7 \quad (3)$
Substitute $2y + 7$ for *x* in the second equation and solve for *y*.
$3x - 21 = 6y \quad (2)$
$3(2y + 7) - 21 = 6y$
$6y + 21 - 21 = 6y$
$6y = 6y$
We have an identity. The equations are dependent, and the solution set is infinite.
$\{(x, y) \mid x = 2y + 7\}$

23. $y = 2x + 5 \quad (1)$
$-2y = -4x - 10 \quad (2)$
Substitute $2x + 5$ for *y* in the second equation and solve for *x*.
$-2y = -4x - 10 \quad (2)$
$-2(2x + 5) = -4x - 10$
$-4x - 10 = -4x - 10$
We have an identity. The equations are dependent, and the solution set is infinite.
$\{(x, y) \mid y = 2x + 5\}$

25. $2x + 3y = -2 \quad (1)$
$2x - y = 9 \quad (2)$
Solve the second equation for *y*.
$2x - y = 9 \quad (2)$
$-y = -2x + 9$
$y = 2x - 9 \quad (3)$
Substitute $2x - 9$ for *y* in the first equation and solve for *x*.
$2x + 3y = -2 \quad (1)$
$2x + 3(2x - 9) = -2$
$2x + 6x - 27 = -2$
$8x - 27 = -2$
$8x = 25$
$x = \frac{25}{8}$
Next substitute $\frac{25}{8}$ for *x* in Equation (3).
$y = 2x - 9 \quad (3)$
$y = 2\left(\frac{25}{8}\right) - 9$
$y = \frac{25}{4} - \frac{36}{4}$
$y = \frac{-11}{4}$
The ordered pair $\left(\frac{25}{8}, \frac{-11}{4}\right)$ works in both equations (1) and (2). It is the solution.

27. $a - b = 6 \quad (1)$
$3a - 4b = 18 \quad (2)$
Solve the first equation for *a*.
$a - b = 6 \quad (1)$
$a = b + 6 \quad (3)$
Substitute $b + 6$ for *a* in the second equation and solve for *b*.

$$\begin{aligned} 3a - 4b &= 18 \quad (2) \\ 3(b+6) - 4b &= 18 \\ 3b + 18 - 4b &= 18 \\ -b + 18 &= 18 \\ -b &= 0 \\ b &= 0 \end{aligned}$$

Next substitute 0 for b in Equation (3) and solve for a.

$$\begin{aligned} a &= b + 6 \quad (3) \\ a &= 0 + 6 \\ a &= 6 \end{aligned}$$

The ordered pair (6, 0) works in both equations (1) and (2). It is the solution.

29. $s = \frac{1}{2}r \quad (1)$

$3r - 4s = 10 \quad (2)$

Substitute $\frac{1}{2}r$ for s in the second equations and solve for s.

$$\begin{aligned} 3r - 4s &= 10 \quad (2) \\ 3r - 4\left(\frac{1}{2}r\right) &= 10 \\ 3r - 2r &= 10 \\ r &= 10 \end{aligned}$$

Next substitute 10 for r in Equation (1).

$$\begin{aligned} s &= \frac{1}{2}r \quad (1) \\ s &= \frac{1}{2}(10) \\ s &= 5 \end{aligned}$$

The ordered pair (10, 5) works in both equations (1) and (2). It is the solution.

31. $8x + 2y = 6 \quad (1)$

$y = 3 - 4x \quad (2)$

Substitute $3 - 4x$ for y in the first equation and solve for x.

$$\begin{aligned} 8x + 2y &= 6 \quad (1) \\ 8x + 2(3 - 4x) &= 6 \\ 8x + 6 - 8x &= 6 \\ 6 &= 6 \end{aligned}$$

We have an identity. The equations are dependent, and the solution set is infinite.

$\{(x, y) \mid y = 3 - 4x\}$

33. $x - 2y = 5 \quad (1)$

$2y - 3x = 1 \quad (2)$

Solve the first equation for x.

$$\begin{aligned} x - 2y &= 5 \quad (1) \\ x &= 2y + 5 \quad (3) \end{aligned}$$

Substitute $2y + 5$ for x in the second equation and solve for y.

$$\begin{aligned} 2y - 3x &= 1 \quad (2) \\ 2y - 3(2y + 5) &= 1 \\ 2y - 6y - 15 &= 1 \\ -4y - 15 &= 1 \\ -4y &= 16 \\ y &= -4 \end{aligned}$$

Next substitute –4 for y in Equation (3).

$$\begin{aligned} x &= 2y + 5 \quad (3) \\ x &= 2(-4) + 5 \\ x &= -8 + 5 \\ x &= -3 \end{aligned}$$

The ordered pair (–3, –4) works in both equations (1) and (2). It is the solution.

35. $2x - y = 0 \quad (1)$

$2x - y = -2 \quad (2)$

$2x - y$ cannot equal 0 [Equation (1)] *and* –2 [Equation (2)]. This system has no solution.

37. **Familiarize.** Let $x =$ the lesser number and $y =$ the greater number.

Translate.

Greater Number	is	5 more than	Lesser Number
↓	↓	↓	↓
y	$=$	$5 \quad +$	x

The sum	is	83
↓	↓	↓
$x + y$	$=$	83

Carry out. Solve the system of equations.

$$\begin{aligned} y &= 5 + x \quad (1) \\ x + y &= 83 \quad (2) \end{aligned}$$

Substitute $5 + x$ for y in Equation (2).

$$\begin{aligned} x + y &= 83 \quad (2) \\ x + (5 + x) &= 83 \\ 2x + 5 &= 83 \\ 2x &= 78 \\ x &= 39 \end{aligned}$$

Substitute 39 for x in Equation (1).

$y = 5 + x \quad (1)$

$y = 5 + 39$

$y = 44.$

Check. 44 is 5 more than 39, and $44 + 39 = 83.$ The numbers check.

State. The two numbers are 39 and 44.

39. Familiarize. Let $x =$ one number and $y =$ the second number.

Translate.

The sum is $93 \rightarrow x + y = 93$

The difference is $9 \rightarrow x - y = 9$

Carry out. Solve the system of equations.

$x + y = 93 \quad (1)$

$x - y = 9 \quad (2)$

Solve the second equation for x.

$x - y = 9 \quad (2)$

$x = y + 9 \quad (3)$

Substitute $y + 9$ for x in Equation (1).

$$\begin{aligned} x + y &= 93 \quad (1) \\ y + 9 + y &= 93 \\ 2y + 9 &= 93 \\ 2y &= 84 \\ y &= 42 \end{aligned}$$

Substitute 42 for y in Equation (3).

$x = y + 9$

$x = 42 + 9$

$x = 51$

Check. $42 + 51 = 93$ and $51 - 42 = 9.$

The numbers check.

State. The two numbers are 42 and 51.

41. Familiarize. Let $x =$ the larger number and $y =$ the smaller number.

Translate.

Difference is $16 \rightarrow x - y = 16$

3	times	larger	is	7	times	smaller
↓	↓	↓	↓	↓	↓	↓
3	·	x	=	7	·	y

Carry out. Solve the system.

$x - y = 16 \quad (1)$

$3x = 7y \quad (2)$

Solve Equation (1) for x.

$x - y = 16 \quad (1)$

$x = y + 16 \quad (3)$

Substitute $y + 16$ for x in Equation (2).

$$\begin{aligned} 3x &= 7y \quad (2) \\ 3(y + 16) &= 7y \\ 3y + 48 &= 7y \\ 48 &= 4y \\ 12 &= y \end{aligned}$$

Substitute 12 for y in Equation (3).

$x = y + 16 \quad (3)$

$x = 12 + 16$

$x = 28$

Check. $28 - 12 = 16$ and $3 \cdot 28 = 84 = 7 \cdot 12.$

The numbers check.

State. The numbers are 12 and 28.

43. Familiarize. Two angles are supplementary if the sum of their measures is 180°. Let x and y represent the measures of the two angles.

Translate. Since the angles are supplementary, $x + y = 180.$

one angle	is	15° more than twice the other angle
↓	↓	↓
y	=	$2x + 15$

Carry out. Solve the system of equations.

$x + y = 180 \quad (1)$

$y = 2x + 15 \quad (2)$

Substitute $2x + 15$ for y in Equation (1).

$$\begin{aligned} x + y &= 180 \\ x + (2x + 15) &= 180 \\ 3x + 15 &= 180 \\ 3x &= 165 \\ x &= 55 \end{aligned}$$

Substitute 55 for x in Equation (2).

$y = 2x + 15 \quad (2)$

$y = 2 \cdot 55 + 15$

$y = 110 + 15$

$y = 125$

Check.

$55 + 125 = 180$ and $2 \cdot 55 + 15 = 110 + 15 = 125.$

The numbers check.

State. The angles have measures of 55° and 125°.

45. Familiarize. Two angles are complementary if the sum of their measures is 90°. Let x and y represent the measures of the two angles.

Translate. Since the angles are complementary $x + y = 90$

Their difference is $18°$

$$\underbrace{x - y}_{\text{Their difference}} \underbrace{=}_{\text{is}} \underbrace{18}_{18°}$$

Carry out. Solve the system of equations.

$x + y = 90 \quad (1)$

$x - y = 18 \quad (2)$

Solve equation (2) for x.

$x - y = 18 \quad (2)$

$x = y + 18 \quad (3)$

Substitute $y + 18$ for x in Equation (1).

$$\begin{aligned} x + y &= 90 \quad (1) \\ (y + 18) + y &= 90 \\ 2y + 18 &= 90 \\ 2y &= 72 \\ y &= 36 \end{aligned}$$

Substitute 36 for y in Equation (3).

$x = y + 18$

$x = 36 + 18$

$x = 54$

Check. $54 + 36 = 90$ and $54 - 36 = 18$.

The numbers check.

State. The angles have measures of 36° and 54°.

47. Familiarize. $p = 2l + 2w$, where l = the length of the poster, in inches and w = the width, in inches.

Translate. Perimeter is 100

$$\underbrace{2l + 2w}_{\text{Perimeter}} \underbrace{=}_{\text{is}} \underbrace{100}_{100}$$

Length is 6 inches more than the width

$$\underbrace{l}_{\text{Length}} \underbrace{=}_{\text{is}} \underbrace{w + 6}_{\text{6 inches more than the width}}$$

Carry out. Solve the system of equations.

$2l + 2w = 100 \quad (1)$

$l = w + 6 \quad (2)$

Substitute $w + 6$ for l in Equation (1).

$$\begin{aligned} 2l + 2w &= 100 \quad (1) \\ 2(w + 6) + 2w &= 100 \\ 2w + 12 + 2w &= 100 \\ 4w + 12 &= 100 \\ 4w &= 88 \\ w &= 22 \end{aligned}$$

Substitute 22 for w in Equation (2) to find l.

$l = w + 6 \quad (2)$

$l = 22 + 6$

$l = 28$

Check. $2 \cdot 28 + 2 \cdot 22 = 56 + 44$

$= 100$

and $28 = 22 + 6$

The numbers check.

State. The length is 28 in., and the width is 22 in.

49. Familiarize. Perimeter $2l + 2w$, where l = the length and w = the width, in miles.

Translate. Perimeter is 1300.

$$\underbrace{2l + 2w}_{\text{Perimeter}} \underbrace{=}_{\text{is}} \underbrace{1300}_{1300}$$

Width is 110 less than the length

$$\underbrace{w}_{\text{Width}} \underbrace{=}_{\text{is}} \underbrace{l - 110}_{\text{110 less than the length}}$$

Carry out. Solve the system of equations.

$2l + 2w = 1300 \quad (1)$

$w = l - 110 \quad (2)$

Substitute $l - 110$ for w in Equation (1).

$$\begin{aligned} 2l + 2w &= 1300 \quad (1) \\ 2l + 2\,(l - 110) &= 1300 \\ 2l + 2l - 220 &= 1300 \\ 4l &= 1520 \\ l &= 380 \end{aligned}$$

Substitute 380 for l in Equation (2)

$w = l - 110 \quad (2)$

$w = 380 - 110$

$w = 270$

Check.

$2(380) + 2(270) = 760 + 540$

$= 1300$ and $380 - 110 = 270$.

The numbers check.

State. Colorado is roughly 380 mi long and 270 mi wide.

51. Familiarize. $P = 2l + 2w$, l and w in yards.
Translate. Perimeter is $280 \rightarrow 2l + 2w = 280$
width is 5 more than half the length

$$\underset{w}{\downarrow} \quad \underset{=}{\downarrow} \quad \underset{\frac{1}{2}l+5}{\downarrow}$$

Carry out. Solve the system of equations.

$$2l + 2w = 280 \quad (1)$$
$$w = \frac{1}{2}l + 5 \quad (2)$$

Substitute $\frac{1}{2}l + 5$ for w in Equation (1).

$$2l + 2w = 280 \quad (1)$$
$$2l + 2\left(\frac{1}{2}l + 5\right) = 280$$
$$2l + l + 10 = 280$$
$$3l + 10 = 280$$
$$3l = 270$$
$$l = 90$$

Substitute 90 for l in Equation (2).

$$w = \frac{1}{2}l + 5 \quad (2)$$
$$w = \frac{1}{2} \cdot 90 + 5$$
$$w = 45 + 5$$
$$w = 50$$

Check.

$$2 \cdot 90 + 2 \cdot 50 = 180 + 100$$
$$= 280 \text{ and } 50 = \frac{1}{2} \cdot 90 + 5.$$

The numbers check.
State. The soccer field is 90 yd long and 50 yd wide.

53. Familiarize. Let h = height and w = width, both in feet.
Translate. Total is $25 \rightarrow h + w = 25$ and height is 4 times width $\rightarrow h = 4w$.
Carry out. Solve the system of equations.

$$h + w = 25 \quad (1)$$
$$h = 4w \quad (2)$$

Substitute $4w$ for h in Equation (1).

$$h + w = 25 \quad (1)$$
$$4w + w = 25$$
$$5w = 25$$
$$w = 5$$

Substitute 5 for w in Equation (2).

$$h = 4w \quad (2)$$
$$h = 4 \cdot 5$$
$$h = 20$$

Check. $5 + 20 = 25$ and $4 \cdot 5 = 20$.
The numbers check.
State. The height is 20 ft; and the width is 5 ft.

55. *Thinking and Writing Exercise.*

57. $2(5x+3y) - 3(5x+3y) = 10x + 6y - 15x - 9y$
$= (10-15)x + (6-9)y$
$= -5x - 3y$

We could also work this problem using the distributive law, since $(5x + 3y)$ is a common factor.

$2(5x+3y) - 3(5x+3y) = (5x+3y)(2-3)$
$= (5x+3y)(-1)$
$= -5x - 3y$

59. $4(5x+6y) - 5(4x+7y)$
$= 20x + 24y - 20x - 35y$
$= (20-20)x + (24-35)y$
$= -11y$

61. $2(5x-3y) - 5(2x+y)$
$= 10x - 6y - 10x - 5y$
$= (10-10)x + (-6-5)y$
$= -11y$

63. *Thinking and Writing Exercise.*

65. $\frac{1}{6}(a+b) = 1 \quad (1)$

$\frac{1}{4}(a-b) = 2 \quad (2)$

Multiply Equation (1) by 6 and Equation (2) by 4 to obtain integer coefficients.

$$6 \cdot \frac{1}{6}(a+b) = 6 \cdot 1$$
$$a + b = 6 \quad (3)$$
$$4 \cdot \frac{1}{4}(a-b) = 4 \cdot 2$$
$$a - b = 8 \quad (4)$$

Solve Equation (4) for a.

$$a - b = 8 \quad (4)$$
$$a = b + 8 \quad (5)$$

Substitute $b + 4$ for a in Equation (3).

$a+b=6 \quad (3)$

$b+8+b=6$

$2b+8=6$

$2b=-2$

$b=-1$

Substitute –1 for b in Equation (5).

$a=b+8 \quad (5)$

$a=-1+8$

$a=7$

The ordered pair (7, –1) checks in both equations. It is the solution.

67. $y+5.97=2.35x \quad (1)$

$2.14y-x=4.88 \quad (2)$

Solve Equation (1) for y.

$y+5.97=2.35x \quad (1)$

$y=2.35x-5.97 \quad (3)$

Substitute $2.35x-5.97$ for y in Equation (2).

$2.14y-x=4.88 \quad (2)$

$2.14(2.35x-5.97)-x=4.88$

$5.029x-12.7758-x=4.88$

$4.029x-12.7758=4.88$

$4.029x=17.6558$

$x\approx 4.38$

Substitute 4.38 for x in Equation (3).

$y=2.35x-5.97 \quad (3)$

$y=2.35(4.38)-5.97$

$y=10.293-5.97$

$y\approx 4.32$

The ordered pair (4.38, 4.32) checks – remember, the answers are approximate values – in both equations. It is the solution.

69. Familiarize. Let $x=$ Trudy's age and $y=$ Dennis' age

Translate.

Trudy's is 20 years younger than Dennis

$\downarrow \quad \downarrow \quad \downarrow$

$x \quad = \quad y-20$

Trudy's age is more than half of Dennis' age

$\downarrow \quad \downarrow \quad \downarrow$

$x \quad = \quad 7+\frac{1}{2}y$

Carry out. Solve the system of equations.

$x=y-20 \quad (1)$

$x=7+\frac{1}{2}y \quad (2)$

Substitute $y-20$ for x is Equation (2).

$x=7+\frac{1}{2}y \quad (2)$

$y-20=7+\frac{1}{2}y$

$y=27+\frac{1}{2}y$

$2\cdot\frac{1}{2}y=27\cdot 2$

$y=54$

Substitute 54 for y in Equation (1).

$x=y-20 \quad (1)$

$x=54-20$

$x=34$

Check. $34=\frac{1}{2}\cdot 54+7$

$=27+7$ and $54-20=34.$

The numbers check.

State. The youngest age at which Trudy can marry Dennis is 34 yr.

