Chapter 4

Polynomials

Exercise Set 4.1

1. By the rule for raising a product to a power on page 232, choice (e) is correct.

3. By the power rule on page 231, choice (b) is correct.

5. By the definition of 0 as an exponent on page 230, choice (g) is correct.

7. By the rule for raising a quotient to a power on page 229, choice (c) is correct.

9. The base is $2x$. The exponent is 5.

11. The base is x. The exponent is 3.

13. The base is $\frac{4}{y}$. The exponent is 7.

15. $d^3 \cdot d^{10} = d^{3+10} = d^{13}$

17. $a^6 \cdot a = a^6 \cdot a^1 = a^{6+1} = a^7$

19. $6^5 \cdot 6^{10} = 6^{5+10} = 6^{15}$

21. $(3y)^4(3y)^8 = (3y)^{4+8} = (3y)^{12}$

23. $(8n)(8n)^9 = (8n)^1(8n)^9 = (8n)^{1+9} = (8n)^{10}$

25.
$$\begin{aligned}(a^2b^7)(a^3b^2) &= a^2b^7a^3b^2 && \text{Using an associative law}\\ &= a^2a^3b^7b^2 && \text{Using a commutative law}\\ &= a^5b^9 && \text{Adding exponents}\end{aligned}$$

27. $(x+3)^5(x+3)^8 = (x+3)^{5+8} = (x+3)^{13}$

29. $r^3 \cdot r^7 \cdot r^0 = r^{3+7+0} = r^{10}$

31.
$$\begin{aligned}(mn^5)(m^3n^4) &= mn^5m^3n^4\\ &= m^1m^3n^5n^4\\ &= m^{1+3}n^{5+4}\\ &= m^4n^9\end{aligned}$$

33. $\frac{7^5}{7^2} = 7^{5-2} = 7^3$ Subtracting exponents

35. $\frac{t^8}{t} = t^{8-1} = t^7$ Subtracting exponents

37. $\frac{(5a)^7}{(5a)^6} = (5a)^{7-6} = (5a)^1 = 5a$

39. $\frac{(x+y)^8}{(x+y)^8}$

Observe that we have an expression divided by itself. Thus, the result is 1.

We could also do this exercise as follows:

$\frac{(x+y)^8}{(x+y)^8} = (x+y)^{8-8} = (x+y)^0 = 1$

41. $\frac{(r+s)^{12}}{(r+s)^4} = (r+s)^{12-4} = (r+s)^8$

43. $\frac{12d^9}{15d^2} = \frac{12}{15}d^{9-2} = \frac{4}{5}d^7$

45. $\frac{8a^9b^7}{2a^2b} = \frac{8}{2} \cdot \frac{a^9}{a^2} \cdot \frac{b^7}{b^1} = 4a^{9-2}b^{7-1} = 4a^7b^6$

47. $\frac{x^{12}y^9}{x^0y^2} = x^{12-0}y^{9-2} = x^{12}y^7$

49. When $t = 15$, $t^0 = 15^0 = 1$. (Any nonzero number raised to the 0 power is 1.)

51. When $x = -22$, $5x^0 = 5(-22)^0 = 5 \cdot 1 = 5$.

53. $7^0 + 4^0 = 1 + 1 = 2$

55. $(-3)^1 - (-3)^0 = -3 - 1 = -4$

57. $(x^3)^{11} = x^{3 \cdot 11} = x^{33}$ Multiplying exponents

59. $(5^8)^4 = 5^{8 \cdot 4} = 5^{32}$ Multiplying exponents

61. $(t^{20})^4 = t^{20 \cdot 4} = t^{80}$

63. $(10x)^2 = 10^2x^2 = 100x^2$

65. $(-2a)^3 = (-2)^3a^3 = -8a^3$

67. $(-5n^7)^2 = (-5)^2(n^7)^2 = 25n^{7 \cdot 2} = 25n^{14}$

69. $(a^2b)^7 = (a^2)^7(b^7) = a^{14}b^7$

71. $(r^5t)^3(r^2t^8) = (r^5)^3(t)^3r^2t^8 = r^{15}t^3r^2t^8 = r^{17}t^{11}$

73. $(2x^5)^3(3x^4) = 2^3(x^5)^3(3x^4) = 8x^{15} \cdot 3x^4 = 24x^{19}$

75. $\left(\frac{x}{5}\right)^3 = \frac{x^3}{5^3} = \frac{x^3}{125}$

77. $\left(\frac{7}{6n}\right)^2 = \frac{7^2}{(6n)^2} = \frac{49}{6^2n^2} = \frac{49}{36n^2}$

79. $\left(\frac{a^3}{b^8}\right)^6 = \frac{(a^3)^6}{(b^8)^6} = \frac{a^{18}}{b^{48}}$

81. $\left(\frac{x^2y}{z^3}\right)^4 = \frac{(x^2y)^4}{(z^3)^4} = \frac{(x^2)^4(y^4)}{z^{12}} = \frac{x^8y^4}{z^{12}}$

83. $\left(\frac{a^3}{-2b^5}\right)^4 = \frac{(a^3)^4}{(-2b^5)^4} = \frac{a^{12}}{(-2)^4(b^5)^4} = \frac{a^{12}}{16b^{20}}$

85. $\left(\frac{5x^7y}{-2z^4}\right)^3 = \frac{(5x^7y)^3}{(-2z^4)^3} = \frac{5^3(x^7)^3y^3}{-2^3(z^4)^3} = \frac{125x^{21}y^3}{-8z^{12}}$

87. $\left(\frac{4x^3y^5}{3z^7}\right)^0$

Observe that for $x \neq 0$, $y \neq 0$, and $z \neq 0$, we have a nonzero number raised to the 0 power. Thus, the result is 1.

89. *Writing Exercise.* -5^2 is the opposite of the square of 5, or the opposite of 25, so it is -25; $(-5)^2$ is the square of the opposite of 5, or $-5(-5)$, so it is 25.

91. $-10-14=-10+(-14)=-24$

93. $-16+5=-11$

95. $-3+(-11)=-14$

97. *Writing Exercise.* Any number raised to an even power is nonnegative. Any nonnegative number raised to an odd power is nonnegative. Any negative number raised to an odd power is negative. Thus, a must be a negative number, and n must be an odd number.

99. *Writing Exercise.* Let $s=$ the length of a side of the smaller square. Then $3s=$ the length of a side of the larger square. The area of the smaller square is s^2, and the area of the larger square is $(3s)^2$, or $9s^2$, so the area of the larger square is 9 times the area of the smaller square.

101. Choose any number except 0. For example, let $x=1$.

$3x^2=3\cdot 1^2=3\cdot 1=3$, but

$(3x)^2=(3\cdot 1)^2=3^2=9$.

103. Choose any number except 0 or 1. For example, let $t=-1$. Then $\dfrac{t^6}{t^2}=\dfrac{(-1)^6}{(-1)^2}=\dfrac{1}{1}=1$, but $t^3=(-1)^3=-1$.

105. $y^{4x}\cdot y^{2x}=y^{4x+2x}=y^{6x}$

107. $\dfrac{x^{5t}(x^t)^2}{(x^{3t})^2}=\dfrac{x^{5t}x^{2t}}{x^{6t}}=\dfrac{x^{7t}}{x^{6t}}=x^t$

109.
$$\begin{aligned}\frac{t^{26}}{t^x}&=t^x\\ t^{26-x}&=t^x\\ 26-x&=x \quad \text{Equating exponents}\\ 26&=2x\\ 13&=x\end{aligned}$$
The solution is 13.

111. Since the bases are the same, the expression with the larger exponent is larger. Thus, $4^2<4^3$.

113. $4^3=64$, $3^4=81$, so $4^3<3^4$.

115.
$$\begin{aligned}25^8&=(5^2)^8=5^{16}\\ 125^5&=(5^3)^5=5^{15}\\ 5^{16}&>5^{15},\ \text{or } 25^8>125^5.\end{aligned}$$

117. $2^{22}=2^{10}\cdot 2^{10}\cdot 2^2\approx 10^3\cdot 10^3\cdot 4\approx 1000\cdot 1000\cdot 4\approx 4,000,000$

Using a calculator, we find that $2^{22}=4,194,304$. The difference between the exact value and the approximation is $4,194,304-4,000,000$, or 194,304.

119. $2^{31}=2^{10}\cdot 2^{10}\cdot 2^{10}\cdot 2\approx 10^3\cdot 10^3\cdot 10^3\cdot 2$
$\approx 1000\cdot 1000\cdot 1000\cdot 2=2,000,000,000$

Using a calculator, we find that $2^{31}=2,147,483,648$. The difference between the exact value and the approximation is $2,147,483,648-2,000,000,000=147,483,648$.