71. $x+y+z=180 \quad (1)$

$x=z-70 \quad (2)$

$2y-z=0 \quad (3)$

Solve Equation (3) for y.

$2y-z=0 \quad (3)$

$2y=z$

$y=\frac{1}{2}z \quad (4)$

Substitute $\frac{1}{2}z$ for y and $z-70$ for x in Equation (1).

$x+y+z=180 \quad (1)$

$(z-70)+\frac{1}{2}z+z=180$

$2\frac{1}{2}z-70=180$

$\frac{2}{5}\cdot\frac{5}{2}z=250\cdot\frac{2}{5}$

$z=100$

Substitute 100 for z in Equation (2).

$x=z-70 \quad (2)$

$x=100-70$

$x=30$

Substitute 100 for z in Equation (4).

$$y=\frac{1}{2}z \quad (4)$$

$$y=\frac{1}{2}\cdot 100$$

$$y=50$$

The ordered triple (30, 50, 100) checks in all three equations. It is the solution.

73. *Thinking and Writing Exercise*.

Exercise Set 4.3

1. False

3. True

5.
$$\begin{array}{rl} x-y &= 6 \\ x+y &= 12 \\ \hline 2x \quad &= 18 \quad \text{Adding} \\ x &= 9 \end{array}$$

$$x+y=12 \rightarrow 9+y=12$$
$$y=3$$

Check.

$$\begin{array}{c|c} x-y & = \\ \hline 9-3 & ?\ 6 \\ 6 & 6 \end{array} \qquad \begin{array}{c|c} x+y & =12 \\ \hline 9+3 & ?\ 12 \\ 12 & 12 \end{array}$$

TRUE TRUE

Since (9, 3) checks, it is the solution.

7.
$$\begin{array}{rl} x+\ \ y &= 6 \\ -x+3y &= -2 \\ \hline 4y &= 4 \quad \text{Adding} \\ y &= 1 \end{array}$$

$$x+y=6 \rightarrow x+1=6$$
$$x=5$$

Check.

$$\begin{array}{c|c} x+y & =6 \\ \hline 5+1 & ?\ 6 \\ 6 & 6 \end{array} \qquad \begin{array}{c|c} -x+3y & =-2 \\ \hline -5+3(1) & ?\ -2 \\ -5+3 & \\ -2 & -2 \end{array}$$

TRUE TRUE

Since (5, 1) checks, it is the solution.

9.
$$\begin{array}{rl} 4x-y &= 1 \\ 3x+y &= 13 \quad \text{Adding} \\ \hline 7x &= 14 \\ x &= 2 \end{array}$$

$$3x+y=13 \rightarrow 3\cdot 2+y=13$$
$$6+y=13$$
$$y=7$$

The ordered pair (2, 7) checks in both equations; it is the solution.

11.
$$\begin{array}{rl} 5a+4b &= 7 \\ -5a+\ \ b &= 8 \\ \hline 5b &= 15 \\ b &= 3 \end{array}$$

$$5a+4b=7 \rightarrow 5a+4\cdot 3=7$$
$$5a+12=7$$
$$5a=-5$$
$$a=-1$$

The ordered pair (–1, 3) checks in both equations; it is the solution.

13.
$$\begin{array}{rl} 8x-5y &= -9 \\ 3x+5y &= -2 \\ \hline 11x &= -11 \\ x &= -1 \end{array}$$

$$3x+5y=-2 \rightarrow 3(-1)+5y=-2$$
$$5y=1$$
$$y=\frac{1}{5}$$

The ordered pair $\left(-1, \frac{1}{5}\right)$ checks in both equations; it is the solution.

15.
$$\begin{array}{rl} 3a-6b &= 8 \\ -3a+6b &= -8 \\ \hline 0 &= 0 \end{array}$$

This is an identity.

$\{(a, b)\,|\,3a-6b=8\}$

17. $-x - y = 8 \quad (1)$
$2x - y = -1 \quad (2)$
Multiply Equation (1) by –1 and add the result to Equation (2).
$$\begin{array}{rcrcr} x & + & y & = & -8 \\ 2x & - & y & = & -1 \\ \hline 3x & & & = & -9 \\ x & & & = & -3 \end{array}$$
$$x + y = -8 \rightarrow -3 + y = -8$$
$$y = -5$$
The ordered pair (–3, –5) checks in both equations; it is the solution.

19. $x + 3y = 19 \quad (1)$
$x - y = -1 \quad (2)$
Multiply Equation (2) by –1 and add the result to Equation (1).
$$\begin{array}{rcrcr} x & + & 3y & = & 19 \\ -x & + & y & = & 1 \\ \hline & & 4y & = & 20 \\ & & y & = & 5 \end{array}$$
$$x - y = -1 \rightarrow x - 5 = -1$$
$$x = 4$$
The ordered pair (4, 5) checks in both equations; it is the solution.

21. $8x - 3y = -6 \quad (1)$
$5x + 6y = 75 \quad (2)$
Multiply Equation (1) by 2, add the result to Equation (2), and solve for *x*.
$$\begin{array}{r} 16x - 6y = -12 \\ 5x + 6y = 75 \\ \hline 21x = 63 \\ x = 3 \end{array}$$
Substitute $x = 3$ into equation (1) or (2) to solve for *y*.
$$5x + 6y = 75 \quad (2)$$
$$5(3) + 6y = 75$$
$$15 + 6y = 75$$
$$6y = 60$$
$$y = 10$$
The ordered pair (3, 10) works in both equations (1) and (2). It is the solution.

23. $2w - 3z = -1 \quad (1)$
$-4w + 6z = 5 \quad (2)$
Multiply Equation (1) by 2 and add the result to Equation (2).
$$\begin{array}{rcrcr} 4w & - & 6z & = & -2 \\ -4w & + & 6z & = & 5 \\ \hline & & 0 & = & 3 \end{array}$$
This is a contradiction. The system has no solution.

25. $4a + 6b = -1 \quad (1)$
$a - 3b = 2 \quad (2)$
Multiply Equation (2) by 2, add the result to Equation (1), and solve for *a*.
$$\begin{array}{r} 4a + 6b = -1 \\ 2a - 6b = 4 \\ \hline 6a = 3 \end{array}$$
$$a = \frac{3}{6} = \frac{1}{2}$$
Substitute $a = \frac{1}{2}$ into equation (1) or (2) to solve for *b*.
$$4a + 6b = -1 \quad (1)$$
$$4\left(\frac{1}{2}\right) + 6b = -1$$
$$2 + 6b = -1$$
$$6b = -3$$
$$b = -\frac{3}{6} = -\frac{1}{2}$$
The ordered pair $\left(\frac{1}{2}, -\frac{1}{2}\right)$ works in both equations (1) and (2). It is the solution.

27. $3y = x \rightarrow x - 3y = 0 \quad (1)$
$5x + 14 = y \rightarrow 5x - y = -14 \quad (2)$
Multiply Equation (2) by –3 and add the result to Equation (1).
$$\begin{array}{rcr} x - 3y & = & 0 \\ -15x + 3y & = & 42 \\ \hline -14x & = & 42 \\ x & = & -3 \end{array}$$
$$3y = x \rightarrow 3y = -3$$
$$y = -1$$
The ordered pair (–3, –1) checks in both of the original equations; it is the solution.

29. $4x-10y=13 \quad (1)$

$-2x+5y = 8 \quad (2)$

Multiply Equation (2) by 2 and add the result to Equation (1).

$$\begin{aligned} 4x-10y &= 13 \\ -4x+10y &= 16 \\ \hline 0 &= 29 \end{aligned}$$

This is a contradiction. The system has no solution.

31. $8n+6-3m=0 \rightarrow -3m+8n = -6 \quad (1)$

$32=m-n \rightarrow m-n = 32 \quad (2)$

Multiply Equation (2) by 3 and add the result to Equation (1)

$$\begin{aligned} -3m+8n &= -6 \\ 3m-3n &= 96 \\ \hline 5n &= 90 \\ n &= 18 \end{aligned}$$

$$\begin{aligned} m-n=32 \rightarrow m-18 &= 32 \\ m &= 50 \end{aligned}$$

The ordered pair (50, 18) checks in both of the original equations; it is the solution.

33. $3x+5y=4 \quad (1)$

$-2x+3y=10 \quad (2)$

Multiply Equation (1) by 2, Equation (2) by 3, and add the resulting equations.

$$\begin{aligned} 6x+10y &= 8 \\ -6x+9y &= 30 \\ \hline 19y &= 38 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} 3x+5y=4 \rightarrow 3x+5\cdot 2 &= 4 \\ 3x+10 &= 4 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

The ordered pair (–2, 2) checks in both equations, it is the solution.

35. $0.06x+0.05y=0.07 \quad (1)$

$0.4x- 0.3y=1.1 \quad (2)$

Multiply Equation (1) by 6 and add the result to Equation (2)

$$\begin{aligned} 0.36x+0.30y &= 0.42 \\ 0.40x-0.30y &= 1.10 \\ \hline 0.76x &= 1.52 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} 0.4x-0.3y=1.1 \rightarrow 0.4\cdot 2-0.3y &= 1.1 \\ 0.8-0.3y &= 1.1 \\ -0.3y &= 0.3 \\ y &= -1 \end{aligned}$$

The ordered pair (2, –1) checks in both equations; it is the solution.

37. $x+\frac{9}{2}y=\frac{15}{4} \quad (1)$

$\frac{9}{10}x-y=\frac{9}{20} \quad (2)$

Multiply Equation (1) by $\frac{2}{9}$ and add the result to Equation (2)

$$\begin{aligned} \frac{2}{9}x+y &= \frac{5}{6} \\ \frac{9}{10}x-y &= \frac{9}{20} \\ \hline \left(\frac{2}{9}+\frac{9}{10}\right)x &= \frac{5}{6}+\frac{9}{20} \\ \frac{20+81}{90}x &= \frac{50+27}{60} \\ \frac{90}{101}\cdot\frac{101}{90}x &= \frac{77}{60}\cdot\frac{90}{101} \\ x &= \frac{77\cdot \cancel{90}\,3}{\underset{2}{\cancel{60}}\cdot 101} \\ x &= \frac{231}{202} \end{aligned}$$

$$\frac{9}{10}x-y=\frac{9}{20} \rightarrow \frac{9}{10}\cdot\frac{231}{202}-y=\frac{9}{20}$$

$$\begin{aligned} \frac{2079}{2020}-y &= \frac{9}{20} \\ -y &= \frac{9}{20}-\frac{2079}{2020} \\ -y &= \frac{909-2079}{2020} \\ -y &= \frac{-1170}{2020} \\ y &= \frac{117}{202} \end{aligned}$$

The ordered pair $\left(\frac{231}{202}, \frac{117}{202}\right)$ checks in both equations; it is the solution.

39. **Familiarize.** Let x = the number of miles, and c = cost, in dollars.
Translate.

$$\underbrace{\text{Cost of cargo van}}\ \text{is}\ \underbrace{\$27}\ \text{plus}\ \underbrace{22¢\cdot\text{miles}}$$
$$\downarrow \qquad \downarrow\ \ \downarrow \quad \downarrow \qquad \downarrow$$
$$c_{\text{van}} = 27 + 0.22x$$

$$\underbrace{\text{Cost of pickup}}\ \text{is}\ \underbrace{\$29}\ \text{plus}\ \underbrace{17¢\cdot\text{miles}}$$
$$\downarrow \qquad \downarrow\ \ \downarrow \quad \downarrow \qquad \downarrow$$
$$c_{\text{truck}} = 29 + 0.17x$$

Carry out. Solve the system.
$$c_{\text{van}} = 27 + 0.22x \quad (1)$$
$$c_{\text{truck}} = 29 + 0.17x \quad (2)$$
Set $c_{\text{van}} = c_{\text{truck}}$ and solve for x.
$$27 + 0.22x = 29 + 0.17x$$
$$(0.22 - 0.17)x = 29 - 27$$
$$.05x = 2$$
$$x = 40$$
Check. The cost of renting the van and driving 40 miles is:
$$27 + 0.22(40) = 27 + 8.8 = \$35.80$$
The cost of renting the truck and driving 40 miles is:
$$29 + 0.17(40) = 29 + 6.8 = \$35.80$$
The costs are equal. The answer checks.
State. The cost will be the same for 40 mi.

41. **Familiarize.** Let x and y equal the measures of the two angles, in degrees. Two angles are complementary if the sum of their measures is 90°.
Translate. Two angles are complementary:
$$x + y = 90° \quad (1)$$
Their difference is 38°:
$$x - y = 38 \quad (2)$$
Carry out. Solve the system.
Add Equation (1) to Equation (2) and solve for x.
$$\begin{aligned} x + y &= 90 \\ x - y &= 38 \\ \hline 2x &= 128 \\ x &= 64 \end{aligned}$$
Then use equation (1) or (2) to find y.
$$x + y = 90 \quad (1)$$
$$64 + y = 90$$
$$y = 26$$
Check. $64 + 26 = 90$, and
$$64 - 26 = 38$$
The numbers check
State. The two angles measure 26° and 64°.

43. **Familiarize.** Let x = the number of minutes and c = the monthly cost, in dollars.
Translate.

$$\underbrace{\text{Monthly PowerNet cost}}\ \text{is}\ \underbrace{\$1.99}\ \text{plus}\ \underbrace{43¢\cdot\text{minutes}}$$
$$\downarrow \qquad \downarrow\ \ \downarrow \quad \downarrow \qquad \downarrow$$
$$c_P = 1.99 + 0.43x$$

$$\underbrace{\text{Monthly AT\&T cost}}\ \text{is}\ \underbrace{\$3.99}\ \text{plus}\ \underbrace{28¢\cdot\text{minutes}}$$
$$\downarrow \qquad \downarrow\ \ \downarrow \quad \downarrow \qquad \downarrow$$
$$c_A = 3.99 + 0.28x$$

Carry out. Set $c_P = c_A$ and solve for x.
$$c_P = c_A$$
$$1.99 + 0.43x = 3.99 + 0.28x$$
$$0.43x - 0.28x = 3.99 - 1.99$$
$$0.15x = 2$$
$$x = \frac{2}{0.15} = \frac{200}{15} = 13\frac{1}{3}$$
Check. PowerNet costs:
$$1.99 + 0.43\left(13\frac{1}{3}\right) = 1\frac{99}{100} + \frac{43}{100}\left(\frac{40}{3}\right)$$
$$= \frac{199}{100} + \frac{43}{100}\left(\frac{40}{3}\right)$$
$$= \frac{199}{100} + \frac{1720}{300}$$
$$= \frac{597}{300} + \frac{1720}{300} = \frac{2317}{300} = \$7.72\overline{3}$$
AT&T costs:

$$3.99+0.28\left(13\frac{1}{3}\right)=3\frac{99}{100}+\frac{28}{100}\left(\frac{40}{3}\right)$$
$$=\frac{399}{100}+\frac{28}{100}\left(\frac{40}{3}\right)$$
$$=\frac{399}{100}+\frac{1120}{300}$$
$$=\frac{1197}{300}+\frac{1120}{300}=\frac{2317}{300}=\$7.72\overline{3}$$

The costs are equal (when rounded to the nearest penny, \$7.72). The answer checks.
State. The costs will be the same for $13\frac{1}{3}$ min $=13$ min 20 sec.

45. **Familiarize.** Let x and y equal the measures of the two angles, in degrees. Two angles are supplementary if the sum of their measures is 180°.
Translate. Supplementary $\rightarrow$ $x+y=180$
Their difference is 68° $\rightarrow$ $x-y=68$
Carry out. Solve the system.
$$x+y=180 \quad (1)$$
$$x-y=68 \quad (2)$$
Add the two equations and solve for x,
$$\begin{aligned} x+y&=180\\ x-y&=68\\ \hline 2x&=248\\ x&=124 \end{aligned}$$
Use equation (1) or (2) to find y.
$$\begin{aligned} x+y&=180 \quad (1)\\ 124+y&=180\\ y&=180-124=56 \end{aligned}$$
Check. $124+56=180$ and $124-56=68$. The numbers check
State. The measures of the two angles are 124° and 56°.

47. **Familiarize.** Let $x=$ the number of acres of Chardonnay grapes and $y=$ the number of acres of Riesling grapes.
Translate. Total acres equal $820 \rightarrow x+y=820$

140 more than	Riesling acres	is	Chardonnay acres
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$140\ +$	y	$=$	x

Carry out. Solve the system.
$$\begin{aligned} x+y&=820 \quad (1)\\ 140+y&=x \quad (2)\\ x-y&=140 \quad (3) \end{aligned}$$
$$\begin{aligned} x+y&=820\\ x-y&=140\\ \hline 2x&=960\\ x&=480 \end{aligned}$$
$$140+y=x \rightarrow 140+y=480$$
$$y=340$$
Check. $480+340=820$ and $140+340=480$. The numbers check.
State. South Wind Vineyards plants 340 acres of Riesling grapes and 480 acres of Chardonnay grapes.

49. **Familiarize.** $P=2l+2w$, where $l=$ the length and $w=$ the width, in feet
Translate. Perimeter is $18 \rightarrow 18=2l+2w$
Length is twice width $\rightarrow l=2w$
Carry out. Solve the system.
$$\begin{aligned} 2l+2w&=18 \quad (1)\\ l&=2w \quad (2)\\ l-2w&=0 \quad (3) \end{aligned}$$
$$\begin{aligned} 2l+2w&=18\\ l-2w&=0\\ \hline 3l&=18\\ l&=6 \end{aligned}$$
$$l=2w \rightarrow 6=2w$$
$$3=w$$
Check. $2\cdot 6+2\cdot 3=12+6=18$ and $12=2\cdot 6$. The numbers check.
State. The dimensions of the frame should be 6 ft long and 3 ft wide.

51. *Thinking and Writing Exercise.*

53. $12.2\%=\dfrac{12.2}{100}=0.122$

55. Translate:

What	percent	of	65	is	26?
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
x	$\left(\frac{1}{100}\right)$	$\cdot$	65	$=$	26

Solve:

$$x\left(\frac{1}{100}\right)\cdot 65 = 26$$

$$x = \frac{26\cdot 100}{65} = 40$$

State:
26 is 40% of 65.

57. Let x equal the unknown number of liters.
Then 12% of the number of liters is:

$$12\cdot\left(\frac{1}{100}\right)\cdot x = 0.12x$$

59. *Thinking and Writing Exercise.*

61. $$x + y = 7$$
$$3(y - x) = 9 \rightarrow y - x = 3$$
$$\begin{array}{rcl} x + y &=& 7 \\ -x + y &=& 3 \\ \hline 2y &=& 10 \\ y &=& 5 \end{array}$$
$$x + y = 7 \rightarrow x + 5 = 7$$
$$x = 2$$
The solution is (2, 5).

63. $$\begin{array}{rcl} 2(5a - 5b) = 10 \rightarrow & 10a - 10b & = 10 \\ -5(2a + 6b) = 10 \rightarrow & -10a - 30b & = 10 \\ \hline & -40b & = 20 \end{array}$$
$$b = -\frac{20}{40} = -\frac{1}{2}$$
$$10a - 10b = 10 \rightarrow a - b = 1$$
$$a - \left(-\frac{1}{2}\right) = 1$$
$$a + \frac{1}{2} = \frac{2}{2}$$
$$a = \frac{1}{2}$$
The solution is $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

65. $$y = \frac{-2}{7}x + 3$$
$$y = \frac{4}{5}x + 3$$
Since $y = y$, $\frac{-2}{7}x + 3 = \frac{4}{5}x + 3$
$$\frac{-2}{7}x = \frac{4}{5}x$$
$$x = 0$$
$$y = \frac{-2}{7}\cdot 0 + 3$$
$$y = 3$$
The solution is (0, 3).
Note: this is the y-intercept of each line.

67. $y = ax + b$ (1)
$y = x + c$ (2)
Substitute $x + c$ for y in Equation (1).
$$x + c = ax + b$$
$$c = ax - x + b$$
$$c - b = ax - x$$
$$c - b = x(a - 1)$$
$$\frac{c - b}{a - 1} = x$$
Substitute $\frac{c - b}{a - 1}$ for x in Equation (2).
$$y = \frac{c - b}{a - 1} + c \qquad \text{LCD is } (a - 1)$$
$$y = \frac{c - b}{a - 1} + \frac{c(a - 1)}{a - 1}$$
$$y = \frac{c - b + ac - c}{a - 1}$$
$$y = \frac{ac - b}{a - 1}$$
$x = \frac{c - b}{a - 1}$ and $y = \frac{ac - b}{a - 1}$.

Substitute 1 for y in Equation (1).
$$ax + by + c = 0$$
$$ax + b + c = 0$$
$$ax = -b - c$$
$x = \frac{-b - c}{a}$ and $y = 1$.

69. Familiarize. Let r = the number of rabbits and p = the number of pheasants. Each rabbit has one head and four feet; each pheasant has one head and two feet.

Translate.

Total number of heads	is	No. of rabbits	plus	No. of pheasants
↓	↓	↓	↓	↓
35	=	r	+	p

Total number of feet	is	4 times no. of rabbits	plus	2 times no. of pheasants
↓	↓	↓	↓	↓
94	=	$4r$	+	$2p$

Carry out. Solve the system.

$r+p=35 \quad (1)$

$4r+2p=94 \quad (2)$

Multiply Equation (1) by –2 and add the result to Equation (2).

$$\begin{aligned} -2r-2p &= -70 \\ 4r+2p &= 94 \\ \hline 2r &= 24 \\ r &= 12 \end{aligned}$$

$r+p=35 \to 12+p = 35$

$p = 23$

Check. Total number of heads is $12+23=46.$ Total number of feet is $4\cdot 12+2\cdot 23=48+46=94.$ The numbers check.

State. There are 12 rabbits and 23 pheasants.

71. Familiarize. Let x = the man's age and y = the daughter's age. 5 years ago, the man's age was $x-5$, and his daughter's was $y-5$.

Translate.

Total of man's age plus 5	divided by 5	is	daughter's age
↓	↓	↓	↓
$(x+5)$	$\div 5$	=	y

$x+5=5y$

$x-5y=-5$

Five years ago:

Man's age	was	8 times daughter's age
↓	↓	↓
$x-5$	=	$8(y-5)$
$x-5$	=	$8y-40$
x	=	$8y-35$

Carry out. Solve the system.

$x-5y=-5 \quad (1)$

$x=8y-35 \quad (2)$

Substitute $8y-35$ for x in Equation (1).

$$\begin{aligned} (8y-35)-5y &= -5 \\ 3y-35 &= -5 \\ 3y &= 30 \\ y &= 10 \end{aligned}$$

$x=8y-35 \to x = 8\cdot 10-35$

$x = 80-35$

$x = 45$

Check. $(45+5)\div 5=50\div 5=10$ and $(45-5)=8(10-5).$ The numbers check.

State. The man is 45 years old, and his daughter is 10 years old.

Mid-Chapter Review

Guided Solutions

1. $2x-3(x-1)=5$ Substituting $x-1$ for y

$2x-3x+3=5$ Using the distributive law

$-x+3=5$ Combining like terms

$-x=2$ Subtracting 3 from both sides

$x=-2$ Dividing both sides by -1

$y=x-1$

$y=-2-1$ Substituting

$y=-3$

The solution is $(-2,-3)$.

2. $$\begin{aligned} 2x-5y &= 1 \\ x+5y &= 8 \\ \hline 3x &= 9 \\ x &= 3 \end{aligned}$$

$$x+5y=8$$
$$3+5y=8 \quad \text{Substituting}$$
$$5y=5$$
$$y=1$$

The solution is (3, 1).

Mixed Review

1. $x = y \quad (1)$

$x+y=2 \quad (2)$

This system is easily solved using substitution because equation (1) already has y solved completely in terms of x. Substitute y for x in equation (2) and solve for y. Since $y = x$ finding the value of x is trivial.

$$x+y=2 \quad (2)$$
$$y+y=2$$
$$2y=2$$
$$y=1$$
$$x=y=1$$

The ordered pair (1, 1) is the solution to the system.

2. $x+y=10 \quad (1)$

$x-y=8 \quad (2)$

This system is easily solved using elimination because the coefficients of y in equations (1) and (2) are opposites. Add equations (1) and (2) and solve for x. Then substitute the value of x into either equation to find y.

$$x+y=10 \quad (1)$$
$$x-y=8 \quad (2)$$
$$2x \quad =18$$
$$x=9$$
$$x+y=10 \quad (1)$$
$$9+y=10$$
$$y=1$$

The ordered pair (9, 1) is the solution to the system.

3. $y=\frac{1}{2}x+1 \quad (1)$

$y=2x-5 \quad (2)$

This system is easily solved using substitution because both equations already have one variable solved completely in terms of the other. Substitute the expression for y, from equation (1) into equation (2) and solve for x. Then substitute the value of x into either equation to find y.

$$y=2x-5 \quad (2)$$
$$\frac{1}{2}x+1=2x-5$$

Multiply both sides of the equation by 2 to clear denominators to find x.

$$2\left[\frac{1}{2}x+1\right]=2[2x-5]$$
$$2\cdot\frac{1}{2}x+2\cdot 1=2\cdot 2x-2\cdot 5$$
$$x+2=4x-10$$
$$2+10=4x-x$$
$$12=3x$$
$$x=4$$
$$y=2x-5 \quad (2)$$
$$y=2(4)-5=8-5=3$$

The ordered pair (4, 3) is the solution to the system.

4. $y=2x-3 \quad (1)$

$x+y=12 \quad (2)$

This system is easily solved using substitution because equation (1) already has y solved completely in terms of x. Substitute $2x-3$ for y into equation (2) and solve for x. Then substitute the value of x into equation (1) to find y.

$$x+y=12 \quad (2)$$
$$x+(2x-3)=12$$
$$x+2x-3=12$$
$$3x-3=12$$
$$3x=15$$
$$x=5$$
$$y=2x-3 \quad (1)$$
$$=2(5)-3=10-3=7$$

The ordered pair (5, 7) is the solution to the system.

5. $x=5$

$y=10$

The system requires no work to solve. The values of x and y are already given. The ordered pair (5, 10) is the solution.

6. $3x+5y=8$ (1)

$3x-5y=4$ (2)

This system is easily solved using elimination because the coefficients of y in equations (1) and (2) are opposites. Add equations (1) and (2) and solve for x. Then substitute the value of x into either equation to find y.

$$\begin{array}{rl} 3x+5y=8 & (1) \\ 3x-5y=4 & (2) \\ \hline 6x \quad\; =12 & \\ x=2 & \end{array}$$

$$\begin{aligned} 3x+5y&=8 \quad (1)\\ 3(2)+5y&=8\\ 6+5y&=8\\ 5y&=2\\ y&=\frac{2}{5} \end{aligned}$$

The ordered pair $\left(2\,,\,\frac{2}{5}\right)$ is the solution to the system.

7. $2x-y=1$ (1)

$2y-4x=3$ (2)

This system is easily solved using substitution because equation (1) is easily solved for y in terms of x. Solve equation (1) for y an substitute the expression for y into equation (2) and solve for x. Then substitute the value of x into either equation to find y.

$$\begin{aligned} 2x-y&=1 \quad (1)\\ -y&=-2x+1\\ y&=2x-1 \end{aligned}$$

$$\begin{aligned} 2y-4x&=3 \quad (2)\\ 2(2x-1)-4x&=3\\ 4x-2-4x&=3\\ -2&=3 \end{aligned}$$

The process led to a contradiction. The system has no solution.

8. $x=2-y$ (1)

$3x+3y=6$ (2)

This system is easily solved using substitution because equation (1) already has x solved completely in terms of y. Substitute the expression for x, from equation (1) into equation (2) and solve for y. Then substitute the value of y into either equation to find x.

$$\begin{aligned} 3x+3y&=6 \quad (2)\\ 3(2-y)+3y&=6\\ 6-3y+3y&=6\\ 6&=6 \end{aligned}$$

The system has an infinite number of solutions: any solution to the first equation will also be a solution to the second. The solution set is given by the ordered pairs $\{(x, y)\,|\,x=2-y\}$, or, equivalently, $(2-y, y)$.

9. $x+2y=3$ (1)

$3x=4-y$ (2)

This system is easily solved using substitution because equation (1) is easily solved for x in terms of y. Solve equation (1) for x. Then substitute the result for x into equation (2) and solve for y. Then substitute the value of y into the new equation to find x.

$$\begin{aligned} x+2y&=3 \quad (1)\\ x&=3-2y \end{aligned}$$

$$\begin{aligned} 3x&=4-y \quad (2)\\ 3(3-2y)&=4-y\\ 9-6y&=4-y\\ 9-4&=-y+6y\\ 5&=5y\\ y&=1 \end{aligned}$$

$$\begin{aligned} x&=3-2y\\ &=3-2(1)=3-2=1 \end{aligned}$$

The ordered pair (1, 1) is the solution to the system.

10. $9x+8y=0$ (1)

$11x-7y=0$ (2)

This system is more easily solved using elimination because solving either equation for x or y would mean introducing fractions into the system. To eliminate y and find x, multiply equation (1) by 7 and equation (2) by 8 and then add the resulting equations.

$$\begin{aligned} 7[9x+8y]&=8[0]\\ 8[11x-7y]&=7[0] \end{aligned}$$

$$\begin{array}{rl} 63x+56y=0 & (3) \\ 88x-56y=0 & (4) \\ \hline 151x \quad\; =0 & \\ x=0 & \end{array}$$

$$9x+8y=0 \quad (1)$$
$$9(0)+8y=0$$
$$0+8y=0$$
$$8y=0$$
$$y=0$$

The ordered pair (0, 0) is the solution to the system.

11. $10x+20y=40 \quad (1)$
$$x-y=7 \quad (2)$$
This system is easily solved using either method if we begin by noting that every term in equation (1) is divisible by 10. We can then replace the original system with the equivalent system:
$$x+2y=4 \quad (3)$$
$$x-y=7 \quad (2)$$
where equation (3) is simply the result of dividing every term in equation (1) by 10. To solve the system using substitution, solve equation (3) for *x*, substitute the result into equation (2) to find *y*, then substitute the value of *y* into equation (3) to find *x*.
$$x+2y=4 \quad (3)$$
$$x=4-2y$$
$$x-y=7 \quad (2)$$
$$(4-2y)-y=7$$
$$4-2y-y=7$$
$$4-3y=7$$
$$-3y=7-4$$
$$-3y=3$$
$$y=-1$$
$$x=4-2y=4-2(-1)$$
$$=4-(-2)=4+2=6$$
The ordered pair (6, -1) is the solution to the system.

12. To solve this system, note that when rearranged it reads:
$$y-\frac{5}{3}x=7 \quad (1)$$
$$y-\frac{5}{3}x=-8 \quad (2)$$
It is impossible for the expression $y-\frac{5}{3}x$ to equal two different values. This system has no solution. Graphically, one can see that the lines have the same slope, but differing y-intercepts. Thus they never cross, confirming the conclusion of no solution.

13. $2x-5y=1 \quad (1)$
$$3x+2y=11 \quad (2)$$
This system is more easily solved using elimination because solving either equation for *x* or *y* would mean introducing fractions into the system. To eliminate *x* and find *y*, multiply equation (1) by 3 and equation (2) by -2 and then add the resulting equations.
$$3[2x-5y]=3[1] \quad (3)$$
$$-2[3x+2y]=-2[11] \quad (4)$$
$$6x-15y=3 \quad (3)$$
$$\underline{-6x-4y=-22} \quad (4)$$
$$-19y=-19$$
$$y=1$$
$$3x+2y=11 \quad (2)$$
$$3x+2(1)=11$$
$$3x+2=11$$
$$3x=9$$
$$x=3$$
The ordered pair (3, 1) is the solution to the system.

14. Begin by clearing denominators in both equations to create an equivalent system of equations: multiply equation (1) by 6, the LCM of 2, 3, and 3, and multiply equation (2) by 20, the LCM of 5, 2, and 4.
$$6\left[\frac{x}{2}+\frac{y}{3}\right]=6\left[\frac{2}{3}\right] \quad (3)$$
$$20\left[\frac{x}{5}+\frac{5y}{2}\right]=20\left[\frac{1}{4}\right] \quad (4)$$
$$6\cdot\frac{x}{2}+6\cdot\frac{y}{3}=6\cdot\frac{2}{3} \quad (3)$$
$$20\cdot\frac{x}{5}+20\cdot\frac{5y}{2}=20\cdot\frac{1}{4} \quad (4)$$
$$\cancel{2}\cdot 3\cdot\frac{x}{\cancel{2}}+2\cdot\cancel{3}\cdot\frac{y}{\cancel{3}}=2\cdot\cancel{3}\cdot\frac{2}{\cancel{3}} \quad (3)$$
$$2\cdot 2\cdot\cancel{5}\cdot\frac{x}{\cancel{5}}+\cancel{2}\cdot 2\cdot 5\cdot\frac{5y}{\cancel{2}}=\cancel{2}\cdot\cancel{2}\cdot 5\cdot\frac{1}{\cancel{2}\cdot\cancel{2}} \quad (4)$$

$$3\cdot x+2\cdot y=2\cdot 2 \quad (3)$$
$$2\cdot 2\cdot x+2\cdot 5\cdot 5y=5\cdot 1 \quad (4)$$
$$3x+2y=4 \quad (3)$$
$$4x+50y=5 \quad (4)$$

The resulting system can then be solved using elimination. Start by finding y by eliminating x. Multiply equation (3) by 4 and equation (4) by -3, then add the results and solve for y.

$$4[3x+2y]=4[4]$$
$$-3[4x+50y]=-3[5]$$
$$\begin{array}{r} 12x+8y=16 \\ -12x-150y=-15 \\ \hline -142y=1 \end{array}$$
$$y=-\frac{1}{142}$$

Since this value is a little unwieldy to substitute back in to find x, one can solve for x by eliminating y if preferred. Start by multiplying equation (3) by -25 and go from there.

$$-25[3x+2y]=-25[4]$$
$$4x+50y=5$$
$$\begin{array}{r} -75x-50y=-100 \\ 4x+50y=5 \\ \hline -71x=-95 \end{array}$$
$$x=\frac{-95}{-71}=\frac{95}{71}$$

The ordered pair $\left(\frac{95}{71},-\frac{1}{142}\right)$ is the solution to the system.

15. One can begin by clearing decimals to create an equivalent system of equations. Since no decimal has a digit beyond the tenths place, multiply both equations by 10.

$$10[1.1x-0.3y]=10[0.8]$$
$$10[2.3x+0.3y]=10[2.6]$$
$$11x-3y=8 \quad (3)$$
$$23x+3y=26 \quad (4)$$

Since the coefficients of y are opposites, one can easily find the solution using elimination.

$$\begin{array}{rl} 11x-3y=8 & (3) \\ 23x+3y=26 & (4) \\ \hline 34x=34 & \end{array}$$
$$x=1$$
$$11x-3y=8 \quad (3)$$
$$11(1)-3y=8$$
$$11-3y=8$$
$$-3y=-3$$
$$y=1$$

The ordered pair (1, 1) is the solution to the system.

16. $y=-3$

$x=11$

The system requires no work to solve. The values of x and y are already given. The ordered pair $(11,-3)$ is the solution.

17. Noting that all the terms in equation (2) are divisible by 3 gives the new system:

$$x-2y=5 \quad (1)$$
$$\frac{1}{3}[3x-15]=\frac{1}{3}[6y] \quad (3)$$
$$x-2y=5 \quad (1)$$
$$x-5=2y \quad (3)$$

Now rearrange equation (3) and note that it is exactly the same equation as (1).

$$\begin{array}{rl} x-5=2y & (3) \\ +5=+5 & \\ \hline x=2y+5 & \\ -2y=-2y & \\ \hline x-2y=5 & \end{array}$$

Thus, the two equations are dependent and the system has an infinite number of solutions. (The same conclusion can be reached using substitution or elimination.) Therefore the set of order pairs $\{(x,y)\,|\,x-2y=5\}$ form the solution to the system.