121.
$$\begin{aligned}1.5\text{ MB}&=1.5\times 1000\text{ KB}\\ &=1.5\times 1000\times 1\times 2^{10}\text{ bytes}\\ &=1,536,000\text{ bytes}\\ &\approx 1,500,000\text{ bytes}\end{aligned}$$

Exercise Set 4.2

1. $\left(\dfrac{x^3}{y^2}\right)^{-2}=\left(\dfrac{y^2}{x^3}\right)^2=\dfrac{(y^2)^2}{(x^3)^2}=\dfrac{y^4}{x^6}\Rightarrow$ (c)

3. $\left(\dfrac{y^{-2}}{x^{-3}}\right)^{-3}=\dfrac{(y^{-2})^{-3}}{(x^{-3})^{-3}}=\dfrac{y^6}{x^9}\Rightarrow$ (a)

5. $2^{-3}=\dfrac{1}{2^3}=\dfrac{1}{8}$

7. $(-2)^{-6}=\dfrac{1}{(-2)^6}=\dfrac{1}{64}$

9. $t^{-9}=\dfrac{1}{t^9}$

11. $xy^{-2}=x\cdot\dfrac{1}{y^2}=\dfrac{x}{y^2}$

13. $r^{-5}t=\dfrac{1}{r^5}\cdot t=\dfrac{t}{r^5}$

15. $\dfrac{1}{a^{-8}}=a^8$

17. $7^{-1}=\dfrac{1}{7^1}=\dfrac{1}{7}$

19. $\left(\dfrac{3}{5}\right)^{-2}=\left(\dfrac{5}{3}\right)^2=\dfrac{5^2}{3^2}=\dfrac{25}{9}$

21. $\left(\dfrac{x}{2}\right)^{-5}=\left(\dfrac{2}{x}\right)^5=\dfrac{2^5}{x^5}=\dfrac{32}{x^5}$

23. $\left(\dfrac{s}{t}\right)^{-7}=\left(\dfrac{t}{s}\right)^7=\dfrac{t^7}{s^7}$

25. $\dfrac{1}{9^2}=9^{-2}$

27. $\dfrac{1}{y^3}=y^{-3}$

29. $\dfrac{1}{5}=\dfrac{1}{5^1}=5^{-1}$

31. $\dfrac{1}{t}=\dfrac{1}{t^1}=t^{-1}$

33. $2^{-5}\cdot 2^8=2^{-5+8}=2^3$, or 8

35. $x^{-3}\cdot x^{-9}=x^{-12}=\dfrac{1}{x^{12}}$

37. $t^{-3}\cdot t=t^{-3}\cdot t^1=t^{-3+1}=t^{-2}=\dfrac{1}{t^2}$

39. $\left(n^{-5}\right)^3=n^{-5\cdot 3}=n^{-15}=\dfrac{1}{n^{15}}$

41. $(t^{-3})^{-6}=t^{-3(-6)}=t^{18}$

43. $(t^4)^{-3} = t^{4(-3)} = t^{-12} = \dfrac{1}{t^{12}}$

45. $(mn)^{-7} = \dfrac{1}{(mn)^7} = \dfrac{1}{m^7n^7}$

47. $(3x^{-4})^2 = 3^2(x^{-4})^2 = 9x^{-8} = \dfrac{9}{x^8}$

49. $(5r^{-4}t^3)^2 = 5^2(r^{-4})^2(t^3)^2 = 25r^{-8}t^6 = \dfrac{25t^6}{r^8}$

51. $\dfrac{t^{12}}{t^{-2}} = t^{12-(-2)} = t^{14}$

53. $\dfrac{y^{-7}}{y^{-3}} = y^{-7-(-3)} = y^{-4} = \dfrac{1}{y^4}$

55. $\dfrac{15y^{-7}}{3y^{-10}} = 5y^{-7-(-10)} = 5y^3$

57. $\dfrac{2x^6}{x} = 2\dfrac{x^6}{x^1} = 2x^{6-1} = 2x^5$

59. $-\dfrac{15a^{-7}}{10b^{-9}} = -\dfrac{3b^9}{2a^7}$

61. $\dfrac{t^{-7}}{t^{-7}}$

Note that we have an expression divided by itself. Thus, the result is 1. We could also find this result as follows:

$$\frac{t^{-7}}{t^{-7}} = t^{-7-(-7)} = t^0 = 1.$$

63. $\dfrac{8x^{-3}}{y^{-7}z^{-1}} = \dfrac{8y^7z}{x^3}$

65. $\dfrac{3t^4}{s^{-2}u^{-4}} = 3s^2t^4u^4$

67. $(x^4y^5)^{-3} = (x^4)^{-3}(y^5)^{-3} = x^{-12}y^{-15} = \dfrac{1}{x^{12}y^{15}}$

69. $\left(3m^{-5}n^{-3}\right)^{-2} = 3^{-2}m^{-5(-2)}n^{-3(-2)} = 3^{-2}m^{10}n^6$
$= \dfrac{m^{10}n^6}{9}$

71. $(a^{-5}b^7c^{-2})(a^{-3}b^{-2}c^6) = a^{-5+(-3)}b^{7+(-2)}c^{-2+6}$
$= a^{-8}b^5c^4 = \dfrac{b^5c^4}{a^8}$

73. $\left(\dfrac{a^4}{3}\right)^{-2} = \left(\dfrac{3}{a^4}\right)^2 = \dfrac{3^2}{(a^4)^2} = \dfrac{9}{a^8}$

75. $\left(\dfrac{m^{-1}}{n^{-4}}\right)^3 = \dfrac{(m^{-1})^3}{(n^{-4})^3} = \dfrac{m^{-3}}{n^{-12}} = \dfrac{n^{12}}{m^3}$

77. $\left(\dfrac{2a^2}{3b^4}\right)^{-3} = \left(\dfrac{3b^4}{2a^2}\right)^3 = \dfrac{(3b^4)^3}{(2a^2)^3} = \dfrac{3^3(b^4)^3}{2^3(a^2)^3} = \dfrac{27b^{12}}{8a^6}$

79. $\left(\dfrac{5x^{-2}}{3y^{-2}z}\right)^0$

Any nonzero expression raised to the 0 power is equal to 1. Thus, the answer is 1.

81. $\dfrac{-6a^3b^{-5}}{-3a^7b^{-8}} = \dfrac{-6}{-3} \cdot \dfrac{a^3b^{-5}}{a^7b^{-8}}$
$= 2 \cdot \dfrac{b^{8-5}}{a^{7-3}}$
$= \dfrac{2b^3}{a^4}$

83. $\dfrac{10x^{-4}yz^7}{8x^7y^{-3}z^{-3}} = \dfrac{10}{8} \cdot \dfrac{y^{1+3}z^{7+3}}{x^{7+4}} = \dfrac{5y^4z^{10}}{2x^{11}}$

85. $4.92 \times 10^3 = 4.920 \times 10^3$ The decimal point moves right three places
$= 4920$

87. 8.92×10^{-3}

Since the exponent is negative, the decimal point will move to the left.

.008.92 The decimal point moves left 3 places.

$8.92 \times 10^{-3} = 0.00892$

89. 9.04×10^8

Since the exponent is positive, the decimal point will move to the right.

9.04000000.

8 places

$9.04 \times 10^8 = 904,000,000$

91. $3.497 \times 10^{-6} = 0000003.497 \times 10^{-6}$ The decimal point left six places
$= 0.000003497$

93. $36,000,000 = 3.6 \times 10^m$ We move the decimal point 7 places to the right. Thus m is 7.
$= 3.6 \times 10^7$

95. $0.00583 = 5.83 \times 10^m$

To write 5.83 as 0.00583 we move the decimal point 3 places to the left. Thus, m is -3 and

$$0.00583 = 5.83 \times 10^{-3}.$$

97. $78,000,000,000 = 7.8 \times 10^m$

To write 7.8 as 78,000,000,000 we move the decimal point 10 places to the right. Thus, m is 10 and

$$78,000,000,000 = 7.8 \times 10^{10}.$$

99. $0.000000527 = 5.27 \times 10^m$

To write 5.27 as 0.000000527 we move the decimal point 7 places to the left. Thus, m is -7 and

$$0.000000527 = 5.27 \times 10^{-7}.$$

101. $0.000001032 = 1.032 \times 10^m$ We move the decimal point 6 places to the left. Thus $m = -6$.
$= 1.032 \times 10^{-6}$

103. $(3\times 10^5)(2\times 10^8) = (3\cdot 2)\times(10^5\cdot 10^8)$
$= 6\times 10^{5+8}$
$= 6\times 10^{13}$

105. $(3.8\times 10^9)(6.5\times 10^{-2}) = (3.8\cdot 6.5)\times(10^9\cdot 10^{-2})$
$= 24.7\times 10^7$

The answer is not yet in scientific notation since 24.7 is not a number between 1 and 10. We convert to scientific notation.