18. $12x-19y=13 \quad (1)$

$8x+19y=7 \quad (2)$

This system is easily solved using elimination because the coefficients of y in equations (1) and (2) are opposites. Add equations (1) and (2) and solve for x. Then substitute the value of x into either equation to find y.

$$\begin{array}{rl} 12x-19y=13 & (1) \\ 8x+19y=7 & (2) \\ \hline 20x \qquad =20 & \end{array}$$
$$x=1$$

$$12x-19y=13 \quad (1)$$
$$12(1)-19y=13$$
$$12-19y=13$$
$$-19y=1$$
$$y=-\frac{1}{19}$$

The ordered pair $\left(1,-\frac{1}{19}\right)$ is the solution to the system.

19. Clear decimals and use elimination:
$$10[0.2x+0.7y]=10[1.2]$$
$$10[0.3x-0.1y]=10[2.7]$$
$$2x+7y=12 \quad (3)$$
$$3x-y=27 \quad (4)$$
$$2x+7y=12 \quad (3)$$
$$7[3x-y]=7[27] \quad (5)$$
$$2x+7y=12 \quad (3)$$
$$\underline{21x-7y=189} \quad (5)$$
$$23x=201$$
$$x=\frac{201}{23}$$
Repeat the process to find y.
$$3[2x+7y]=3[12] \quad (6)$$
$$-2[3x-y]=-2[27] \quad (7)$$
$$6x+21y=36 \quad (6)$$
$$\underline{-6x+2y=-54} \quad (7)$$
$$23y=-18$$
$$y=-\frac{18}{23}$$
The ordered pair $\left(\frac{201}{23},-\frac{18}{23}\right)$ is the solution to the system.

20. Clear denominators and use elimination.
$$12\left[\frac{1}{4}x\right]-12\left[\frac{1}{3}y\right]=0 \quad (3)$$
$$30\left[\frac{1}{2}x-\frac{1}{15}y\right]=30[2] \quad (4)$$
$$3x-4y=0 \quad (3)$$
$$15x-2y=60 \quad (4)$$
$$3x-4y=0 \quad (3)$$
$$-2[15x-2y]=-2[60] \quad (5)$$
$$3x-4y=0 \quad (3)$$
$$\underline{-30x+4y=-120} \quad (5)$$
$$-27x=-120$$
$$x=\frac{-120}{-27}=\frac{\cancel{-3}\cdot 40}{\cancel{-3}\cdot 9}=\frac{40}{9}$$
Find y using equation (1).
$$\frac{1}{4}x=\frac{1}{3}y$$
$$\frac{1}{4}\left(\frac{40}{9}\right)\cdot 3=\frac{1}{\cancel{3}}y\cdot\cancel{3}$$
$$\frac{1}{\cancel{4}}\left(\frac{\cancel{4}\cdot 10}{3\cdot\cancel{3}}\right)\cdot\cancel{3}=y$$
$$\frac{10}{3}=y$$
The ordered pair $\left(\frac{40}{9},\frac{10}{3}\right)$ is the solution to the system.

Exercise Set 4.4

1. Let p = the number of endangered plant species, and a = the number of endangered animal species. Then :
$$p+a=1010 \quad (1)$$
$$p-a=192 \quad (2)$$
The system can be solved using elimination since the coefficients of a are opposites.
$$p+a=1010$$
$$\underline{p-a=192}$$
$$2p \quad =1202$$
$$p=601$$
$$p+a=1010 \quad (1)$$
$$601+a=1010$$
$$a=1010-601=409$$
In 2009 there were 601 endangered plant species and 409 endangered animal species.

3. Let f = the number of Facebook users, in millions, and m = the number of MySpace users, in millions. Then:
$$f+m=160 \quad (1)$$
$$f=2m-8 \quad (2)$$
The system can be solved using substitution since f is solved for in equation (2).

$f + m = 160 \quad (1)$

$f = 2m - 8 \quad (2)$

$(2m-8) + m = 160$

$2m - 8 + m = 160$

$3m - 8 = 160$

$3m = 160 + 8 = 168$

$m = 56$

$f = 2m - 8 \quad (2)$

$f = 2(56) - 8 = 112 - 8 = 104$

In 2009 there were 104 million Facebook users and 56 million MySpace users.

5. Let x = the measure of one angle, in degrees, and y = the measure of the second angle, in degrees. Then:

$x + y = 180 \quad (1)$

$x = 2y - 3 \quad (2)$

The system can be solved using substitution since x is solved for in equation (2).

$x + y = 180 \quad (1)$

$x = 2y - 3 \quad (2)$

$(2y-3) + y = 180$

$2y - 3 + y = 180$

$3y - 3 = 180$

$3y = 180 + 3 = 183$

$y = \frac{183}{3} = 61$

$x = 2y - 3 \quad (2)$

$x = 2(61) - 3 = 122 - 3 = 119$

The two angle measures are 119° and 61°.

7. Let $x =$ the number of 3-credit courses and $y =$ the number of 4-credit courses. Then:

Total number of courses is 48.

$\downarrow \quad \downarrow \quad \downarrow$

$x + y = 48$

Total number of credits is 155.

$\downarrow \quad \downarrow \quad \downarrow$

$3x + 4y = 155$

Solve the system using elimination:

$x + y = 48 \quad (1)$

$3x + 4y = 155 \quad (2)$

Multiply Equation (1) by –3 and add the result to Equation (2).

$$\begin{array}{rcr} -3x - 3y & = & -144 \\ 3x + 4y & = & 155 \\ \hline y & = & 11 \end{array}$$

$x + y = 48 \rightarrow x + 11 = 48$

$x = 37$

Check. $37 + 11 = 48$, and

$37 \cdot 3 + 11 \cdot 4 = 111 + 44 = 155.$

The numbers check.

The members of the swim team are taking 37 3-credit courses and 11 4-credit courses.

9. Let $x =$ the number of $5¢$ bottles and $y =$ the number of $10¢$ bottles. Then:

Total number of bottles is 430.

$\downarrow \quad \downarrow \quad \downarrow$

$x + y = 430$

Total value is \$26.20

$\downarrow \quad \downarrow \quad \downarrow$

$0.05x + 0.10y = \$26.20$

Solve the system using elimination:

$x + y = 430 \quad (1)$

$0.05x + 0.10y = 26.50 \quad (2)$

Multiply Equation (1) by –0.05 and add the result to Equation (2).

$$\begin{array}{rcr} -0.05x - 0.05y & = & -21.50 \\ 0.05x + 0.10y & = & 26.20 \\ \hline 0.05y & = & 4.7 \\ y & = & 94 \end{array}$$

$x + y = 430 \rightarrow x + 94 = 430$

$x = 336$

Check.

$336 + 94 = 430$ and $0.05 \cdot 336 + 0.10 \cdot 94$

$= 16.80 + 9.40 = 26.20.$

The numbers check.

The Daycare collected 336 $5¢$ bottles and 94 $10¢$ bottles.

11. Let x = the number of cars and y = the number of motorcycles. Then:

$$x + y = 5950 \quad (1)$$
$$25x + 20y = 137,625 \quad (2)$$

Since only the value of y, the number motorcycles, is asked for, eliminate x by multiplying equation (1) by –25 and add the result to equation (2).

$$-25x - 25y = -25 \cdot 5950$$
$$25x + 20y = 137,625$$
$$-5y = 137,625 - 25 \cdot 5950$$
$$= 137,325 - 148,750$$
$$= -11,125$$
$$y = \frac{-11125}{-5} = 2225$$

2225 motorcycles entered Yellowstone National Park.

13. Let x = the number of sheets of regular papers used, and y = the number of recycled papers used. Then:

$$x + y = 150 \quad (1)$$
$$1.9x + 2.4y = 341 \quad (2)$$

Note that the price per page was given in cents, and therefore equation (2) is set to 341¢.
The system can be solved using elimination by multiplying equation (1) by (-1.9) and adding the result to equation (2) to find y, then finding x.

$$-1.9[x + y] = -1.9[150]$$
$$\to -1.9x - 1.9y = -285$$
$$-1.9x - 1.9y = -285$$
$$1.9x + 2.4y = 341$$
$$0.5y = 56$$
$$y = \frac{56}{0.5} = 112$$

Now find x using equation (1).

$$x + y = 150 \quad (1)$$
$$x + 112 = 150$$
$$x = 150 - 112 = 38$$

38 regular sheets and 112 recycled paper sheets were used.

15. Let x = the number of silver beads purchased, and y = the number of gemstone beads purchased. Then:

$$x + y = 80 \quad (1)$$
$$40x + 65y = 3900 \quad (2)$$

Note that the price per bead was given in cents, and therefore equation (2) is set to 3900¢.
The system can be solved using elimination by multiplying equation (1) by (-40) and adding the result to equation (2) to find y, then finding x.

$$-40[x + y] = -40[80] \to -40x - 40y = -3200$$
$$-40x - 40y = -3200$$
$$40x + 65y = 3900$$
$$25y = 700$$
$$y = \frac{700}{25} = 28$$

Now find x using equation (1).

$$x + y = 80 \quad (1)$$
$$x + 28 = 80$$
$$x = 80 - 28 = 52$$

Alicia purchased 52 silver beads and 28 gemstone beads.

17. Let x = the number of Epson cartridges purchased, and y = the number of HP cartridges purchased. Then:

$$x + y = 50 \quad (1)$$
$$1699x + 2599y = 98,450 \quad (2)$$

Note all prices in equation (2) are shown in cents.
The system can be solved using elimination by multiplying equation (1) by (-1699) and adding the result to equation (2) to find y, then finding x.

$$-1699[x + y] = -1699[50]$$
$$\to -1699x - 1699y = -84,950$$
$$-1699x - 1699y = -84,950$$
$$1699x + 2599y = 98,450$$
$$9\cancel{0}\cancel{0}y = 13,5\cancel{0}\cancel{0}$$
$$y = \frac{135}{9} = 15$$

Now find x using equation (1).

$$x + y = 50 \quad (1)$$
$$x + 15 = 50$$
$$x = 50 - 15 = 35$$

Office Depot sold 35 Epson cartridges and 15 HP cartridges.

19. Let x = the number of pounds of Mexican coffee used in the blend, and y = the number of pounds of Peruvian used. Then:

$$x+y=28 \quad (1)$$
$$13x+11y=12\cdot 28=336 \quad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-13) and adding the result to equation (2) to find y, then finding x.

$$-13[x+y]=-13[28]\rightarrow -13x-13y=-364$$
$$\begin{array}{r} -13x-13y=-364 \\ 13x+11y=336 \\ \hline -2y=-28 \end{array}$$
$$y=\frac{-28}{-2}=14$$

Now find x using equation (1).

$$x+y=28 \quad (1)$$
$$x+14=14$$
$$x=28-14=14$$

The coffee blend should be made using 14 pounds of Mexican coffee and 14 pounds of Peruvian coffee.

21. Let x = the number of ounces of sumac in the blend, and y = the number of ounces of thyme used. Then:

$$x+y=20 \quad (1)$$
$$1.35x+1.85y=1.65\cdot 20=33 \quad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-1.35) and adding the result to equation (2) to find y, then finding x.

$$-1.35[x+y]=-1.35[20]$$
$$\rightarrow -1.35x-1.35y=-27$$
$$\begin{array}{r} -1.35x-1.35y=-27 \\ 1.35x+1.85y=33 \\ \hline 0.5y=6 \end{array}$$
$$y=\frac{6}{0.5}=\frac{60}{5}=12$$

Now find x using equation (1).

$$x+y=20 \quad (1)$$
$$x+12=20$$
$$x=20-12=8$$

The Zahtar seasoning should be made using 8 ounces of sumac and 12 ounces of thyme.

23. Let x = the number of mL of 50%-acid solution and y = the number of mL of the 80% acid solution.
Complete the table with the given information.
*Note: Completing the table is part of the exercise.

Type of Solution	50%-Acid	80%-Acid	68%-Acid Mix
Amount of Solution	x	y	200
Percent Acid	50%	80%	68%
Amount of Acid in Solution	$0.5x$	$0.8y$	136

From the table we have an equation for the total mL: $x+y=200$.

We also have an equation for the total amount of mL of acid:
$0.5x+0.8y=136$.

Solve the system.

$$x+y=200 \quad (1)$$
$$0.5x+0.8y=136 \quad (2)$$

Multiply Equation (1) by –0.5 and add the result to Equation (2).

$$\begin{array}{rcl} -0.5x-0.5y &=& 100 \\ 0.5x+0.8y &=& 136 \\ \hline 0.3y &=& 36 \\ y &=& 120 \end{array}$$
$$x+y=200\ \rightarrow\ x+120 = 200$$
$$x = 80$$

Check.
$80+120=200,$ and
$0.5(80)+0.8(120)=40+96=136.$
The numbers check.
State. Jerome should mix 80 mL of the 50%-acid solution and 120 mL of the 80%-acid solution.

25. Let x = the number of pounds of the 50% chocolate mix used in the mix, and y = the number of pounds of the 10% chocolate mix used. Then:

$$x+y=20 \quad (1)$$
$$50x+10y=25\cdot 20=500 \quad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-50) and adding the result to equation (2) to find y, then finding x.

$-50[x+y]=-50[20] \to -50x-50y=-1000$

$$\begin{array}{r} -50x-50y=-1000 \\ 50x+10y=500 \\ \hline -40y=-500 \end{array}$$

$$y=\frac{-500}{-40}=12\frac{1}{2}$$

Now find x using equation (1).

$x+y=20 \quad (1)$

$x+12\frac{1}{2}=20$

$x=20-12\frac{1}{2}=7\frac{1}{2}$

The mix should be made using 7.5 pounds of the 50% chocolate mix and 12.5 pounds of the 10% chocolate mix.

27. Let x = the amount borrowed at 6.5%, and y = the amount borrowed at 7.2%. Then:

$$\begin{aligned} x+y&=12{,}000 \quad (1) \\ 0.065x+0.072y&=811.50 \quad (2) \end{aligned}$$

Note that since the interest on the loans (the right side of equation (2)) is not represented as a percent of the total amount borrowed, the percentage rates (shown on the left side of equation (2)) need to be converted to decimals (or the interest could be multiplied by 100: 81,150).

The system can be solved using elimination by multiplying equation (1) by (-0.065) and adding the result to equation (2) to find y, then finding x.

$-0.065[x+y]=-0.065[12{,}000]$

$\to -0.065x-0.065y=-780$

$$\begin{array}{r} -0.065x-0.065y=-780 \\ 0.065x+0.072y=811.50 \\ \hline 0.007y=31.50 \end{array}$$

$$y=\frac{31.50}{0.007}=\frac{31{,}500}{7}=4500$$

Now find x using equation (1).

$x+y=12{,}000 \quad (1)$

$x+4500=12{,}000$

$x=12{,}000-4500=7500$

Asel borrowed \$7500 at 6.5% interest and \$4500 at 7.2%.

29. Let x = the number of liters of the 18% alcohol antifreeze used, and y = the number of liters of the 10% alcohol antifreeze used. Then:

$$\begin{aligned} x+y&=20 \quad (1) \\ 18x+10y&=15\cdot 20=300 \quad (2) \end{aligned}$$

The system can be solved using elimination by multiplying equation (1) by (-18) and adding the result to equation (2) to find y, then finding x.

$-18[x+y]=-18[20] \to -18x-18y=-360$

$$\begin{array}{r} -18x-18y=-360 \\ 18x+10y=300 \\ \hline -8y=-60 \end{array}$$

$$y=\frac{-60}{-8}=7\frac{1}{2}$$

Now find x using equation (1).

$x+y=20 \quad (1)$

$x+7\frac{1}{2}=20$

$x=20-7\frac{1}{2}=12\frac{1}{2}$

The mix should be made using 12.5 L of the 18% alcohol solution and 7.5 L of the 10% alcohol solution.

31. Let x = the number of gallons of the 87-octane gasoline used, and y = the number of gallons of the 95-octane gasoline used. Then:

$$\begin{aligned} x+y&=10 \quad (1) \\ 87x+95y&=93\cdot 10=930 \quad (2) \end{aligned}$$

The system can be solved using elimination by multiplying equation (1) by (-87) and adding the result to equation (2) to find y, then finding x.

$-87[x+y]=-87[10] \to -87x-87y=-870$

$$\begin{array}{r} -87x-87y=-870 \\ 87x+95y=930 \\ \hline 8y=60 \end{array}$$

$$y=\frac{60}{8}=7\frac{1}{2}$$

Now find x using equation (1).

$x+y=10 \quad (1)$

$x+7\frac{1}{2}=10$

$x=10-7\frac{1}{2}=2\frac{1}{2}$

The new 93-octane should be made using 2.5 gallons of the 87-octane gasoline and 7.5 gallons of the 95-octane gasoline.

33. Let x = the number of pounds of the whole milk (4% milk fat) used, and y = the number of pounds of the cream (30% milk fat) used. Then:

$$x+y=200 \qquad (1)$$
$$4x+30y=8\cdot 200=1600 \qquad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-4) and adding the result to equation (2) to find y, then finding x.

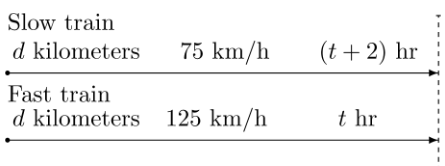
$$-4[x+y]=-4[200]\rightarrow -4x-4y=-800$$
$$-4x-4y=-800$$
$$4x+30y=1600$$
$$26y=800$$
$$y=\frac{800}{26}=30\frac{20}{26}=30\frac{10}{13}$$

Now find x using equation (1).

$$x+y=200 \qquad (1)$$
$$x+30\frac{10}{13}=200$$
$$x=200-30\frac{10}{13}=169\frac{3}{13}$$

The cream cheese milk should be made using $169\frac{3}{13}$ pounds of whole milk and $30\frac{10}{13}$ pounds of cream.