$24.7\times 10^7 = (2.47\times 10)\times 10^7 = 2.47\times 10^8$

107. $(8.7\times 10^{-12})(4.5\times 10^{-5})$
$= (8.7\cdot 4.5)\times(10^{-12}\cdot 10^{-5})$
$= 39.15\times 10^{-17}$

The answer is not yet in scientific notation since 39.15 is not a number between 1 and 10. We convert to scientific notation.

$39.15\times 10^{-17} = (3.915\times 10)\times 10^{-17} = 3.915\times 10^{-16}$

109. $\dfrac{8.5\times 10^8}{3.4\times 10^{-5}} = \dfrac{8.5}{3.4}\times\dfrac{10^8}{10^{-5}}$
$= 2.5\times 10^{8-(-5)}$
$= 2.5\times 10^{13}$

111. $(4.0\times 10^3)\div(8.0\times 10^8) = \dfrac{4.0}{8.0}\times\dfrac{10^3}{10^8}$
$= 0.5\times 10^{3-8}$
$= 0.5\times 10^{-5}$
$= 5.0\times 10^{-6}$

113. $\dfrac{7.5\times 10^{-9}}{2.5\times 10^{12}} = \dfrac{7.5}{2.5}\times\dfrac{10^{-9}}{10^{12}}$
$= 3.0\times 10^{-9-12}$
$= 3.0\times 10^{-21}$

115. *Writing Exercise.* $3^{-29} = \dfrac{1}{3^{29}}$ and $2^{-29} = \dfrac{1}{2^{29}}$. Since $3^{29} > 2^{29}$, we have $\dfrac{1}{3^{29}} < \dfrac{1}{2^{29}}$.

117. $9x + 2y - x - 2y = 9x - x + 2y - 2y = 8x$

119. $-3x + (-2) - 5 - (-x)$
$= -3x + x - 2 - 5$
$= -2x - 7$

121. $4 + x^3 = 4 + 10^3 = 4 + 1000 = 1004$

123. *Writing Exercise.* x^{-n} represents a negative integer when x is negative, $\dfrac{1}{x}$ is an integer, and n is an odd number.

125. $\dfrac{1}{1.25\times 10^{-6}} = \dfrac{1}{1.25}\times\dfrac{1}{10^{-6}} = 0.8\times 10^6$
$= (8\times 10^{-1})\times 10^6 = 8\times 10^5$

127. $8^{-3}\cdot 32\div 16^2 = (2^3)^{-3}\cdot 2^5\div(2^4)^2$
$= 2^{-9}\cdot 2^5\div 2^8 = 2^{-4}\div 2^8 = 2^{-12}$

129. $\dfrac{125^{-4}(25^2)^4}{125} = \dfrac{(5^3)^{-4}((5^2)^2)^4}{5^3}$
$= \dfrac{5^{-12}(5^4)^4}{5^3} = \dfrac{5^{-12}\cdot 5^{16}}{5^3} = \dfrac{5^4}{5^3} = 5^1 = 5$

131. $\left[\left(5^{-3}\right)^2\right]^{-1} = 5^{(-3)(2)(-1)} = 5^6$

133. $3^{-1} + 4^{-1} = \dfrac{1}{3} + \dfrac{1}{4} = \dfrac{4}{12} + \dfrac{3}{12} = \dfrac{7}{12}$

135. $\dfrac{27^{-2}(81^2)^3}{9^8} = \dfrac{(3^3)^{-2}((3^4)^2)^3}{(3^2)^8}$
$= \dfrac{3^{-6}\cdot 3^{24}}{3^{16}} = \dfrac{3^{18}}{3^{16}} = 3^2 = 9$

137. $\dfrac{5.8\times 10^{17}}{(4.0\times 10^{-13})(2.3\times 10^4)}$
$= \dfrac{5.8}{(4.0\cdot 2.3)}\times\dfrac{10^{17}}{(10^{-13}\cdot 10^4)}$
$\approx 0.6304347826\times 10^{17-(-13)-4}$
$\approx (6.304347826\times 10^{-1})\times 10^{26}$
$\approx 6.304347826\times 10^{25}$

139. $\dfrac{(2.5\times 10^{-8})(6.1\times 10^{-11})}{1.28\times 10^{-3}}$
$= \dfrac{(2.5\cdot 6.1)}{1.28}\times\dfrac{(10^{-8}\cdot 10^{-11})}{10^{-3}}$
$= 11.9140625\times 10^{-8+(-11)-(-3)}$
$= 11.9140625\times 10^{-16}$
$= (1.19140625\times 10)\times 10^{-16}$
$= 1.19140625\times 10^{-15}$

141. ***Familiarize***. Let $n =$ the number of car miles.

Translate. We reword the problem.

$$\begin{array}{ccccc} \underbrace{\text{Miles for one tree}} & \text{times} & \underbrace{\text{the number of trees}} & \text{is} & \underbrace{n} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 500 & \times & 600,000 & = & n \end{array}$$

Carry out. We solve the equation.

$500\times 600,000 = n$

$300,000,000 = n$

$n = 3\times 10^8$

Check. Review the computation, The answer is reasonable.

State. Trees can clean 3×10^8 miles of car traffic in a year.

143. ***Familiarize***. Let $c =$ the total cost of the condominiums.

Translate. We reword the problem.

$$\begin{array}{cccccc} \underbrace{\text{What}} & \text{is} & \underbrace{\text{price } \$2100 \text{ per ft}^2} & \text{times} & \underbrace{\text{space } 110{,}000 \text{ ft}^2} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ c & = & (2100) & \times & (110,000) \end{array}$$

Carry out.

$c = 2100\times 1.1\times 10^5$

$c = 231,000,000$

Check. We review calculations.

State. $c = \$2.31\times 10^8$ is the total cost.

145. ***Familiarize***. Let m = the number of minutes.

Convert. 3.3 hours = $3.3 \times 60 = 198$ minutes

Translate.

$$\underbrace{\text{What}}\ \text{is}\ \underbrace{\text{number of patients}}\ \text{times}\ \underbrace{\text{average time in minutes}}$$
$$\downarrow\quad\downarrow\qquad\downarrow\qquad\downarrow\qquad\downarrow$$
$$m \quad = 115,000,000 \quad \times \quad 198$$

$m = 2.277 \times 10^{10}$

Check. We review calculations.

State. In 2005 patients spent 2.277 $\times 10^{10}$ minutes in emergency rooms.

4.2 Connecting the Concepts

1. $x^4x^{10} = x^{4+10} = x^{14}$

3. $\dfrac{x^{-4}}{x^{10}} = \dfrac{1}{x^{10+4}} = \dfrac{1}{x^{14}}$

5. $(x^{-4})^{-10} = x^{40}$

7. $\dfrac{1}{c^{-8}} = c^8$

9. $(2x^3y)^4 = 2^4x^{3\cdot 4}y^4 = 16x^{12}y^4$

11. $(3xy^{-1}z^5)^0 = 1$

13. $\left(\dfrac{a^3}{b^4}\right)^5 = \dfrac{a^{15}}{b^{20}}$

15. $\dfrac{30x^4y^3}{12xy^7} = \dfrac{30}{12}x^{4-1}y^{3-7} = \dfrac{5x^3}{2y^4}$

17. $\dfrac{7p^{-5}}{xt^{-6}} = \dfrac{7t^6}{xp^5}$

19. $(2p^2q^4)(3pq^5)^2 = 2p^2q^4(9p^2q^{10}) = 18p^4q^{14}$

Exercise Set 4.3

1. The only expression with 4 terms is (b).

3. Expression (h) has three terms and they are written in descending order.

5. Expression (g) has two terms, and the degree of the leading term is 7.

7. Expression (a) has two terms, but it is not a binomial because $\dfrac{2}{x^2}$ is not a monomial.

9. $8x^3 - 11x^2 + 6x + 1$

The terms are $8x^3$, $-11x^2$, $6x$, and 1.

11. $-t^6 - 3t^3 + 9t - 4$

The terms are $-t^6$, $-3t^3$, $9t$, and -4.

13. $8x^4 + 2x$

Term	Coefficient	Degree
$8x^4$	8	4
$2x$	2	1

15. $9t^2 - 3t + 4$

Term	Coefficient	Degree
$9t^2$	9	2
$-3t$	-3	1
4	4	0

17. $6a^5 + 9a + a^3$

Term	Coefficient	Degree
$6a^5$	6	5
$9a$	9	1
a^3	1	3

19. $x^4 - x^3 + 4x - 3$

Term	Coefficient	Degree
x^4	1	4
$-x^3$	-1	3
$4x$	4	1
-3	-3	0

21. $5t + t^3 + 8t^4$

a)

Term	$5t$	t^3	$8t^4$
Degree	1	3	4

b) The term of highest degree is $8t^4$. This is the leading term. Then the leading coefficient is 8.

c) Since the term of highest degree is $8t^4$, the degree of the polynomial is 4.

23. $3a^2 - 7 + 2a^4$

a)

Term	$3a^2$	-7	$2a^4$
Degree	2	0	4

b) The term of highest degree is $2a^4$. This is the leading term. Then the leading coefficient is 2.

c) Since the term of highest degree is $2a^4$, the degree of the polynomial is 4.

25. $8 + 6x^2 - 3x - x^5$

a)

Term	8	$6x^2$	$-3x$	$-x^5$
Degree	0	2	1	5

b) The term of highest degree is $-x^5$. This is the leading term. Then the leading coefficient is -1 since $-x^5 = -1 \cdot x^5$.

c) Since the term of highest degree is $-x^5$, the degree of the polynomial is 5.

27. $7x^2 + 8x^5 - 4x^3 + 6 - \frac{1}{2}x^4$

Term	Coefficient	Degree of Term	Degree of Polynomial
$8x^5$	8	5	
$-\frac{1}{2}x^4$	$-\frac{1}{2}$	4	
$-4x^3$	-4	3	5
$7x^2$	7	2	
6	6	0	

29. Three monomials are added, so $x^2 - 23x + 17$ is a trinomial.

31. The polynomial $x^3 - 7x + 2x^2 - 4$ is a polynomial with no special name because it is composed of four monomials.

33. Two monomials are added, so $y + 8$ is a binomial.

35. The polynomial 17 is a monomial because it is the product of a constant and a variable raised to a whole number power. (In this case the variable is raised to the power 0.)

37. $5n^2 + n + 6n^2 = (5+6)n^2 + n = 11n^2 + n$

39. $3a^4 - 2a + 2a + a^4 = (3+1)a^4 + (-2+2)a$
$= 4a^4 + 0a = 4a^4$

41. $7x^3 - 11x + 5x + x^2 = 7x^3 + x^2 + (-11+5)x$
$= 7x^3 + x^2 - 6x$

43. $4b^3 + 5b + 7b^3 + b^2 - 6b$
$= (4+7)b^3 + b^2 + (5-6)b$
$= 11b^3 + b^2 - b$

45. $10x^2 + 2x^3 - 3x^3 - 4x^2 - 6x^2 - x^4$
$= -x^4 + (2-3)x^3 + (10-4-6)x^2 = -x^4 - x^3$

47. $\frac{1}{5}x^4 + 7 - 2x^2 + 3 - \frac{2}{15}x^4 + 2x^2$
$= \left(\frac{1}{5} - \frac{2}{15}\right)x^4 + (-2+2)x^2 + (7+3)$
$= \left(\frac{3}{15} - \frac{2}{15}\right)x^4 + 0x^2 + 10 = \frac{1}{15}x^4 + 10$

49. $8.3a^2 + 3.7a - 8 - 9.4a^2 + 1.6a + 0.5$
$= (8.3 - 9.4)a^2 + (3.7 + 1.6)a - 8 + 0.5$
$= -1.1a^2 + 5.3a - 7.5$

51. For $x = 3$: $-4x + 9 = -4 \cdot 3 + 9$
$= -12 + 9$
$= -3$

For $x = -3$: $-4x + 9 = -4(-3) + 9$
$= 12 + 9$
$= 21$

53. For $x = 3$: $2x^2 - 3x + 7 = 2 \cdot 3^2 - 3 \cdot 3 + 7$
$= 2 \cdot 9 - 3 \cdot 3 + 7$
$= 18 - 9 + 7$
$= 16$

For $x = -3$: $2x^2 - 3x + 7 = 2(-3)^2 - 3(-3) + 7$
$= 2 \cdot 9 - 3(-3) + 7$
$= 18 + 9 + 7$
$= 34$

55. For $x = 3$:
$-3x^3 + 7x^2 - 4x - 8 = -3 \cdot 3^3 + 7 \cdot 3^2 - 4 \cdot 3 - 8$
$= -3 \cdot 27 + 7 \cdot 9 - 12 - 8$
$= -81 + 63 - 12 - 8$
$= -38$

For $x = -3$:
$-3x^3 + 7x^2 - 4x - 8 = -3(-3)^3 + (-3)^2 - 4(-3) - 8$
$= -3(-27) + 7 \cdot 9 + 12 - 8$
$= 148$

57. For $x = 3$: $2x^4 - \frac{1}{9}x^3 = 2 \cdot 3^4 - \frac{1}{9} \cdot 3^3$
$= 2 \cdot 81 - \frac{1}{9} \cdot 27 = 162 - 3 = 159$

For $x = -3$: $2x^4 - \frac{1}{9}x^3 = 2(-3)^4 - \frac{1}{9}(-3)^3$
$= 2 \cdot 81 - \frac{1}{9}(-27)$
$= 162 + 3$
$= 165$

59. For $x = 3$: $-x - x^2 - x^3 = -3 - 3^2 - 3^3 = -3 - 9 - 27 = -39$
For $x = -3$: $-x - x^2 - x^3 = -(-3) - (-3)^2 - (-3)^3$
$= 3 - 9 + 27 = 21$

61. Since 2006 is 2 years after 2004, we evaluate the polynomial for $t = 2$.
$0.4t + 1.13 = 0.4(2) + 1.13$
$= 0.8 + 1.13$
$= 1.93$

The amount spent on shoes for college in 2006 is about \$1.93 billion.

63. $11.12t^2 = 11.12(10)^2 = 11.12(100) = 1112$

A skydiver has fallen approximately 1112 ft 10 seconds after jumping from a plane.

65. $2\pi r = 2(3.14)(10)$ Substituting 3.14 for π and 10 for r
$= 62.8$

The circumference is 62.8 cm.

67. $\pi r^2 = 3.14(7)^2$ Substituting 3.14 for π and 7 for r
$= 3.14(49)$
$= 153.86$

The area is 153.86 m^2.

69. Since 2006 is 3 years after 2003, we first locate 3 on the horizontal axis. From there we move vertically to the graph and then horizontally to the K-axis. This locates an K-value of about 75. Thus the number of kidney-paired donations in 2006 is about 75 donations.

71. Locate 10 on the horizontal axis. From there move vertically to the graph and then horizontally to the M-axis. This locates an M-value of about 9. Thus, about 9 words were memorized in 10 minutes.

73. Locate 8 on the horizontal axis. From there move vertically to the graph and then horizontally to the M-axis. This locates an M-value of about 6. Thus, the value of $-0.001t^3 + 0.1t^2$ for $t = 8$ is approximately 6.

75. Locate 4 on the horizontal axis. From there move vertically to the graph and then horizontally to the B-axis. This locates an BMI-value of about 16.

Locate 14 on the horizontal axis. From there move vertically to the graph and then horizontally to the B-axis. This locates an BMI-value of about 19.

77. *Writing Exercise.* A term is a number, a variable, or a product of numbers and variables which may be raised to powers whereas a monomial is a number, a variable, or a product of numbers and variables raised to *whole number* powers. For example, the term $5x^{-2}y^4$ is not a monomial.

79. $2x + 5 - (x + 8) = 2x + 5 - x - 8 = x - 3$

81. $4a + 3 - (-2a + 6) = 4a + 3 + 2a - 6 = 6a - 3$

83. $4t^4 + 8t - (5t^4 - 9t) = 4t^4 + 8t - 5t^4 + 9t = -t^4 + 17t$

85. *Writing Exercise.* Yes; the evaluation will yield a sum of products of integers which must be an integer.

87. Answers may vary. Choose an ax^5-term where a is an even integer. Then choose three other terms with different degrees, each less than degree 5, and coefficients $a+2$, $a+4$, and $a+6$, respectively, when the polynomial is written in descending order. One such polynomial is $2x^5+4x^4+6x^3+8$.