35. Familiarize. We first make a drawing.



From the drawing, we see that the distances are the same. Now complete the chart.

$d \quad = \quad r \quad \cdot \quad t$

	Distance	Rate	Time	
Slow train	d	75	$t+2$	$\rightarrow d = 75(t+2)$
Fast train	d	125	t	$\rightarrow d=125t$

Translate. Using $d=rt$ in each row of the table, we get a system of equations.

$$d=75(t+2)$$
$$d=125t$$

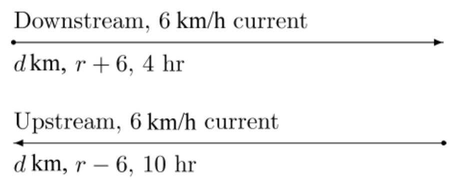
Carry out. Solve the system of equations.

$$125t=75(t+2) \quad \text{Using substitution}$$
$$125t=75t+150$$
$$50t=150$$
$$t=3$$

Check. At 125 km/h, in 3 hr the fast train will travel $125\cdot 3=375$ km. At 75 km/h, in 3 + 2, or 5 hr, the slow train will travel $75\cdot 5=375$ km. The numbers check.

State. The trains will meet 375 km from the station.

37. Familiarize. We first make a drawing. Let d = the distance and r = the speed of the canoe in still water. Then when the canoe travels downstream its speed is $r+6$, and its speed upstream is $r-6$. From the drawing, we see that the distances are the same.



Organize the information in a table.

$d \quad = \quad r \quad \cdot \quad t$

	Distance	Rate	Time
Down-stream	d	$r+6$	4
Up-stream	d	$r-6$	10

Translate. Using $d=rt$ in each row of the table, we get a system of equations.

$$d=4(r+6)$$
$$d=10(r-6)$$

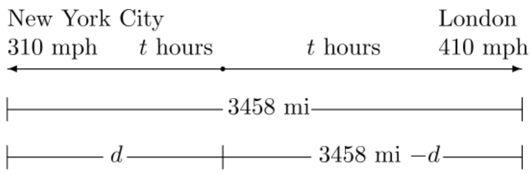
Carry out. Solve the system of equations.

$$4(r+6)=10(r-6) \quad \text{Substitution}$$
$$4r+24=10r-60$$
$$84=6r$$
$$14=r$$

Check. Going downstream, the speed of the canoe would be $r+6=14+6=20$ km/h, and in 4 hours, it would travel $4\cdot 20=80$ km. Going upstream, the speed of the canoe would be $r-6=14-6=8$ km/h, and in 10 hours it would travel $10\cdot 8=80$ km. The numbers check.

State. The speed of the canoe in still water is 14 km/h.

39. Familiarize. We make a drawing. Note that the plane's speed traveling toward London is 360 + 50, or 410 mph, and the speed traveling toward New York City is 360 – 50, or 310 mph. Also, when the plane is d mi from New York City, it is 3458 – d mi from London.



Organize this information in a table.

	Distance	Rate	Time
Toward NYC	d	310	t
Toward London	$3458-d$	410	t

Translate. Using $d = rt$ in each row of the table, we get a system of equations.

$$d = 310t \quad (1)$$
$$3458 - d = 410t \quad (2)$$

Carry out. Solve the system of equations.

$$3458 - 310t = 410t \quad \text{Using substitution}$$
$$3458 = 720t$$
$$4.8028 \approx t$$

Substitute 4.8028 for t in (1).

$$d \approx 310(4.8028) \approx 1489$$

Check. If the plane is 1489 mi from New York City, it can return to New York City, flying at 310 mph, in 1489/310 ≈ 4.8 hr. If the plane is 3458 – 1489, or 1969 mi from London, it can fly to London, traveling at 410 mph in 1969/410 ≈ 4.8 hr. Since the times are the same, the answer checks.

State. The point of no return is about 1489 mi from New York City.

41. Let x = the number of foul shots made, and y = the number of 2-point shots make. Then:

$$x + y = 64 \quad (1)$$
$$1x + 2y = 100 \quad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-1) and adding the result to equation (2) to find y, then finding x.

$$-1[x + y] = -1[64] \to -x - y = -64$$

$$-x - y = -64$$
$$\underline{1x + 2y = 100}$$
$$y = 36$$

Now find x using equation (1).

$$x + y = 64 \quad (1)$$
$$x + 36 = 64$$
$$x = 64 - 36 = 28$$

Wilt Chamberlain made 28 foul shots and 36 two-point shots.

43. Let x = the number of minutes used for landline calls, and y = the number of minutes used for wireless calls. Then:

$$x + y = 400 \quad (1)$$
$$9x + 15y + 399 = 5889$$
$$9x + 15y = 5889 - 399 = 5490 \quad (2)$$

Note that the price per minute was given in cents, and therefore equation (2) is set to the monthly charge and the bill amount were given in cents in equation (2).

The system can be solved using elimination by multiplying equation (1) by (-9) and adding the result to equation (2) to find y, then finding x.

$$-9[x + y] = -9[400] \to -9x - 9y = -3600$$

$$-9x - 9y = -3600$$
$$\underline{9x + 15y = 5490}$$
$$6y = 1890$$
$$y = \frac{1890}{6} = 315$$

Now find x using equation (1).

$$x + y = 400 \quad (1)$$
$$x + 315 = 400$$
$$x = 400 - 315 = 85$$

Kim made 85 minutes worth of landline calls and 315 minutes worth of wireless calls.

45. Familiarize. Monica tendered \$20 to pay for a purchase amounting to \$9.25, so she should receive \$10.75 in change to be paid back in quarters and fifty-cent pieces. Let x = the number of quarters and y = the number of fifty-cent pieces.

Translate. We organize the information in a table.

	Quarters	Fifty-cent pieces	Total
Number of coins	x	y	30
Value of the coin	\$0.25	\$0.50	
Total change	$0.25x$	$0.5y$	\$10.75

From the "Number of coins" line in the table, we have the equation:
$x + y = 30$.
From the "Total change" line in the table, we have the equation:
$0.25x + 0.50y = 10.75$
After clearing decimals, we have translated to a system of equations:

$$x + y = 30 \quad (1)$$
$$25x + 50y = 1075 \quad (2)$$

Carry out. We can use elimination to solve the system of equations.

$$\begin{aligned} -25x - 25y &= -750 \quad \text{Multiply (1) by } -25 \\ 25x + 50y &= 1075 \\ \hline 25y &= 325 \\ y &= 13 \end{aligned}$$

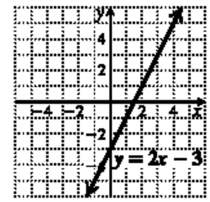
Now substitute 13 for y in Equation (1) to solve for x.

$$\begin{aligned} x + y &= 30 \\ x + 13 &= 30 \\ x &= 17 \end{aligned}$$

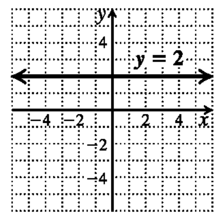
Check. There are a total of 17 + 13 = 30 coins. The total value of the coins is 17(\$0.25) + 13(\$0.50) = \$4.25 + \$6.50 = \$10.75 The answer checks.
State. Monica will receive 17 quarters and 13 fifty-cent pieces in change amounting to \$10.75.

47. *Thinking and Writing Exercise.*

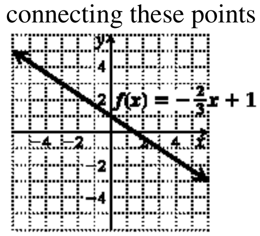
49. $y = 2x - 3$
Slope: 2; y-intercept: $(0,-3)$
We begin by plotting the y-intercept $(0,-3)$ and thinking of the slope as $\frac{2}{1}$, we move up 2 units and right 1 unit to the point $(1,-1)$. Thinking of the slope as $\frac{2}{1}$ we can begin at $(1,-1)$ and move up 2 units and right 1 unit to the point $(2,1)$. We finish by connecting these points to form a line.



51. $y = 2$
We can write this equation as $0 \cdot x + y = 2$. No matter what number we choose for x, we find that y must be 2.



53. $f(x) = -\frac{2}{3}x + 1 \to y = -\frac{2}{3}x + 1$
Slope: $-\frac{2}{3}$; y-intercept: $(0,1)$
We begin by plotting the y-intercept $(0,1)$ and thinking of the slope as $\frac{-2}{3}$, we move down 2 units and right 3 units to the point $(3,-1)$. Thinking of the slope as $\frac{2}{-3}$ we can begin at $(0,1)$ and move up 2 units and left 3 units to the point $(-3,3)$. We finish by connecting these points to form a line.



55. *Thinking and Writing Exercise.*

57. Familiarize In this problem, there is only one unknown amount, the amount of pure silver that must be added to the amount of metal in the coin.
Let x = the number of ounces of pure silver that must be added to the metal in the coin. Then the total amount of metal after the pure silver is added will be: $x + 32$ ounces.

Translate. We organize the information in a table.

	coin silver	pure silver	sterling silver
%	90%	100%	92.5%
amount	32 oz	x	$x+32$
Total silver	$0.90\cdot 32$	$1.00x$	$0.925(x+32)$

Therefore, we get:

$0.90\cdot 32+1.00x=0.925(x+32)$

Carry out. Solve the equation for x.

$$0.90\cdot 32+1.00x=0.925(x+32)$$
$$28.8+x=0.925x+29.6$$
$$(1-0.925)x=29.6-28.8$$
$$0.075x=0.8$$
$$x=\frac{0.8}{0.075}=\frac{800}{75}=10\frac{50}{75}=10\frac{2}{3}$$

Check. There are a total of 32 ounces + $10\frac{2}{3}$ ounces, or $42\frac{2}{3}$ ounces. So $0.925\left(42\frac{2}{3}\right)\approx 39.4667$ is the total amount of silver which is $0.90\cdot 32$ + $10\frac{2}{3}$ which is about 39.4667. The answer checks.

State. $10\frac{2}{3}$ ounces of pure silver must be added to the metal in the coin to get a mixture that is sterling silver, or, equivalently, 92.5% pure silver.

59. Familiarize. Let x = the amount of the original solution that remains after some of the original solution is drained and replaced with pure antifreeze. Let y = the amount of the original solution that is drained and replaced with pure antifreeze. We organize the information in a table. Keep in mind that the table contains information regarding the solution *after* some of the original solution is drained and replaced with pure antifreeze.

Translate. We organize the information in a table.

	Original solution	Pure Anti-freeze	New Mixture
Amount of solution	x	y	6.3 L
Percent of antifreeze	30%	100%	50%
Amount of antifreeze in solution	$0.3x$	$1\cdot y$, or y	$0.5(6.3)$, or 3.15

We get one equation from the "Amount of solution" row of the table:

$x+y=6.3$

The last row of the table gives us a second equation:

$0.3x+y=3.15$

After clearing the decimal we have the problem translated to a system of equations:

$$10x+10y=63 \quad (1)$$
$$30x+100y=315 \quad (2)$$

Carry out. Solve the system of equations using the elimination method.

$$\begin{aligned} -30x-\ 30y &= -189 \quad \text{Multiply (1) by } -3\\ 30x+100y &= \ \ 315\\ \hline 70y &= \ \ 126\\ y &= 1.8 \end{aligned}$$

Now substitute 1.8 for y in Equation (1) to solve for x.

$$x+y=6.3$$
$$x+1.8=6.3$$
$$x=4.5$$

Check. The total amount in the mixture is 4.5 L + 1.8 L, or 6.3 L. The amount of antifreeze in the mixture is 30%(4.5) + 100%(1.8) = 1.35 L + 1.8 L = 3.15 L. The answer checks.

State. Michelle should drain 1.8 L of radiator fluid and replace it with 1.8 L of pure antifreeze

61. Familiarize. Let x = the number of individual volumes purchased, and y = the number of 3-volume sets purchased. Then:

Translate. $x+3y=51$ (1)

$39x+88y=1641$ (2)

Carry out. y can be found by multiplying equation (1) by (-39) and adding the result to equation (2) to eliminate x.

$-39[x+3y]=-39[51]$

$\rightarrow -39x-117y=-1989$

$$\begin{array}{r} -39x-117y=-1989 \\ 39x+88y=1641 \\ \hline -29y=-348 \end{array}$$

$$y=\frac{-348}{-29}=12$$

State. 12 three-volume sets were purchased.

63. Familiarize. Let x = the number of gallons of pure brown and y = the number of gallons of neutral stain that should be added to the original 0.5 gal. Note that the total of 1 gal of stain needs to be added to bring the amount of stain up to 1.5 gal. The original 0.5 gal of stain contains 20%(0.5 gal), or 0.2(0.5 gal) = 0.1 gal of brown stain. The final solution contains 60%(1.5 gal), or 0.6(1.5 gal) = 0.9 gal and the x gal that are added.

Translate.

The amount of stain added was 1 gal.

$\downarrow$ $\downarrow$ $\downarrow$

$x+y$ $=$ 1

The amount of brown stain in the final solution is 0.9 gal.

$\downarrow$ $\downarrow$ $\downarrow$

$0.1+x$ $=$ 0.9

We have a system of equations.

$x+y=1$ (1)

$0.1+x=0.9$ (2)

Carry out. First Solve (2) for x.

$0.1+x=0.9$

$x=0.8$

Then substitute 0.8 for x in (1) and solve for y.

$0.8+y=1$

$y=0.2$

Check.

Total amount of stain: 0.5 + 0.8 + 0.2 = 1.5 gal.

Total amount of brown stain: 0.1 + 0.8 = 0.9 gal.

Total amount of neutral stain: 0.8(0.5) + 0.2 = 0.4 + 0.2 = 0.6 gal = 0.4(1.5 gal).

The answer checks.

State. 0.8 gal of pure brown and 0.2 gal of neutral stain should be added.

65. Familiarize. Let x = the number of miles driven in the city and let y = the number of miles driven on the highway.

Translate. We organize this information in a table.

	City driving	Highway driving	
No. of miles	x	y	465
MPG	18	24	
Gallons used	$\frac{x}{18}$	$\frac{y}{24}$	23

We get one equation from the "No. of miles" row in the table.

$x+y=465$

We get a second equation from the "Gallons used" row in the table.

$$\frac{x}{18}+\frac{y}{24}=23$$

We have a system of equations:

$x+y=465$ (1)

$\frac{x}{18}+\frac{y}{24}=23$ (2)

Carry out. We use substitution to solve the system of equations. We begin by solving Equation (1) for x.

$x+y=465$

$x=465-y$

We then substitute $465-y$ for x in Equation (2) and solve for y.

$$\frac{x}{18}+\frac{y}{24}=465$$

$$\frac{465-y}{18}+\frac{y}{24}=23 \quad \text{Substituting}$$

$$4(465-y)+3y=72(23) \quad \text{Mult. by 72}$$

$$1860-4y+3y=1656$$

$$1860-y=1656$$

$$204=y$$

Substituting 204 for y in (1) and solving for x, we have:

$$\begin{aligned} x+y&=465\\ x+204&=465\\ x&=261 \end{aligned}$$

Check. The total number of miles drive is 261 mi + 204 mi, or 465 mi. If the car is driven 261 miles in the city at 18 miles per gallon, the car would use 261/18 = 14.5 gal of fuel. If the car is driven 204 miles on the highway at 24 miles per gallon, it would use 8.5 gal of fuel. The total amount of fuel used would be 14.5 gal + 8.5 gal, or 23 gal. These numbers check.

State. The car was driven 261 miles in the city and 204 miles on the highway.

67. Let x = the number of 2-count pencil packs purchased, and y = the number of 12-count pencil packs purchased. Then:

$$\begin{aligned} 2x+12y&=138 \qquad (1)\\ 599x+749y&=15,726 \qquad (2) \end{aligned}$$

Note that all prices in equation (2) are given in cents.

y can be found by multiplying equation (1) by $\left(-\frac{599}{2}\right)$ and adding the result to equation (2) to eliminate x. Note that since all the coefficients in equation (1) are even, multiplying it by $\left(-\frac{599}{2}\right)$ will not introduce any fractions once each term is simplified.

$$\left(-\frac{599}{2}\right)[2x+12y]=\left(-\frac{599}{2}\right)[138]$$

$$\rightarrow -\frac{599}{\cancel{2}}\cdot\cancel{2}x-\frac{599}{\cancel{2}}\cdot\cancel{2}\cdot 6y=-\frac{599}{\cancel{2}}\cdot\cancel{2}\cdot 69$$

$$\rightarrow -599x-3594y=-41,331$$

$$\begin{array}{r} -599x-3594y=-41,331\\ 599x+749y=15726\\ \hline -2845y=-25605 \end{array}$$

$$y=\frac{-25605}{-2845}=9$$

Now find x using equation (1).

$$\begin{aligned} 2x+12y&=138 \qquad (1)\\ 2x+12(9)&=138\\ 2x+108&=138\\ 2x&=138-108=30\\ x&=\frac{30}{2}=15 \end{aligned}$$

15 two-count packs and 9 twelve-count packs were purchased.

Exercise Set 4.5

1. e

3. f

5. a

7. $2x-1=-5$

The solution is the x-coordinate of the point of intersection of the graph $f(x)=2x-1$ and $g(x)=-5$. Inspecting the graph suggests that –2 is the solution.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{2x-1=-5}\\ \hline 2(-2)-1 & -5\\ -4-1 & -5\\ \multicolumn{2}{c}{-5=-5 \text{ TRUE}} \end{array}$$

The solution is –2.

9. $2x+3=x-1$

The solution is the x-coordinate of the point of intersection of the graph $f(x)=2x+3$ and $g(x)=x-1$. Inspecting the graph suggests that –4 is the solution.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{2x+3=x-1}\\ \hline 2(-4)+3 & -4-1\\ -8+3 & -5\\ \multicolumn{2}{c}{-5=-5 \quad \text{TRUE}} \end{array}$$

The solution is –4.