89. Find the total revenue from the sale of 30 monitors:

$$\begin{aligned}250x - 0.5x^2 &= 250(30) - 0.5(30)^2\\ &= 250(30) - 0.5(900)\\ &= 7500 - 450\\ &= \$7050\end{aligned}$$

Find the total cost of producing 30 monitors:

$$\begin{aligned}4000 + 0.6x^2 &= 4000 + 0.6(30)^2\\ &= 4000 + 0.6(900)\\ &= 4000 + 540\\ &= \$4540\end{aligned}$$

Subtract the cost from the revenue to find the profit: $\$7050 - \$4540 = \$2510$

91.
$$\begin{aligned}&(3x^2)^3 + 4x^2 \cdot 4x^4 - x^4(2x)^2 + [(2x)^2]^3 - \\ &\quad 100x^2(x^2)^2\\ &= 27x^6 + 4x^2 \cdot 4x^4 - x^4 \cdot 4x^2 + (2x)^6 - 100x^2 \cdot x^4\\ &= 27x^6 + 16x^6 - 4x^6 + 64x^6 - 100x^6\\ &= 3x^6\end{aligned}$$

93. First locate 16 on the vertical axis. Then move horizontally to the graph. We meet the curve at 2 places. At each place move down vertically to the horizontal axis and read the corresponding x-value. We see that the ages for a 16 BMI are 3 and 8.

95. We first find q, the quiz average, and t, the test average.

$$q = \frac{60 + 85 + 72 + 91}{4} = \frac{308}{4} = 77$$

$$t = \frac{89 + 93 + 90}{3} = \frac{272}{3} \approx 90.7$$

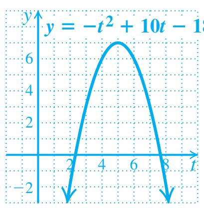
Now we substitute in the polynomial.

$$\begin{aligned}A &= 0.3q + 0.4t + 0.2f + 0.1h\\ &= 0.3(77) + 0.4(90.7) + 0.2(84) + 0.1(88)\\ &= 23.1 + 36.28 + 16.8 + 8.8\\ &= 84.98\\ &\approx 85.0\end{aligned}$$

97. When $t = 3$, $-t^2 + 10t - 18 = -3^2 + 10 \cdot 3 - 18$
$= -9 + 30 - 18 = 3.$
When $t = 4$, $-t^2 + 10t - 18 = -4^2 + 10 \cdot 4 - 18$
$= -16 + 40 - 18 = 6.$
When $t = 5$, $-t^2 + 10t - 18 = -5^2 + 10 \cdot 5 - 18$
$= -25 + 50 - 18 = 7.$
When $t = 6$, $-t^2 + 10t - 18 = -6^2 + 10 \cdot 6 - 18$
$= -36 + 60 - 18 = 6.$
When $t = 7$, $-t^2 + 10t - 18 = -7^2 + 10 \cdot 7 - 18$
$= -49 + 70 - 18 = 3.$

We complete the table. Then we plot the points and connect them with a smooth curve.

t	$-t^2 + 10t - 18$
3	3
4	6
5	7
6	6
7	3



Exercise Set 4.4

1. Since the right-hand side has collected like terms, the correct expression is x^2 to make

$$(3x^2 + 2) + (6x^2 + 7) = (3 + 6)x^2 + (2 + 7).$$

3. Since the right-hand side is the result of using subtraction (the distributive law), the correct operation is $-$ to make

$$(9x^3 - x^2) - (3x^2 + x^2) = 9x^3 - x^2 - 3x^2 - x^2.$$

5. $(3x+2)+(x+7) = (3+1)x+(2+7) = 4x+9$

7. $(2t+7)+(-8t+1) = (2-8)t+(7+1) = -6t+8$

9. $(x^2+6x+3)+(-4x^2-5) = (1-4)x^2+6x+(3-5)$
$= -3x^2+6x-2$

11. $(7t^2-3t-6)+(2t^2+4t+9)$
$= (7+2)t^2+(-3+4)t+(-6+9) = 9t^2+t+3$

13. $(4m^3-7m^2+m-5)+(4m^3+7m^2-4m-2)$
$= (4+4)m^3+(-7+7)m^2+(1-4)m+(-5-2)$
$= 8m^3-3m-7$

15. $(3+6a+7a^2+a^3)+(4+7a-8a^2+6a^3)$
$= (1+6)a^3+(7-8)a^2+(6+7)a+(3+4)$
$= 7a^3-a^2+13a+7$

17. $(3x^6+2x^4-x^3+5x)+(-x^6+3x^3-4x^2+7x^4)$
$= (3-1)x^6+(2+7)x^4+(-1+3)x^3-4x^2+5x$
$= 2x^6+9x^4+2x^3-4x^2+5x$

19. $\left(\frac{3}{5}x^4+\frac{1}{2}x^3-\frac{2}{3}x+3\right)+\left(\frac{2}{5}x^4-\frac{1}{4}x^3-\frac{3}{4}x^2-\frac{1}{6}x\right)$
$= \left(\frac{3}{5}+\frac{2}{5}\right)x^4+\left(\frac{1}{2}-\frac{1}{4}\right)x^3-\frac{3}{4}x^2+\left(-\frac{2}{3}-\frac{1}{6}\right)x+3$
$= x^4+\left(\frac{2}{4}-\frac{1}{4}\right)x^3-\frac{3}{4}x^2+\left(\frac{-4}{6}-\frac{1}{6}\right)x+3$
$= x^4+\frac{1}{4}x^3-\frac{3}{4}x^2-\frac{5}{6}x+3$

21. $(5.3t^2-6.4t-9.1)+(4.2t^3-1.8t^2+7.3)$
$= 4.2t^3+(5.3-1.8)t^2-6.4t+(-9.1+7.3)$
$= 4.2t^3+3.5t^2-6.4t-1.8$

23.
$$\begin{array}{r}
-4x^3 + 8x^2 + 3x - 2\\
- 4x^2 + 3x + 2\\
\hline
-4x^3 + 4x^2 + 6x + 0\\
-4x^3 + 4x^2 + 6x
\end{array}$$

25.
$$\begin{array}{rrrrr}
0.05x^4 & +\,0.12x^3 & -\,0.5x^2 & & \\
 & -\,0.02x^3 & +\,0.02x^2 & +\,2x & \\
1.5x^4 & & +\,0.01x^2 & & +\,0.15\\
 & 0.25x^3 & & & +\,0.85\\
-0.25x^4 & & +\,10x^2 & & -\,0.04\\
\hline
1.3x^4 & +\,0.35x^3 & +\,9.53x^2 & +\,2x & +\,0.96
\end{array}$$

27. Two forms of the opposite of $-3t^3+4t-7$ are
i) $-(-3t^3+4t-7)$ and
ii) $3t^3-4t+7$. (Changing the sign of every term.)

29. Two forms for the opposite of x^4-8x^3+6x are
i) $-(x^4-8x^3+6x)$ and
ii) $-x^4+8x^3-6x$. (Changing the sign of every term)

31. We change the sign of every term inside parentheses.
$-(9x-10) = -9x+10$

33. We change the sign of every term inside parentheses.
$-(3a^4-5a^2+1.2) = -3a^4+5a^2-1.2$

35. We change the sign of every term inside parentheses.
$-\left(-4x^4+6x^2+\frac{3}{4}x-8\right) = 4x^4-6x^2-\frac{3}{4}x+8$

37. $(3x+1)-(5x+8)$
$= 3x+1-5x-8$ Changing the sign of every term inside parentheses
$= -2x-7$

39. $(-9t+12)-(t^2+3t-1)$
$= -9t+12-t^2-3t+1$
$= -t^2-12t+13$

41. $(4a^2+a-7)-(3-8a^3-4a^2) = 4a^2+a-7-3+8a^3+4a^2$
$= 8a^3+8a^2+a-10$

43. $(1.2x^3+4.5x^2-3.8x)-(-3.4x^3-4.7x^2+23)$
$= 1.2x^3+4.5x^2-3.8x+3.4x^3+4.7x^2-23$
$= 4.6x^3+9.2x^2-3.8x-23$

45. $(7x^3-2x^2+6)-(6-2x^2+7x^3)$
Observe that we are subtracting the polynomial $7x^3-2x^2+6$ from itself. The result is 0.

47. $(3+5a+3a^2-a^3)-(2+4a-9a^2+2a^3)$
$= 3+5a+3a^2-a^3-2-4a+9a^2-2a^3$
$= 1+a+12a^2-3a^3$

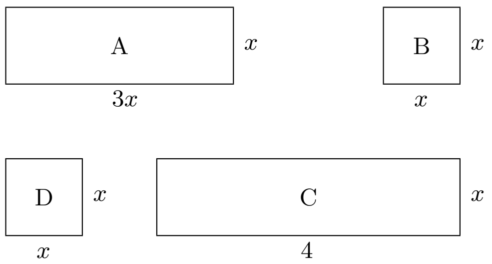
49. $\left(\frac{5}{8}x^3-\frac{1}{4}x-\frac{1}{3}\right)-\left(-\frac{1}{2}x^3+\frac{1}{4}x-\frac{1}{3}\right)$
$= \frac{5}{8}x^3-\frac{1}{4}x-\frac{1}{3}+\frac{1}{2}x^3-\frac{1}{4}x+\frac{1}{3}$
$= \frac{9}{8}x^3-\frac{2}{4}x$
$= \frac{9}{8}x^3-\frac{1}{2}x$

51. $(0.07t^3-0.03t^2+0.01t)-(0.02t^3+0.04t^2-1)$
$= 0.07t^3-0.03t^2+0.01t-0.02t^3-0.04t^2+1$
$= 0.05t^3-0.07t^2+0.01t+1$

53.
$$\begin{array}{rrr}
x^3 + 3x^2 + 1\\
-(x^3 + x^2 - 5)\\
\hline
\end{array}$$
$$\begin{array}{r}
x^3 + 3x^2 + 1\\
-x^3 - x^2 + 5\\
\hline
2x^2 + 6
\end{array}$$

55.
$$\begin{array}{r}
4x^4 - 2x^3\\
-(7x^4 + 6x^3 + 7x^2)\\
\hline
\end{array}$$
$$\begin{array}{r}
4x^4 - 2x^3\\
-7x^4 - 6x^3 - 7x^2\\
\hline
-3x^4 - 8x^3 - 7x^2
\end{array}$$

57. a)



Familiarize. The area of a rectangle is the product of the length and the width.

Translate. The sum of the areas is found as follows:

$$\begin{array}{cccccccc} & \text{Area of } A & + & \text{Area of } B & + & \text{Area of } C & + & \text{Area of } D \\ = & 3x\cdot x & + & x\cdot x & + & 4\cdot x & + & x\cdot x \end{array}$$

Carry out. We collect like terms.

$3x^2 + x^2 + 4x + x^2 = 5x^2 + 4x$

Check. We can go over our calculations. We can also assign some value to x, say 2, and carry out the computation of the area in two ways.

Sum of areas: $3\cdot 2\cdot 2 + 2\cdot 2 + 4\cdot 2 + 2\cdot 2$
$= 12 + 4 + 8 + 4 = 28$

Substituting in the polynomial:
$5(2)^2 + 4\cdot 2 = 20 + 8 = 28$

Since the results are the same, our solution is probably correct.

State. A polynomial for the sum of the areas is $5x^2 + 4x$.

b) For $x = 5$: $5x^2 + 4x = 5\cdot 5^2 + 4\cdot 5$
$= 5\cdot 25 + 4\cdot 5 = 125 + 20 = 145$

When $x = 5$, the sum of the areas is 145 square units.

For $x = 7$: $5x^2 + 4x = 5\cdot 7^2 + 4\cdot 7$
$= 5\cdot 49 + 4\cdot 7 = 245 + 28 = 273$

When $x = 7$, the sum of the areas is 273 square units.

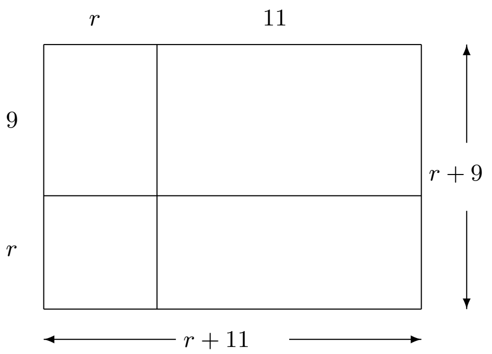
59. The perimeter is the sum of the lengths of the sides.

$4y + 4 + 7 + 2y + 7 + 6 + (3y + 2) + 7y$

$= (4 + 2 + 3 + 7)y + (4 + 7 + 7 + 6 + 2)$

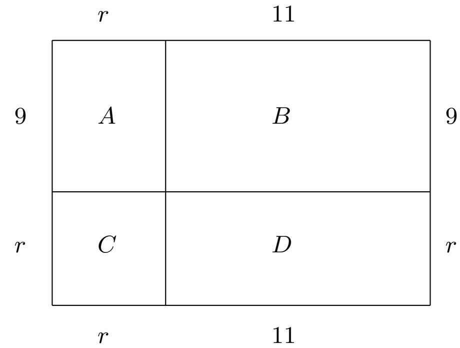
$= 16y + 26$

61.



The length and width of the figure can be expressed as $r + 11$ and $r + 9$, respectively. The area of this figure (a rectangle) is the product of the length and width. An algebraic expression for the area is $(r + 11)\cdot(r + 9)$.

The algebraic expressions $9r + 99 + r^2 + 11r$ and $(r + 11)\cdot(r + 9)$ represent the same area.

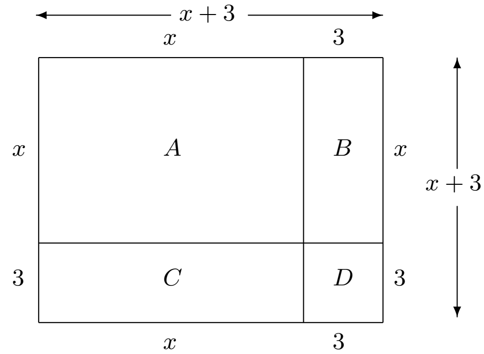


The area of the figure can be found by adding the areas of the four rectangles A, B, C, and D. The area of a rectangle is the product of the length and the width.

$$\begin{array}{cccccccc} & \text{Area of } A & + & \text{Area of } B & + & \text{Area of } C & + & \text{Area of } D \\ = & 9\cdot r & + & 11\cdot 9 & + & r\cdot r & + & 11\cdot r \\ = & 9r & + & 99 & + & r^2 & + & 11r \end{array}$$

An algebraic expression for the area of the figure is $9r + 99 + r^2 + 11r$.

63.



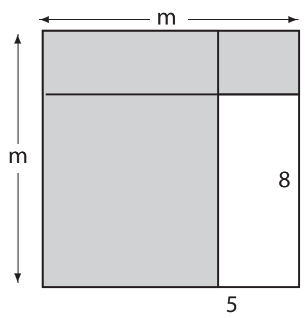
The length and width of the figure can each be expressed as $x + 3$. The area can be expressed as $(x + 3)\cdot(x + 3)$, or $(x + 3)^2$. Another way to express the area is to find an expression for the sum of the areas of the four rectangles A, B, C, and D. The area of each rectangle is the product of its length and width.

$$\begin{array}{cccccccc} & \text{Area of } A & + & \text{Area of } B & + & \text{Area of } C & + & \text{Area of } D \\ = & x\cdot x & + & 3\cdot x & + & 3\cdot x & + & 3\cdot 3 \\ = & x^2 & + & 3x & + & 3x & + & 9 \end{array}$$

The algebraic expressions $(x + 3)^2$ and $x^2 + 3x + 3x + 9$ represent the same area.

$(x + 3)^2 = x^2 + 3x + 3x + 9$

65. Recall that the area of a rectangle is length times width.



Area of entire square	$-$	Area not shaded	$=$	Shaded area
$m \cdot m$	$-$	$8 \cdot 5$	$=$	Shaded area
		$m^2 - 40$	$=$	Shaded area

67. Recall that the area of a circle is the product of π and the square of the radius, r^2.

$$A = \pi r^2.$$

The area of a square is the length of on side, s, squared.

$$A = s^2.$$

Area of circle	$-$	Area of square	$=$	Shaded area
πr^2	$-$	$7 \cdot 7$	$=$	Shaded area
πr^2	$-$	49	$=$	Shaded area

69. ***Familiarize***. Recall that the area of a rectangle is the product of the length and the width and that, consequently, the area of a square with side s is s^2. The remaining floor area is the area of the entire floor less the area of the bath enclosure, in square feet.

Translate.

Area of entire floor	$-$	Area of bath enclosure	$=$	Remaining floor area
x^2	$-$	$2 \cdot 6$	$=$	Remaining floor area

Carry out. We simplify the expression.

$$x^2 - 2 \cdot 6 = x^2 - 12$$

Check. We go over the calculations. The answer checks.

State. A polynomial for the remaining floor area is $(x^2 - 12)$ ft^2.

71. ***Familiarize***. Recall that the area of a square with side z is z^2. Recall that the area of a circle with radius 6, or half the diameter is πr^2 or $\pi \cdot 6^2$ The remaining area of the garden is the area of the garden less the area of the patio, in square feet.

Translate.

Area of garden	$-$	Area of patio	is	Remaining garden area
z^2	$-$	$\pi \cdot 6^2$	$=$	Remaining garden area

Carry out. We simplify the expression.

$$\left(z^2 - 36\pi\right)\text{ft}^2.$$

Check. We go over the calculations. The answer checks.

State. A polynomial for the remaining area of the garden is $(z^2 - 36\pi)$ ft^2.