11. $\frac{1}{2}x+3=x-1$

The solution is the x-coordinate of the point of intersection of the graph $f(x)=\frac{1}{2}x+3$ and $g(x)=x-1$. Inspecting the graph suggests that 8 is the solution.

Check:
$$\begin{array}{c|c} \frac{1}{2}x+3 & x-1 \\ \hline \frac{1}{2}(8)+3 & 8-1 \\ 4+3 & 7 \end{array}$$
$7 = 7$ TRUE

The solution is 8.

13. $f(x) = g(x)$
The solution is the x-coordinate of the point of intersection of the graph $f(x)$ and $g(x)$. Inspecting the graph suggests that 0 is the solution.

15. $y_1 = y_2$
The solution is the x-coordinate of the point of intersection of the graph y_1 and y_2. Inspecting the graph suggests that 5 is the solution.

17. The graph of the function crosses the x-axis at $x = -2$. -2 is zero.

19. The graph of the function does not cross the x-axis. There are no zeros.

21. The graph of the function crosses the x-axis at $x = -2$ and $x = 2$. -2 and 2 are the zeros.

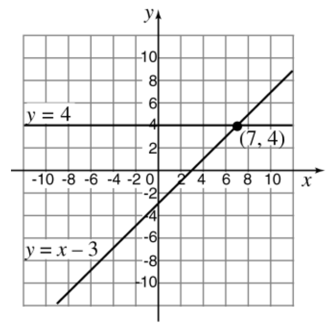
23. Determine the x-intercept of $f(x) = x - 5$. Let $f(x) = 0$ and solve for x.
$0 = x - 5$
$5 = x$ 5 is the zero.

25. Determine the x-intercept of $f(x) = \frac{1}{2}x + 10$.
Let $f(x) = 0$ and solve for x.
$$0 = \frac{1}{2}x + 10$$
$$-10 = \frac{1}{2}x$$
$$2 \cdot (-10) = 2 \cdot \frac{1}{2}x$$
$$-20 = x$$ -20 is the zero.

27. Determine the x-intercept of $f(x) = 2.7 - x$.
Let $f(x) = 0$ and solve for x.
$0 = 2.7 - x$
$x = 2.7$ 2.7 is the zero.

29. Determine the x-intercept of $f(x) = 3x + 7$.
Let $f(x) = 0$ and solve for x.
$0 = 3x + 7$
$-7 = 3x$
$-\frac{7}{3} = x$ $-\frac{7}{3}$ is the zero.

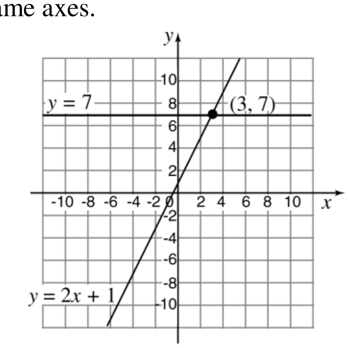
31. Solve $x - 3 = 4$.
We graph $f(x) = x - 3$ and $g(x) = 4$ on the same axes.



It appears that the lines intersect at $(7, 4)$.
$$\begin{array}{c|c} x-3 & 4 \\ \hline 7-3 & 4 \\ 4 & 4 \end{array}$$ TRUE

The solution is 7.

33. Solve $2x + 1 = 7$.
We graph $f(x) = 2x + 1$ and $g(x) = 7$ on the same axes.

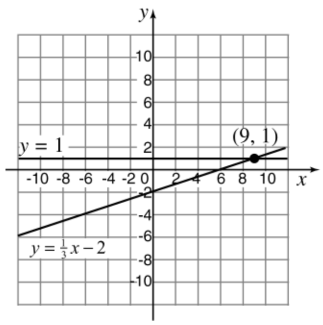


It appears that the lines intersect at $(3, 7)$.
$$\begin{array}{c|c} 2x+1 & 7 \\ \hline 2(3)+1 & 7 \\ 7 & 7 \end{array}$$ TRUE

The solution is 3.

35. Solve $\frac{1}{3}x-2=1$.

We graph $f(x)=\frac{1}{3}x-2$ and $g(x)=1$ on the same axes.



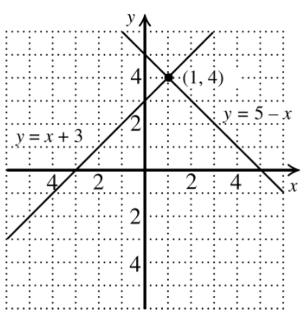
It appears that the lines intersect at $(9,1)$.

$$\begin{array}{r|l} \multicolumn{2}{c}{\frac{1}{3}x-2=1} \\ \hline \frac{1}{3}(9)-2 & 1 \\ 3-2 & 1 \quad \text{TRUE} \end{array}$$

The solution is 9.

37. Solve $x+3=5-x$.

We graph $f(x)=x+3$ and $g(x)=5-x$ on the same axes.



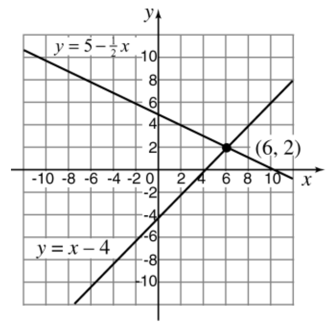
It appears that the lines intersect at $(1,4)$.

$$\begin{array}{c|c} \multicolumn{2}{c}{x+3=5-x} \\ \hline 1+3 & 5-1 \\ 4 & 4 \quad \text{TRUE} \end{array}$$

The solution is 1.

39. Solve $5-\frac{1}{2}x=x-4$.

We graph $f(x)=5-\frac{1}{2}x$ and $g(x)=x-4$ on the same axes.



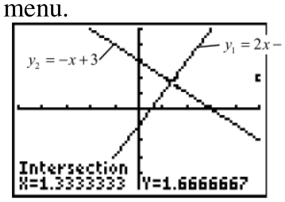
It appears that the lines intersect at $(6,2)$.

$$\begin{array}{c|c} \multicolumn{2}{c}{5-\frac{1}{2}x=x-4} \\ \hline 5-\frac{1}{2}(6) & 6-4 \\ 5-3 & 2 \\ \multicolumn{2}{c}{2=2 \qquad \text{TRUE}} \end{array}$$

The solution is 6.

41. Solve $2x-1=-x+3$.

We graph $y_1=2x-1$ and $y_2=-x+3$ on the same axes using a graphing calculator. We then use the INTERSECT option from the CALC menu.



It the lines intersect at $\left(1\frac{1}{3},1\frac{2}{3}\right)$. The solution is $1\frac{1}{3}$.

43. Familiarize. Let C = the cost to the patient and let b = the hospital bill. The problem asks that we determine how much the patient's hospital bill exceeded $5000 if the patient's cost was $6350.

Translate.

Patient cost is $5000 plus 30% of amount over $5000.

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ C & = & 5000 & + & 0.30 & \cdot & (b-5000) \end{array}$$

where $b \geq 5000$ since the bill cannot be negative.

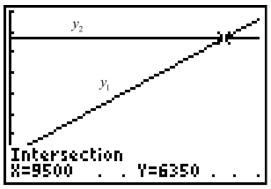
Carry out. To estimate the amount of the hospital bill that would have resulted in the stated cost to the patient, we need to estimate the solution of

$6350 = 5000 + 0.30(b - 5000)$,

We do this by graphing

$y_1 = 5000 + 0.30(x - 5000)$ and $y_2 = 6350$

on a graphing calculator, and finding the point of intersection: $(9500,\ 6350)$.



Thus, we estimate that a hospital bill of $9500 would result in a patient's after-insurance bill to be $6350.

Check. We evaluate:

$$
\begin{aligned}
C(9500) &= 5000 + 0.30(9500 - 5000)\\
&= 5000 + 0.3(4500)\\
&= 5000 + 1350\\
&= 6350
\end{aligned}
$$

Our estimate turns out to be precise.

State. The amount of the hospital bill that resulted in an after-insurance charge of $6350 was a total of $9500. So the excess of that bill over the first $5000 was $4500.

45. Familiarize. Let t = the number of months and let C = the total cost of the telephone bill. We are to determine the number of months that would result in cumulative telephone bill of $275.

Translate.

Phone bill is $100 plus $35 · months.

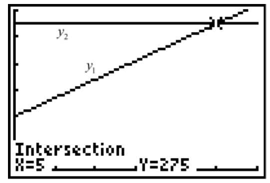
$$
\begin{array}{ccccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\
C & = & 100 & + & 35 & \cdot & t
\end{array}
$$

where $t \ge 0$ since months cannot be negative.

Carry out. To estimate the number of months resulting in a cumulative phone bill of $275, we need to estimate the solution of

$275 = 100 + 35t$,

We do this by graphing $y_1 = 100 + 35x$ and $y_2 = 275$ on a graphing calculator, and we find the point of intersection at $(5,\ 275)$.



Thus, we estimate that after 5 months, the phone bill under the described plan will be $275.

Check. We evaluate:

$$
\begin{aligned}
C(5) &= 100 + 35(5)\\
&= 100 + 175\\
&= 275
\end{aligned}
$$

Our estimate turns out to be precise.

State. The time required for the cumulative phone bill to equal $275 is 5 months.

47. Familiarize. Let x = the number of 15-minute units of time a person is parked. Let F = the parking fee. We are to determine the time a person was parked resulting in a fee of $7.50.

Translate.

Parking fee is $3.00 plus $0.50 per 15-min unit of time.

$$
\begin{array}{cccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\
F & = & 3.00 & + & 0.50 & \cdot x
\end{array}
$$

where $x \ge 0$ since time units cannot be negative.

Rewriting the equation to eliminate decimals we have:

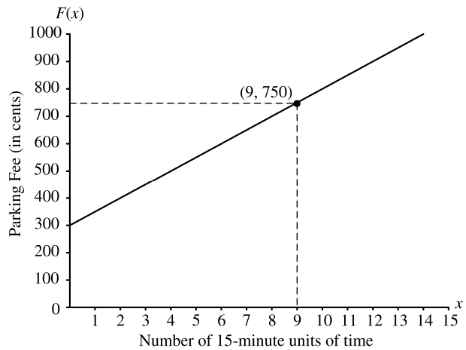
$F = 300 + 50x$

where x is in 15-minute units of time and F is parking fee in cents.

Carry out. To estimate the number of 15-minute units of time resulting in a parking fee of $7.50, or 750 cents, we need to estimate the solution of

$F(x) = 300 + 50x$,

replacing $F(x)$ with 750. We do this by graphing $F(x) = 300 + 50x$, and we find the point of intersection at $(9, 750)$.



Thus, we estimate that after 9 15-minute units of time, the parking fee will be 750 cents, or \$7.50.

Check. We evaluate:

$$F(9) = 300 + 50(9)$$
$$= 300 + 450$$
$$= 750$$

Our estimate turns out to be precise.

State. A parking fee of 750 cents, or \$7.50, would result from parking a total of 9 15-minute units of time, or 2 hr and 15 minutes.

49. Familiarize. Let p = the weight of the package, in pounds. Let C = the cost to ship the package. We are to determine the weight of a package that costs \$325 to ship.

Translate.

Shipping charge is \$130 plus \$1.30 for each pound over 100.

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ C & = & 130 & + & 1.30(p-100) \end{array}$$

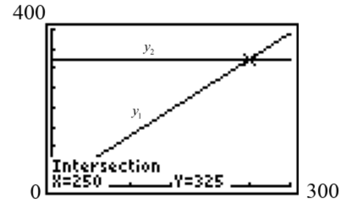
where $p \geq 100$ since weight cannot be negative.

Carry out. To estimate the weight of a package that cost \$325 to ship, we need to estimate the solution of

$325 = 130 + 1.30(p-100)$,

We do this by graphing

$y_1 = 130 + 1.30(x-100)$, $y_2 = 325$ and graphing their intersection on a graphing calculator. We find the point of intersection at $(250, 325)$



Thus, we estimate that a package weighing 250 lb would cost \$325 to ship.

Check.

$$C(250) = 130 + 1.30(250-100)$$
$$= 130 + 1.30(150)$$
$$= 130 + 195$$
$$= 325$$

Our estimate turns out to be precise.

State. The weight of a package that cost \$325 to ship is 250 lb.

51. *Thinking and Writing Exercise.*

53. $(-5)^3 = -5^3 = -125$

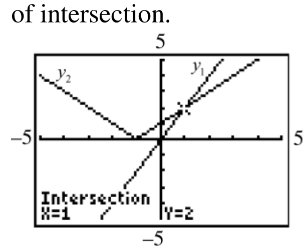
55. $-2^6 = -64$

57. $2-(3-2^2)+10 \square 2 \cdot 5$
$= 2-(3-4)+5 \cdot 5$
$= 2-(-1)+25 = 2+1+25 = 28$

59. *Thinking and Writing Exercise.*

61. $f(x) = g(x)$ is true at those points where the graphs of f and g intersect. The solutions are the first coordinate of each of the points of intersection, or –2, and 2. The solution set is $\{-2, 2\}$.

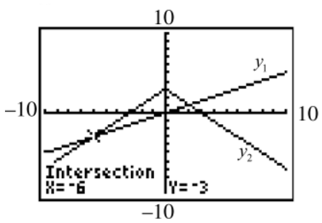
63. Solve $2x = |x+1|$. Let $y_1 = 2x$ and $y_2 = |x+1|$. Graph both in the same window on a graphing calculator and locate the point of intersection.



From the graph the solution is 1.

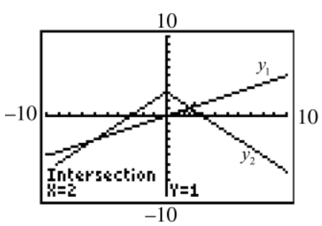
Check.
$$\begin{array}{c|c} \multicolumn{2}{c}{2x = |x+2|} \\ \hline 2\cdot 1 & |1+2| \\ 2 & 2 \end{array}$$
$2 = 2$ is TRUE

65. Solve $\frac{1}{2}x = 3 - |x|$. Let $y_1 = \frac{1}{2}x$ and $y_2 = 3 - |x|$. Graph both in the same window on a graphing calculator and locate the points of intersection.



From the graph, the first solution is –6.

Check.
$$\begin{array}{c|c} \multicolumn{2}{c}{\frac{1}{2}x = 3 - |x|} \\ \hline \frac{1}{2}(-6) & 3 - |-6| \\ -3 & 3-6 \\ -3 & -3 \end{array}$$
$-3 = -3$ is TRUE

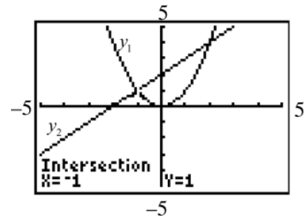


From the graph, the second solution is 2.

Check.
$$\begin{array}{c|c} \multicolumn{2}{c}{\frac{1}{2}x = 3 - |x|} \\ \hline \frac{1}{2}(2) & 3 - |2| \\ 1 & 3-2 \\ 1 & 1 \end{array}$$
$1 = 1$ is TRUE

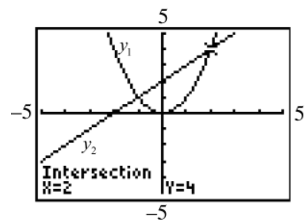
The two solutions are –6 and 2.

67. Solve $x^2 = x + 2$. Let $y_1 = x^2$ and $y_2 = x + 2$. Graph both in the same window on a graphing calculator and locate the points of intersection.



From the graph, the first solution is –1.

Check.
$$\begin{array}{c|c} \multicolumn{2}{c}{x^2 = x+2} \\ \hline (-1)^2 & -1+2 \\ 1 & 1 \end{array}$$
$1 = 1$ is TRUE

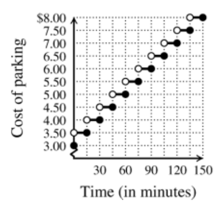


From the graph, the second solution is 2.

Check.
$$\begin{array}{c|c} \multicolumn{2}{c}{x^2 = x+2} \\ \hline 2^2 & 2+2 \\ 4 & 4 \end{array}$$
$4 = 4$ is TRUE

The two solutions are –1 and 2.

69.



Chapter 4 Study Summary

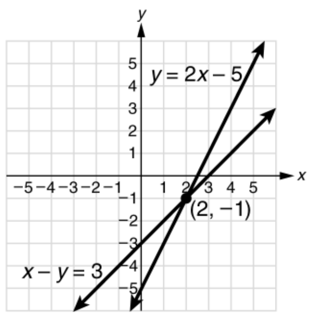
1. $x - y = 3$ (1)

   $y = 2x - 5$ (2)

   The graph of a line can be made by determining two points that lie on the line using an *x*-*y* table. For the first line, defined by $x - y = 3$, substituting $x = 0$ into the equation and solving for *y* gives the point $(0, -3)$. And substituting $y = 0$ into the equation gives the point $(3, 0)$.

   For the second line, defined by $y = 2x - 5$, substituting $x = 0$ into the equation and solving for *y* gives the

point $(0,-5)$. Substituting $x = 3$ into the equation gives the point $(3,1)$.
Using these pairs of points to graph the lines, the intersection appears to be the point $(2,-1)$.



We can verify this by testing the point in each equation. For the first equation, we have:

$$\begin{array}{r|l} x-y & 3 \\ \hline (2)-(-1) & 3 \\ 2+1 & 3 \\ 3 & 3 \end{array}$$

For the second equation, we have:

$$\begin{array}{r|l} y & 2x-5 \\ \hline -1 & 2(2)-5 \\ -1 & 4-5 \\ -1 & -1 \end{array}$$

The ordered pair (2, −1) is the solution to the system.