73. ***Familiarize***. Recall that the area of a square with side s is s^2 and the area of a circle with radius r is πr^2. The radius of the circle is half the diameter, or $\frac{d}{2}$ m. The area of the mat outside the circle is the area of the entire mat less the area of the circle, in square meters.

Translate.

Area of mat	$-$	Area of circle	is	Area outside the circle
12^2	$-$	$\pi \cdot \left(\frac{d}{2}\right)^2$	$=$	Area outside the circle

Carry out. We simplify the expression.

$$12^2 - \pi \cdot \left(\frac{d}{2}\right)^2 = 144 - \pi \cdot \frac{d^2}{4} = 144 - \frac{d^2}{4}\pi$$

Check. We go over the calculations. The answer checks.

State. A polynomial for the area of the mat outside the wrestling circle is $\left(144 - \frac{d^2}{4}\pi\right)$ m^2.

75. *Writing Exercise*. We use the parentheses in $\left(x^2 - 64\pi\right)\text{ft}^2$ to indicate that the units, square feet, apply to the entire quantity in the expression $x^2 - 64\pi$.

77. $2\left(x^2 - x + 3\right) = 2x^2 - 2x + 6$

79. $x^2 \cdot x^6 = x^{2+6} = x^8$

81. $2n \cdot n^2 = 2n^{1+2} = 2n^3$.

83. *Writing Exercise*. The polynomials are opposites.

85.
$$\begin{aligned} &(6t^2 - 7t) + (3t^2 - 4t + 5) - (9t - 6) \\ &= 6t^2 - 7t + 3t^2 - 4t + 5 - 9t + 6 \\ &= 9t^2 - 20t + 11 \end{aligned}$$

87.
$$\begin{aligned} &4\left(x^2 - x + 3\right) - 2\left(2x^2 + x - 1\right) \\ &= 4x^2 - 4x + 12 - 4x^2 - 2x + 2 \\ &= (4 - 4)x^2 + (-4 - 2)x + (12 + 2) \\ &= -6x + 14 \end{aligned}$$

89.
$$\begin{aligned} &(345.099x^3 - 6.178x) - (94.508x^3 - 8.99x) \\ &= 345.099x^3 - 6.178x - 94.508x^3 + 8.99x \\ &= 250.591x^3 + 2.812x \end{aligned}$$

91. ***Familiarize***. The surface area is $2lw + 2lh + 2wh$, where l = length, w = width, and h = height of the rectangular solid. Here we have $l = 3$, $w = w$, and $h = 7$.

Translate. We substitute in the formula above.

$$2 \cdot 3 \cdot w + 2 \cdot 3 \cdot 7 + 2 \cdot w \cdot 7$$

Carry out. We simplify the expression.

$$\begin{aligned} &2 \cdot 3 \cdot w + 2 \cdot 3 \cdot 7 + 2 \cdot w \cdot 7 \\ &= 6w + 42 + 14w \\ &= 20w + 42 \end{aligned}$$

Check. We can go over the calculations. We can also assign some value to w, say 6, and carry out the computation in two ways.

Using the formula: $2 \cdot 3 \cdot 6 + 2 \cdot 3 \cdot 7 + 2 \cdot 6 \cdot 7 = 36 + 42 + 84 = 162$

Substituting in the polynomial: $20 \cdot 6 + 42 =$
$120 + 42 = 162$

Since the results are the same, our solution is probably correct.

State. A polynomial for the surface area is $20w + 42$.

93. ***Familiarize***. The surface area is $2lw + 2lh + 2wh$, where $l =$ length, $w =$ width, and $h =$ height of the rectangular solid. Here we have $l = x$, $w = x$, and $h = 5$.

Translate. We substitute in the formula above.

$$2 \cdot x \cdot x + 2 \cdot x \cdot 5 + 2 \cdot x \cdot 5$$

Carry out. We simplify the expression.

$$\begin{aligned} &2 \cdot x \cdot x + 2 \cdot x \cdot 5 + 2 \cdot x \cdot 5 \\ &= 2x^2 + 10x + 10x \\ &= 2x^2 + 20x \end{aligned}$$

Check. We can go over the calculations. We can also assign some value to x, say 3, and carry out the computation in two ways.

Using the formula: $2 \cdot 3 \cdot 3 + 2 \cdot 3 \cdot 5 + 2 \cdot 3 \cdot 5 =$
$18 + 30 + 30 = 78$

Substituting in the polynomial: $2 \cdot 3^2 + 20 \cdot 3 =$
$2 \cdot 9 + 60 = 18 + 60 = 78$

Since the results are the same, our solution is probably correct.

State. A polynomial for the surface area is $2x^2 + 20x$.

95. Length of top edges: $x + 6 + x + 6$, or $2x + 12$

Length of bottom edges: $x + 6 + x + 6$, or $2x + 12$

Length of vertical edges: $4 \cdot x$, or $4x$

Total length of edges: $2x + 12 + 2x + 12 + 4x = 8x + 24$

97. *Writing Exercise*. Yes; $4(-x)^7 - 6(-x)^3 + 2(-x) =$
$-4x^7 + 6x^3 - 2x = -(4x^7 - 6x^3 + 2x)$.

Exercise Set 4.5

1. $3x^2 \cdot 2x^4 = (3 \cdot 2)(x^2 \cdot x^4) = 6x^6$

Choice (c) is correct.

3. $4x^3 \cdot 2x^5 = (4 \cdot 2)(x^3 \cdot x^5) = 8x^8$

Choice (d) is correct.

5. $4x^6 + 2x^6 = (4 + 2)x^6 = 6x^6$

Choice (c) is correct.

7. $(3x^5)7 = (3 \cdot 7)x^5 = 21x^5$

9. $(-x^3)(x^4) = (-1 \cdot x^3)(x^4) = -1(x^3 \cdot x^4) = -1 \cdot x^7 = -x^7$

11. $(-x^6)(-x^2) = (-1 \cdot x^6)(-1 \cdot x^2) = (-1)(-1)(x^6 \cdot x^2) = x^8$

13. $4t^2(9t^2) = (4 \cdot 9)(t^2 \cdot t^2) = 36t^4$

15. $(0.3x^3)(-0.4x^6) = 0.3(-0.4)(x^3 \cdot x^6) = -0.12x^9$

17. $\left(-\frac{1}{4}x^4\right)\left(\frac{1}{5}x^8\right) = \left(-\frac{1}{4} \cdot \frac{1}{5}\right)(x^4 \cdot x^8) = -\frac{1}{20}x^{12}$

19. $(-5n^3)(-1) = (-5)(-1)n^3 = 5n^3$

21. $11x^5(-4x^5) = (-11 \cdot 4)(x^5 \cdot x^5) = -44x^{10}$

23. $(-4y^5)(6y^2)(-3y^3) = -4(6)(-3)(y^5 \cdot y^2 \cdot y^3) = 72y^{10}$

25. $5x(4x + 1) = 5x(4x) + 5x(1) = 20x^2 + 5x$

27. $(a - 9)3a = a \cdot 3a - 9 \cdot 3a = 3a^2 - 27a$

29.
$$\begin{aligned} x^2(x^3 + 1) &= x^2(x^3) + x^2(1) \\ &= x^5 + x^2 \end{aligned}$$

31.
$$\begin{aligned} &-3n(2n^2 - 8n + 1) \\ &= (-3n)(2n^2) + (-3n)(-8n) + (-3n)(1) \\ &= -6n^3 + 24n^2 - 3n \end{aligned}$$

33. $-5t^2(3t + 6) = -5t^2(3t) - 5t^2(6) = -15t^3 - 30t^2$

35.
$$\begin{aligned} &\frac{2}{3}a^4\left(6a^5 - 12a^3 - \frac{5}{8}\right) \\ &= \frac{2}{3}a^4(6a^5) - \frac{2}{3}a^4(12a^3) - \frac{2}{3}a^4\left(\frac{5}{8}\right) \\ &= \frac{12}{3}a^9 - \frac{24}{3}a^7 - \frac{10}{24}a^4 \\ &= 4a^9 - 8a^7 - \frac{5}{12}a^4 \end{aligned}$$

37.
$$\begin{aligned} (x + 3)(x + 4) &= (x + 3)x + (x + 3)4 \\ &= x \cdot x + 3 \cdot x + x \cdot 4 + 3 \cdot 4 \\ &= x^2 + 3x + 4x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

39.
$$\begin{aligned} (t + 7)(t - 3) &= (t + 7)t + (t + 7)(-3) \\ &= t \cdot t + 7 \cdot t + t(-3) + 7(-3) \\ &= t^2 + 7t - 3t - 21 \\ &= t^2 + 4t - 21 \end{aligned}$$