2. $x = 3y - 2 \quad (1)$
$y - x = 1 \quad (2)$
Equation (1) is already solved for *x*.
Substituting the result into equation (2) gives:

$$\begin{aligned} y - x &= 1 \quad (2) \\ y-(3y-2) &= 1 \\ y-3y+2 &= 1 \\ -2y+2 &= 1 \\ -2y &= 1-2 = -1 \\ y &= \frac{-1}{-2} = \frac{1}{2} \end{aligned}$$

Substituting the value of *y* back into equation (1) gives the value of *x*,

$$\begin{aligned} x &= 3y-2 \quad (1) \\ x &= 3\left(\frac{1}{2}\right)-2 \\ &= \frac{3}{2}-2 \\ &= \frac{3}{2}-\frac{4}{2} = -\frac{1}{2} \end{aligned}$$

The ordered pair $\left(-\frac{1}{2},\frac{1}{2}\right)$ is the solution to the system.

3. $2x - y = 5 \quad (1)$
$x + 3y = 1 \quad (2)$
The system can be solved using elimination by multiplying equation (2) by (−2) and adding the result to equation (1) to find *y*, then finding *x*.
$-2[x+3y] = -2[1] \rightarrow -2x-6y = -2$

$$\begin{array}{r} 2x-y=5 \\ -2x-6y=-2 \\ \hline -7y=3 \end{array}$$

$$y = \frac{3}{-7} = -\frac{3}{7}$$

Now find *x* using equation (2).

$$\begin{aligned} x+3y &= 1 \quad (2) \\ x+3\left(-\frac{3}{7}\right) &= 1 \\ x-\frac{9}{7} &= 1 \\ x &= 1+\frac{9}{7} = 1\frac{9}{7} = \frac{16}{7} \end{aligned}$$

The ordered pair $\left(\frac{16}{7},-\frac{3}{7}\right)$ is the solution to the system.

4. Let *x* = the number of boxes of Roller Grip™ pens purchased, and *y* = the number of boxes of GEL™ pens purchased. Then:

$$\begin{aligned} x+y &= 120 \quad (1) \\ 1749x+1649y &= 201{,}080 \quad (2) \end{aligned}$$

Note that all prices shown in equation (2), and the total spent, are in cents.
The system can be solved using elimination by multiplying equation (1) by (−1749) and adding the result to equation (2) to find *y*, then using equation (1) to find *x*.

$$-1749[x+y] = -1749[120]$$
$$\rightarrow -1749x - 1749y = -209,880$$
$$\begin{array}{r} -1749x - 1749y = -209,880 \\ 1749x + 1649y = 201,080 \\ \hline -100y = -8800 \end{array}$$
$$y = \frac{-8800}{-100} = 88$$

Now find x using equation (1).

$$x + y = 120 \qquad (1)$$
$$x + 88 = 120$$
$$x = 120 - 88 = 32$$

Barlow's Office Supply purchased 32 boxes of Roller Grip™ pens and 88 boxes of GEL™ pens.

5. Let x = the amount, in liters, of 40% nitric acid solution used, and y = the amount, in liters, of 15% nitric acid solution used. Then:

$$x + y = 2 \qquad (1)$$
$$40x + 15y = 25 \cdot 2 = 50 \qquad (2)$$

The system can be solved using elimination by multiplying equation (1) by (-40) and adding the result to equation (2) to find y, then finding x.

$$-40[x+y] = -40[2] \rightarrow -40x - 40y = -80$$
$$\begin{array}{r} -40x - 40y = -80 \\ 40x + 15y = 50 \\ \hline -25y = -30 \end{array}$$
$$y = \frac{-30}{-25} = 1\frac{5}{25} = 1\frac{1}{5} = 1.2$$

Now find x using equation (1).

$$x + y = 2 \qquad (1)$$
$$x + 1\frac{1}{5} = 2$$
$$x = 2 - 1\frac{1}{5} = \frac{4}{5} = 0.8$$

To produce 2 L of a 25% mixture, 0.8 L of 40% solution should be added to 1.2 L of 15% solution.

6. Let x = the speed of Ruth paddles in still water. Then, using the equation $d = rt$ for the time traveling with the current gives:

$$d_{\text{with current}} = (x+2) \cdot 1.5$$

and for the time traveling against the current gives:

$$d_{\text{against current}} = (x-2) \cdot 2.5$$

Since the distance traveled with the current is the same as the distance traveled against it, we have:

$$d_{\text{with current}} = d_{\text{against current}}$$
$$(x+2) \cdot 1.5 = (x-2) \cdot 2.5$$
$$1.5x + 3 = 2.5x - 5$$
$$5 + 3 = 2.5x - 1.5x$$
$$8 = x$$

Ruth paddles at a speed of 8 mph in still water.

7. The "zero" of a function is just the value of the x for which $f(x) = 0$. Therefore:

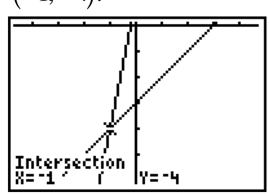
$$f(x) = 0 \rightarrow 8x - 1 = 0 \rightarrow 8x = 1 \rightarrow x = \frac{1}{8}.$$

8. Let $y_1 = x - 3$ and $y_2 = 5x + 1$

The graph of a line can be made by determining two points that lie on the line using an x-y table. For the first line, defined by $y = x - 3$, substituting $x = 0$ into the equation and solving for y gives the point $(0, -3)$. And substituting $y = 0$ into the equation gives the point $(3,\ 0)$.

For the second line, defined by $y = 5x + 1$, substituting $x = 0$ into the equation and solving for y gives the point $(0, 1)$. Substituting $x = 2$ into the equation gives the point $(2,\ 11)$

Using these pairs of points to graph the lines, the intersection appears to be the point $(-1, -4)$.



We can verify this by testing the point in each equation. For the first equation, we have:

$$\begin{array}{r|l} x - 3 & -4 \\ \hline (-1) - 3 & -4 \\ -1 - 3 & -4 \\ -4 & -4 \end{array}$$

For the second equation, we have:

$5x+1$	-4
$5(-1)+1$	-4
$-5+1$	-4
-4	-4

The value $x=-1$ is the solution to the equation.

Chapter 4 Review Exercises

1. substitution

2. elimination

3. graphical

4. dependent

5. inconsistent

6. contradiction

7. parallel

8. zero

9. alphabetical

10. x-coordinate

11. Graph each line and determine the point of intersection.

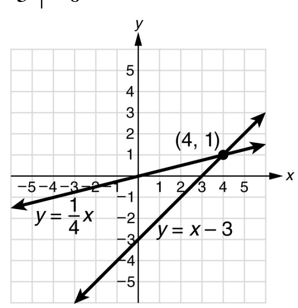
$y=x-3$

x	y
0	–3
3	0

$y=\frac{1}{4}x$

$m=\frac{1}{4}$

y-intercept is (0, 0)



The point of intersection is (4, 1).

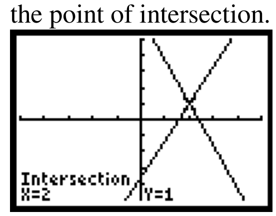
12. We first solve each equation for y.

$$16x-7y=25 \qquad 8x+3y=19$$
$$-7y=-16x+25 \qquad 3y=-8x+19$$
$$y=\frac{16}{7}x-\frac{25}{7} \qquad y=-\frac{8}{3}x+\frac{19}{3}$$

Then we enter and graph $y_1=\frac{16}{7}x-\frac{25}{7}$

and $y_2=-\frac{8}{3}x+\frac{19}{3}$

Using the INTERSECT option, we determine the point of intersection.



[–5, 5, Xscl = 1, –5, 5, Yscl = 1]

The point of intersection is (2, 1).

13. $x-y=8 \quad (1)$

$y=3x+2 \quad (2)$

Substitute $3x+2$ for y in Equation (1).

$$x-(3x+2)=8$$
$$x-3x-2=8$$
$$-2x=8+2=10$$
$$x=\frac{10}{-2}=-5$$

Substitute -5 for x in Equation (1).

$$x-y=8 \quad (1)$$
$$(-5)-y=8$$
$$-5-y=8$$
$$-y=8+5=13$$
$$y=-13$$

The ordered pair (-5 , -13) is the solution to the system.

14. $y=x+2 \quad (1)$

$y-x=8 \quad (2)$

Substitute $x+2$ for y in equation (2).

$$(x+2)-x=8$$
$$2=8$$

This is a contradiction. The system has no solution.

15. $x-3y=-2 \quad (1)$

$7y-4x=6 \quad (2)$

Solve equation (1) for x.

$$x-3y=-2$$
$$x=3y-2 \quad (3)$$

Substitute $3y-2$ for x in Equation (2).

$$7y-4(3y-2)=6$$
$$7y-12y+8=6$$
$$-5y=-2$$
$$y=\frac{2}{5}$$

Substitute $\frac{2}{5}$ for y in Equation (3).

$$x=3\left(\frac{2}{5}\right)-2$$
$$x=\frac{6}{5}-\frac{10}{5}$$
$$x=-\frac{4}{5}$$

The solution is $\left(-\frac{4}{5},\frac{2}{5}\right)$.

16. First, rearrange the second equation to align the x's and y's.

$$2x-5y=11 \quad (1)$$
$$y-2x=5$$
$$-2x+y=5 \quad (2)$$

The coefficients of x are already opposites, so add the equations to solve for y. Then solve for x.

$$\begin{array}{r} 2x-5y=11 \\ -2x+y=5 \\ \hline -4y=16 \end{array}$$
$$y=\frac{16}{-4}=-4$$

Now find x using equation (1).

$$2x-5y=11$$
$$2x-5(-4)=11$$
$$2x+20=11$$
$$2x=11-20=-9$$
$$x=\frac{-9}{2}=-\frac{9}{2}$$

The ordered pair $\left(-\frac{9}{2},-4\right)$ is the solution to the system.

17. $4x-\ 7y=18 \quad (1)$

$9x+14y=40 \quad (2)$

Multiply equation (1) by 2 and add the result to Equation (2).

$$\begin{array}{rcrcr} 8x & - & 14y & = & 36 \\ 9x & + & 14y & = & 40 \\ \hline 17x & & & = & 76 \end{array}$$
$$x = \frac{76}{17}$$

Substitute $\frac{76}{17}$ for x in Equation (1).

$$4\left(\frac{76}{17}\right)-7y=18$$
$$\frac{304}{17}-7y=\frac{306}{17}$$
$$\frac{-1}{7}\cdot(-7y)=\frac{2}{17}\cdot\frac{-1}{7}$$
$$y=\frac{-2}{119}$$

The solution is $\left(\frac{76}{17},-\frac{2}{119}\right)$.

18. $3x-5y=9 \quad (1)$

$5x-3y=-1 \quad (2)$

Multiply Equation (1) by -5, Equation (2) by 3, and add the resulting equations.

$$\begin{array}{rcrcr} -15x & + & 25y & = & -45 \\ 15x & - & 9y & = & -3 \\ \hline & & 16y & = & -48 \\ & & y & = & -3 \end{array}$$

Substitute -3 for y in Equation (2).

$$5x-3(-3)=-1$$
$$5x+9=-1$$
$$5x=-1-9=-10$$
$$x=\frac{-10}{5}=-2$$

The ordered pair $(-2, -3)$ is the solution to the system.

19. $1.5x-3=-2y \rightarrow 1.5x+2y=3 \quad (1)$

$3x+4y=6 \quad (2)$

Multiply Equation (1) by –2 and add the result to Equation (2).

$$\begin{array}{rcrcr} -3x & - & 4y & = & -6 \\ 3x & + & 4y & = & 6 \\ \hline & & 0 & = & 0 \end{array}$$

This is an identity.

$\{(x, y) \mid 3x+4y=6\}$

20. Familiarize. $P = 2l + 2w; l =$ the length and $w =$ the width, both in feet.
Translate. Perimeter is $860 \to 2l + 2w = 860$
Length is 100 more than width $\to l = w + 100$
Carryout. Solve the system.
$2l + 2w = 860$ (1)
$l = w + 100$ (2)
Substitute $w+100$ for l in Equation (1).
$$2(w+100)+2w=860$$
$$2w+200+2w=860$$
$$4w+200=860$$
$$4w=660$$
$$w=165$$
Substitute 165 for w in Equation (2).
$l = 165 + 100 = 265$
Check.: $2 \cdot 265 + 2 \cdot 165 = 530 + 330 = 860$ and $165 = 65 + 100.$ The numbers check.
State. The John Hancock building is 265 ft long and 165 ft wide.

21. Let x = the amount, in ounces, of lemon juice used, and y = the amount, in ounces, of linseed oil used. Then:
$x + y = 32$ (1)
$y = 2x$ (2)
Find x by substituting $2x$ for y in equation (1). Then use equation (2) to find y.
$$x + y = 32 \quad (1)$$
$$x + (2x) = 32$$
$$3x = 32$$
$$x = \frac{32}{3} = 10\frac{2}{3}$$
Now find y using equation (2).
$$y = 2x \quad (2)$$
$$y = 2\left(\frac{32}{3}\right) = \frac{64}{3} = 21\frac{1}{3}$$
$10\frac{2}{3}$ ounces of lemon juice and $21\frac{1}{3}$ ounces of linseed oil should be used.

22. Familiarize. Let $x =$ the number of students taking private lessons and $y =$ the number of students taking group lessons.
Translate.
Total # of students is $12 \to x + y = 12$
Total \$ is \$265 $\to \$25x + \$18y = \$265$
Carry out. Solve the system.
$x + y = 12$ (1)
$25x + 18y = 265$ (2)
Multiply equation (1) by –18 and add the result to equation (2)
$$\begin{array}{rcrcr} -18x & - & 18y & = & -216 \\ 25x & + & 18y & = & 265 \\ \hline 7x & & & = & 49 \\ x & & & = & 7 \end{array}$$
$$x + y = 12 \to 7 + y = 12$$
$$y = 5$$
Check.
$7 + 5 = 12,$ and
$25 \cdot 7 + 18 \cdot 5 = 175 + 90 = 265.$
The numbers check.
State. Jillian had 7 students who took private lessons and 5 students who took group lessons.

23. Familiarize. Two angles are supplementary if the sum of their measures is 180°. Let x and y equal the measures of the two angles in degrees.
Translate. Supplementary $\to x + y = 180$
One angle is 7° less than 10 times the other
$$\downarrow \quad \downarrow \quad \downarrow$$
$$x \quad = \quad 10y - 7$$
Carry out. Solve the system.
$x + y = 180$ (1)
$x = 10y - 7$ (2)
Substitute $10y - 7$ for x in Equation (1).
$$(10y - 7) + y = 180$$
$$11y - 7 = 180$$
$$11y = 187$$
$$y = 17$$
$$x = 10y - 7 \to x = 10(17) - 7$$
$$x = 170 - 7 = 163$$
Check. $163 + 17 = 180$ and $163 = 10(17) - 7$
The numbers check.
State. The angles measure 163° and 17°.

24. Familiarize**.** We will summarize the information given in a chart and determine the equation using $D = R \cdot T$.
Translate.

	Distance	Rate	Time
Freight Train	d	44 mph	t
Passenger Train	d	55 mph	$t - 1$

We have two equations:
$d = 44t$
$d = 55(t-1)$
Carry out. Solve the system.
$d = 44t$ (1)
$d = 55(t-1) \rightarrow d = 55t - 55$ (2)
Substitute $44t$ for d in Equation (2).
$44t = 55t - 55$
$-11t = -55$
$t = 5$
$t - 1 = 5 - 1 = 4$
Check. The freight train's distance $= 5 \cdot 44 =$ 220 m., and the passenger train's distance $= 4 \cdot 55 = 220$ m. The distances are equal; the numbers check.
State. The passenger train will travel 4 hr before it overtakes the freight train.

25. Familiarize. Let $x =$ the number of liters of the 15% juice punch and $y =$ the number of liters of the 8% juice punch.
Translate**.**
Total liters is $14 \rightarrow x + y = 14$
Total amount of juice is 10% of total liters

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 0.15x + 0.08y & = & 0.10 \cdot 14 \\ 0.15x + 0.08y & = & 1.4 \end{array}$$

Carry out. Solve the system.
$x + y = 14$ (1)
$0.15x + 0.08y = 1.4$ (2)
Multiply Equation (1) by –0.08 and add the result to Equation (2).

$$\begin{array}{rcrcr} -0.08x & - & 0.08y & = & -1.12 \\ 0.15x & + & 0.08y & = & 1.40 \\ \hline 0.07x & & & = & 0.28 \\ x & & & = & 4 \end{array}$$

$x + y = 14 \rightarrow 4 + y = 14$
$y = 10$
Check.
$4 + 10 = 14$, and
$0.15 \cdot 4 + 0.08 \cdot 10 = 0.6 + 0.8$
$= 1.4 = 0.10 \cdot 14$
The numbers check.
State. D'Andre should purchase 4 L of the 15% juice punch and 10 L of the 8% juice punch.

26. Familiarize**.** Let $x =$ the number of 1300 words/page pages and $y =$ the number of 1850 words/page pages.
Translate.
Total number of pages is $12 \rightarrow x + y = 12$
Total number of words is $18{,}350 \rightarrow 1300x + 1850y = 18{,}350$
Carry out. Solve the system.
$x + y = 12$ (1)
$1300x + 1850y = 18{,}350$ (2)
Multiply Equation (1) by –1300 and add the result to Equation (2).

$$\begin{array}{rcrcr} -1300x & - & 1300y & = & -15{,}600 \\ 1300x & + & 1850y & = & 18{,}350 \\ \hline & & 550y & = & 2750 \\ & & y & = & 5 \end{array}$$

$x + y = 12 \rightarrow x + 5 = 12$
$x = 7$
Check. $7 + 5 = 12$ and $1300 \cdot 7 + 1850 \cdot 5$
$= 9100 + 9250 = 18{,}350.$
The numbers check.
State. The typesetter had 7 of the 1300-word pages and 5 of the 1850-word pages.

27. Using the graph, we determine the x-coordinate of the point of intersection.
$x = -2$

28. Using the graph, we determine the x-coordinate of the point where $f(x)$ crosses the x-axis. $x = 1$ 1 is the zero.

29. $f(x) = 4 - 7x$
Determine the x-intercept of $f(x)$. Let $f(x) = 0$ and solve for x.

$0 = 4 - 7x$

$7x = 4$

$x = \frac{4}{7}$ $\quad \frac{4}{7}$ is the zero.

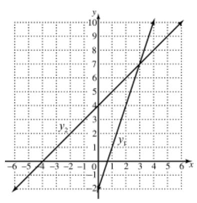
30. $3x - 2 = x + 4$

Graph: $y = 3x - 2$ and $y = x + 4$ and determine the x-coordinate of the point of intersection.

$y = 3x - 2 \qquad y = x + 4$

$m = 3 \qquad m = 1$

y-intercept is $(0, -2)$ y-intercept is $(0, 4)$

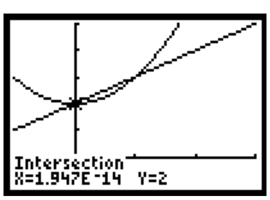


The x-coordinate of the point of intersection is 3; this is the solution.

31. *Thinking and Writing Exercise.* The solution of a system of two equations is an ordered pair that makes both equations true. The graph of an equation represents all ordered pairs that make that equation true. In order for an ordered pair to make both equations true, it must be on both graphs.

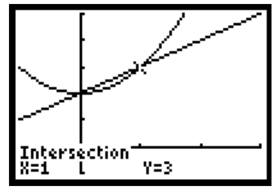
32. *Thinking and Writing Exercise.* Both methods involve finding the coordinates of the point of intersection of two graphs. The solution of a system of equations is the ordered pair at the point of intersection; the solution of an equation is the x-coordinate of the point of intersection.

33. Enter and graph $y_1 = x + 2$ and $y_2 = x^2 + 2$. Using the INTERSECT option, determine the points of intersection.



[–1, 3, Xscl=1, –1, 5, Yscl=1]

Note: 1.947E-14 is extremely close to 0.



[–1, 3, Xscl=1, –1, 5, Yscl=1]

The points of intersection are $(0, 2)$ and $(1, 3)$.

34. Let $x = 6$ and $y = 2$.

$$\begin{aligned} 2(6) - D(2) &= 6 \\ 12 - 2D &= 6 \\ -2D &= -6 \\ D &= 3 \end{aligned}$$

$$\begin{aligned} C(6) + 4(2) &= 14 \\ 6C + 8 &= 14 \\ 6C &= 6 \\ C &= 1 \end{aligned}$$

35. Let $t =$ the tens digit and $u =$ the ones digit. The sum of the digits is 6, so $t + u = 6$. The value of the original number is $10t + u$ and the value of the new number is $10u + t$, using place-value. The new number is 18 more than the original number, so

$$\begin{aligned} 10u + t &= 18 + 10t + u \\ 9u - 9t &= 18 \\ 9(u - t) &= 9 \cdot 2 \\ u - t &= 2 \end{aligned}$$

Solve the system.

$$\begin{array}{rcrcrcr} t + u = 6 \rightarrow & u & + & t & = & 6 \\ & u & - & t & = & 2 \\ \hline & 2u & & & = & 8 \\ & u & & & = & 4 \end{array}$$

$t + u = 6 \rightarrow t + 4 = 6$

$t = 2$

The original number was 24.

Note: $24 + 18 = 42$

36. Let x = the value of the computer, in dollars. 7 mos. is $\frac{7}{12}$ of a year. Since the full value of the computer was included in the prorated package, we have:

$\frac{7}{12}(42{,}000+x)=23{,}750+x$

$12\left[\frac{7}{12}(42{,}000+x)\right]=12[23{,}750+x]$

$7(42{,}000+x)=285{,}000+12x$

$294{,}000+7x=285{,}000+12x$

$294{,}000-285{,}000=12x-7x$

$9000=5x$

$x=\frac{9000}{5}=1800$

The value of the computer was \$1800.

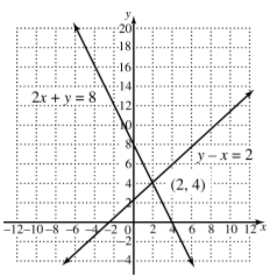
Chapter 4 Test

1. Graph each line and determine the points of intersection.

$2x+y=8$ $y-x=2$

x	y
0	8
4	0

x	y
–2	0
0	2



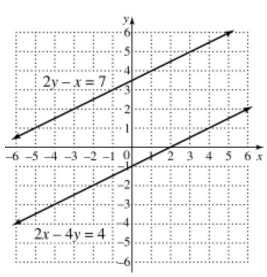
The point of intersection is (2, 4).

2. Graph each line and determine the point of intersection.

$2y-x=7$ $2x-4y=4$

x	y
0	$\frac{7}{2}$
–7	0

x	y
0	–1
2	0



The lines are parallel; they do not intersection. The system has no solution.

3. $x+3y=-8$ (1)

$4x-3y=23$ (2)

Solve Equation (2) for 3y.

$4x-3y=23$

$-3y=-4x+23$

$3y=4x-23$

Substitute $4x-23$ for 3y in Equation (1).

$x+(4x-23)=-8$

$5x-23=-8$

$5x=15$

$x=3$

$x+3y=-8\rightarrow 3+3y=-8$

$3y=-11$

$y=-\frac{11}{3}$

The solution is $\left(3,-\frac{11}{3}\right)$.

4. $2x+4y=-6$ (1)

$y=3x-9$ (2)

Substitute $3x-9$ for y in Equation (1).

$2x+4(3x-9)=-6$

$2x+12x-36=-6$

$14x=30$

$x=\frac{30}{14}=\frac{15}{7}$

$y=3x-9\rightarrow y=3\left(\frac{15}{7}\right)-9$

$y=\frac{45}{7}-\frac{63}{7}$

$y=-\frac{18}{7}$

The solution is $\left(\frac{15}{7},-\frac{18}{7}\right)$.

5. $x=5y-10$ (1)

$15y=3x+30\rightarrow 5y=x+10$ (2)

Substitute $5y-10$ for x in Equation (2).

$5y=(5y-10)+10$

$5y=5y$

This is an identity.

$\{(x,y)\mid x=5y-10\}$

6. $3x - y = 7 \quad (1)$

$x + y = 1 \quad (2)$

The coefficients of y are opposites. Add the two equations to find x. Then use equation (2) to find y.

$$\begin{array}{rcrcl} 3x & - & y & = & 7 \\ x & + & y & = & 1 \\ \hline 4x & & & = & 8 \end{array}$$

$$x = \frac{8}{4} = 2$$

Substitute 2 for x in Equation (2) to find y..

$x + y = 1$

$(2) + y = 1$

$y = 1 - 2 = -1$

The ordered pair (2, -1) is the solution to the system.

7. $4y + 2x = 18 \rightarrow 2x + 4y = 18 \quad (1)$

$3x + 6y = 26 \quad (2)$

Multiply Equation (1) by –3, Equation (2) by 2, and add the resulting equations.

$$\begin{array}{rcrcr} -6x & - & 12y & = & -54 \\ 6x & + & 12y & = & 52 \\ \hline & & 0 & = & -2 \end{array}$$

This is a contradiction. The system has no solution.

8. $4x - 6y = 3 \quad (1)$

$6x - 4y = -3 \quad (2)$

Multiply Equation (1) by 3, Equation (2) by -2, and add the resulting equations.

$$\begin{array}{rcrcr} 12x & - & 18y & = & 9 \\ -12x & + & 8y & = & 6 \\ \hline & - & 10y & = & 15 \end{array}$$

$$y = \frac{-15}{10} = \frac{-3}{2}$$

$$4x - 6y = 3 \rightarrow 4x - 6\left(\frac{-3}{2}\right) = 3$$

$$4x + 9 = 3$$

$$4x = -6$$

$$x = \frac{-6}{4} = -\frac{3}{2}$$

The solution is $\left(-\frac{3}{2}, -\frac{3}{2}\right)$.

9. $4x + 5y = 5 \quad (1)$

$6x + 7y = 7 \quad (2)$

Multiply Equation (1) by 3, Equation (2) by -2, and add the resulting equations.

$$\begin{array}{rcrcr} 12x & + & 15y & = & 15 \\ -12x & - & 14y & = & -14 \\ \hline & & y & = & 1 \end{array}$$

$4x + 5y = 5 \rightarrow 4x + 5 \cdot 1 = 5$

$4x = 0$

$x = 0$

The solution is (0, 1).

10. Familiarize. $P = 2l + 2w$, where $l =$ the length and $w =$ the width, both in feet

Translate.

Perimeter is 66 feet.

$2l + 2w = 66$

Length is 1 foot longer than three times the width $l = 3w + 1$

Carry out. Solve the system.

$2l + 2w = 66 \quad (1)$

$l = 3w + 1 \quad (2)$

Substitute $3w + 1$ for l in Equation (1).

$2(3w + 1) + 2w = 66$

$6w + 2 + 2w = 66$

$8w + 2 = 66$

$8w = 64$

$$w = \frac{64}{8} = 8$$

Use equation (2) to find l.

$l = 3w + 1 = 3(8) + 1 = 24 + 1 = 25$

Check. $2 \cdot 25 + 2 \cdot 8 = 50 + 16 = 66$

and $25 = 3 \cdot 8 + 1$.

The numbers check.

State. The dimensions of a garden are 25 ft long by 8 ft wide.

11. Familiarize. Two angles are complementary if the sum of their measures is 90°. Let x and y equal the measures of the angles, in degrees.

Translate. Complementary $\rightarrow x + y = 90$

Sum of first angle and half the second is 64°

$\downarrow \qquad \downarrow \quad \downarrow$

$x + \frac{1}{2}y \qquad = 64$

Carry out. Solve the system.

$$x + y = 90 \quad (1)$$
$$x + \frac{1}{2}y = 64 \quad (2)$$

Multiply Equation (2) by –1 and add the result to Equation (1).

$$\begin{aligned} x + y &= 90 \\ -x - \tfrac{1}{2}y &= -64 \\ 2 \cdot \tfrac{1}{2}y &= 26 \cdot 2 \\ y &= 52 \end{aligned}$$

$$x + y = 90 \rightarrow x + 52 = 90$$
$$x = 38$$

Check. $38 + 52 = 90$ and $38 + \frac{1}{2} \cdot 52$
$= 38 + 26 = 64.$
The numbers check.
State. The angles measure 38° and 52°.

12. Let x = the number of Nintendo Wii game machines sold, in millions, and y = the number of PlayStation 3 consoles sold, in millions. Then:
$$x + y = 4.84 \quad (1)$$
$$x = 3y \quad (2)$$
Substitute $3y$ for x in equation (1) to find y, then use equation (2) to find x.
$$x + y = 4.84 \quad (1)$$
$$3y + y = 4.84$$
$$4y = 4.84$$
$$y = \frac{4.84}{4} = 1.21$$
Now find x using equation (2).
$$x = 3y = 3(1.21) = 3.63$$
There were 3.63 million Nintendos Wii game machines and 1.21 million PlayStation 3 consoles sold.

13. Familiarize. Let $x =$ the number of hard-backs and $y =$ the number of paperbacks.
Translate.
Total # of books is $23 \rightarrow x + y = 23$
Total cost is $28.25.
$\rightarrow \$1.75x + \$0.75y = \$28.25$
Carry out. Solve the system.
$$x + y = 23 \quad (1)$$
$$1.75x + 0.75y = 28.25 \quad (2)$$
Multiply Equation (1) by –0.75 and add the result to Equation (2).
$$\begin{aligned} -0.75x - 0.75y &= -17.25 \\ 1.75x + 0.75y &= 28.25 \\ \hline 1.00x &= 11 \\ x &= 11 \end{aligned}$$
$$x + y = 23 \rightarrow 11 + y = 23$$
$$y = 12$$
Check. 11 + 12 = 23, and
1.75(11) + 0.75(12) = 19.25 + 9 = 28.25.
These numbers check.
State. Keith purchased 11 hardbacks and 12 paperbacks.

14. Familiarize. Let x = the number of grams of Pepperidge Farm® Goldfish and let y = the number of grams of Rold Gold® Pretzels.
Translate. Total grams is 620
$\rightarrow x + y = 620$.
Total fat calories can be found in two ways, thus giving us an equation:
40% of Goldfish grams + 9% of Pretzel grams, or 15% of total grams
$$0.4x + 0.09y = 0.15(620)$$
Carry out. Solve the system.
$$x + y = 620 \ (1)$$
$$0.4x + 0.09y = 93 \quad (2)$$
Multiply Equation (1) by –0.09, and add the result to Equation (2).
$$\begin{aligned} -0.09x - 0.09y &= -55.8 \\ 0.4x + 0.09y &= 93.0 \\ \hline 0.31x &= 37.2 \\ x &= 120 \end{aligned}$$
$$x + y = 620 \rightarrow 120 + y = 620 \rightarrow y = 500$$
Check. 120 + 500 = 620, and 0.4(120) + 0.09(500) = 48 + 45 = 93. These numbers check.
State. 120 g of Pepperidge Farm® Goldfish and 500 g of Rold Gold® Pretzels should be mixed.

15. Familiarize. Let $s =$ speed of boat, in mph.
Translate. We make a table

	Distance	Rate	Time
With current	d	$s+5$	3
Against current	d	$s-5$	5

We have two equations:
$$d = 3(s+5) \rightarrow d = 3s + 15 \quad (1)$$
$$d = 5(s-5) \rightarrow d = 5s - 25 \quad (2)$$

Carry out. Solve the system.
Substitute $3s+15$ for d in Equation (2).

$3s+15=5s-25$

$3s+40=5s$

$40=2s$

$20=s$

Check. Downstream distance is $3(20+5)=3\cdot 25=75$; upstream distance is $5(20-5)=5\cdot 15=75.$ The distances are the same; our solution checks.
State. The speed of the boat is 20 mph.

16. Familiarize. Let $x=$ the number of 25¢ and $y=$ the number of 5¢.
Translate.
Total # of coins
$13 \rightarrow x+y=13$
Total value of coins is
$\$1.25 \rightarrow 0.25x+0.05y=\$1.25$
Carry out. Solve the system.

$$\begin{aligned} x + y &= 13 \quad (1)\\ 0.25x + 0.05y &= 1.25 \quad (2)\end{aligned}$$

Multiply Equation (1) by –0.05 and add the result to Equation (2).

$$\begin{array}{r} -0.05x - 0.05y = -0.65\\ 0.25x + 0.05y = 1.25\\ \hline 0.2x = 0.6\\ x = 3\end{array}$$

$x+y=13 \rightarrow 3+y=13$

$y=10$

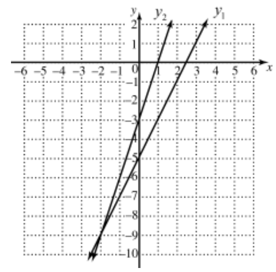
Check.
$3+10=13,$ and
$0.25\cdot 3+0.05\cdot 10=0.75+0.50=1.25.$
The numbers check.
State. There are 3 quarters and 10 nickels in the collection.

17. $2x-5=3x-3$
Graph: $y=2x-5$ and $y=3x-3$ and determine the x-coordinate of the point of intersection.

$y=2x-5$ $\qquad$ $y=3x-3$

$m=2$ $\qquad$ $m=3$

y-intercept is $(0,-5)$; y-intercept is (0, –3).



The x-coordinate of the point of intersection is –2; this is the solution.

18. $f(x)=\frac{1}{2}x-5$
Let $f(x)=0$ and solve for x.

$0=\frac{1}{2}x-5$

$2\cdot 5=\frac{1}{2}x\cdot 2$

$10=x$ $\quad$ 10 is the zero.

19. $3(x-y)=4+x$ $\qquad$ $x=5y+2$

$3x-3y=4+x$

$2x-3y=4$

Substitute $5y+2$ for x.

$2(5y+2)-3y=4$

$10y+4-3y=4$

$7y=0$

$y=0$

$x=5y+2 \rightarrow x=5\cdot 0+2=2$

The solution is (2, 0).

20. $\frac{3}{2}x-y=24$ (1)

$2x+\frac{3}{2}y=15$ (2)

Multiply equation (1) by $\frac{3}{2}$ and add the result to equation (2).

$$\begin{array}{r} \frac{9}{4}x - \frac{3}{2}y = 36\\ 2x + \frac{3}{2}y = 15\\ \hline (2+\frac{9}{4})x = 51\end{array}$$

$\frac{8+9}{4}x=51$

$\frac{4}{17}\cdot\frac{17}{4}x=51\cdot\frac{4}{17}$

$x=12$

$$\frac{3}{2}x - y = 24 \rightarrow \frac{3}{2} \cdot 12 - y = 24$$
$$18 - y = 24$$
$$-y = 6$$
$$y = -6$$

The solution is (12, –6).

21. Let $x =$ the number of people behind you, then $x + 2 =$ the number of people ahead of you. There are 3 times the number of people behind, or $3x$ total people, in line. Another way to express the total is $x + (x + 2) + 1$ (the 1 represents you, as you are in line also). So, we have

$$3x = x + x + 2 + 1$$
$$3x = 2x + 3$$
$$x = 3$$

The total number of people in line is $3 \cdot 3$, or 9 people.

22. $f(x) = mx + b$

$$(-1, 3) \rightarrow f(-1) = m(-1) + b$$
$$3 = -m + b$$
$$(-2, -4) \rightarrow f(-2) = m(-2) + b$$
$$-4 = -2m + b$$

Solve the system:

$$\begin{aligned} 3 &= -m + b \quad (1) \\ -4 &= -2m + b \quad (2) \end{aligned}$$

Multiply Equation (1) by –2 and add the result to Equation (2).

$$\begin{array}{rcrcr} -6 & = & 2m & - & 2b \\ -4 & = & -2m & + & b \\ \hline -10 & = & & - & b \\ & & 10 & = & b \end{array}$$

$$3 = -m + b$$
$$3 = -m + 10$$
$$-7 = -m$$
$$7 = m$$