41.
$$\begin{aligned} (a - 0.6)(a - 0.7) &= (a - 0.6)a + (a - 6)(-0.7) \\ &= a \cdot a - 0.6 \cdot a + a(-0.7) + (-0.6)(-0.7 \\ &= a^2 - 0.6a - 0.7a + 0.42 \\ &= a^2 - 1.3a + 0.42 \end{aligned}$$

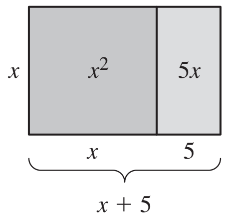
43.
$$\begin{aligned} (x + 3)(x - 3) &= (x + 3)x + (x + 3)(-3) \\ &= x \cdot x + 3 \cdot x + x(-3) + 3(-3) \\ &= x^2 + 3x - 3x - 9 \\ &= x^2 - 9 \end{aligned}$$

45.
$$\begin{aligned} (4 - x)(7 - 2x) &= (4 - x)7 + (4 - x)(-2x) \\ &= 4 \cdot 7 - x \cdot 7 + 4(-2x) - x(-2x) \\ &= 28 - 7x - 8x + 2x^2 \\ &= 28 - 15x + 2x^2 \end{aligned}$$

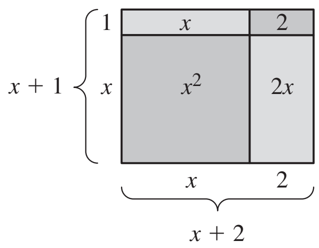
47.
$$\begin{aligned} \left(t + \frac{3}{2}\right)\left(t + \frac{4}{3}\right) &= \left(t + \frac{3}{2}\right)t + \left(t + \frac{3}{2}\right)\left(\frac{4}{3}\right) \\ &= t \cdot t + \frac{3}{2} \cdot t + t \cdot \frac{4}{3} + \frac{3}{2} \cdot \frac{4}{3} \\ &= t^2 + \frac{3}{2}t + \frac{4}{3}t + 2 \\ &= t^2 + \frac{9}{6}t + \frac{8}{6}t + 2 \\ &= t^2 + \frac{17}{6}t + 2 \end{aligned}$$

49. $\left(\frac{1}{4}a+2\right)\left(\frac{3}{4}a-1\right)$
$= \left(\frac{1}{4}a+2\right)\left(\frac{3}{4}a\right) + \left(\frac{1}{4}a+2\right)(-1)$
$= \frac{1}{4}a\left(\frac{3}{4}a\right) + 2\cdot\frac{3}{4}a + \frac{1}{4}a(-1) + 2(-1)$
$= \frac{3}{16}a^2 + \frac{3}{2}a - \frac{1}{4}a - 2$
$= \frac{3}{16}a^2 + \frac{6}{4}a - \frac{1}{4}a - 2$
$= \frac{3}{16}a^2 + \frac{5}{4}a - 2$

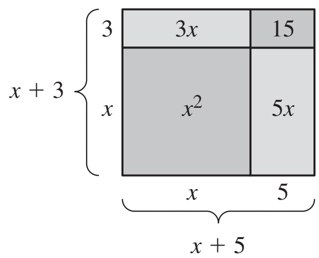
51. Illustrate $x(x+5)$ as the area of a rectangle with width x and length $x+5$.



53. Illustrate $(x+1)(x+2)$ as the area of a rectangle with width $x+1$ and length $x+2$.



55. Illustrate $(x+5)(x+3)$ as the area of a rectangle with length $x+5$ and width $x+3$.



57. $(x^2-x+3)(x+1)$
$= (x^2-x+3)x + (x^2-x+3)1$
$= x^3 - x^2 + 3x + x^2 - x + 3$
$= x^3 + 2x + 3$

A partial check can be made by selecting a convenient replacement for x, say 1, and comparing the values of the original expression and the result.

$(1^2-1+3)(1+1)$	$1^3+6\cdot 1+3$
$=(1-1+3)(1+1)$	$=1+6+3$
$=3\cdot 2$	$=6$
$=6$	

Since the value of both expressions is 6, the multiplication is very likely correct.

59. $(2a+5)(a^2-3a+2)$
$= (2a+5)a^2 - (2a+5)(3a) + (2a+5)2$
$= 2a\cdot a^2 + 5\cdot a^2 - 2a\cdot 3a - 5\cdot 3a + 2a\cdot 2 + 5\cdot 2$
$= 2a^3 + 5a^2 - 6a^2 - 15a + 4a + 10$
$= 2a^3 - a^2 - 11a + 10$

A partial check can be made as in Exercise 57.

61. $(y^2-7)(3y^4+y+2)$
$= (y^2-7)(3y^4) + (y^2-7)y + (y^2-7)(2)$
$= y^2\cdot 3y^4 - 7\cdot 3y^4 + y^2\cdot y - 7\cdot y + y^2\cdot 2 - 7\cdot 2$
$= 3y^6 - 21y^4 + y^3 - 7y + 2y^2 - 14$
$= 3y^6 - 21y^4 + y^3 + 2y^2 - 7y - 14$

A partial check can be made as in Exercise 57.

63. $(3x+2)(7x+4x+1) = (3x+2)(11x+1)$
$= (3x+2)(11x) + (3x+2)(1)$
$= 3x\cdot 11x + 2\cdot 11x + 3x\cdot 1 + 2\cdot 1$
$= 33x^2 + 22x + 3x + 2$
$= 33x^2 + 25x + 2$

65.
$$\begin{array}{rrrrrl}
 & x^2 & +\ 5x & -\ 1 & & \text{Line up like terms} \\
 & x^2 & -\ \ x & +\ 3 & & \text{in columns} \\ \hline
 & 3x^2 & +\ 15x & -\ 3 & & \text{Multiplying by 3} \\
-\ x^3 & -\ 5x^2 & +\ \ x & & & \text{Multiplying by } -x \\
x^4 + 5x^3 & -\ \ x^2 & & & & \text{Multiplying by } x^2 \\ \hline
x^4 + 4x^3 & -\ 3x^2 & +\ 16x & -\ 3 & &
\end{array}$$

A partial check can be made as in Exercise 57.

67.
$$\begin{array}{rl}
5t^2 - t + \frac{1}{2} & \\
2t^2 + t - 4 & \\ \hline
-20t^2 + 4t - 2 & \text{Multiplying by } -4 \\
5t^3 - t^2 + \frac{1}{2}t \quad & \text{Multiplying by } t \\
10t^4 - 2t^3 + t^2 \qquad\quad & \text{Multiplying by } 2t^2 \\ \hline
10t^4 + 3t^3 - 20t^2 + \frac{9}{2}t - 2 &
\end{array}$$

A partial check can be made as in Exercise 57.

69. We will multiply horizontally while still aligning like terms.

$(x+1)(x^3+7x^2+5x+4)$

$$\begin{array}{rrrrrl}
= x^4 & +\ 7x^3 & +\ 5x^2 & +\ 4x & & \text{Multiplying by } x \\
 & +\ x^3 & +\ 7x^2 & +\ 5x & +\ 4 & \text{Multiplying by 1} \\
= x^4 & +\ 8x^3 & +\ 12x^2 & +\ 9x & +\ 4 &
\end{array}$$

A partial check can be made as in Exercise 57.

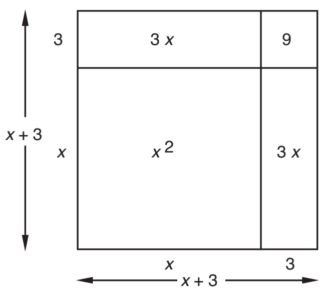
71. *Writing Exercise.* No; the distributive law is the basis for polynomial multiplication.

73. $(9-3)(9+3)+3^2-9^2 = (6)(12)+3^2-9^2$
$= (6)(12)+9-81 = 72+9-81 = 0$

75. $5+\dfrac{7+4+2\cdot 5}{7}$
$= 5+\dfrac{7+4+10}{7}$
$= 5+\dfrac{21}{7}$
$= 5+3$
$= 8$

77. $(4+3\cdot 5+5)\div 3\cdot 4$
$= (4+15+5)\div 3\cdot 4$
$= 24\div 3\cdot 4$
$= 8\cdot 4$
$= 32$

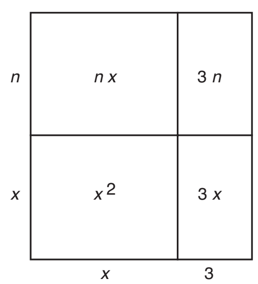
79. *Writing Exercise.* $(A+B)(C+D)$ will be a trinomial when there is exactly one pair of like terms among AC, AD, BC, and BD.



Then we see that the area of the figure is $(x+3)^2$, or $x^2+3x+3x+9 \neq x^2+9$.

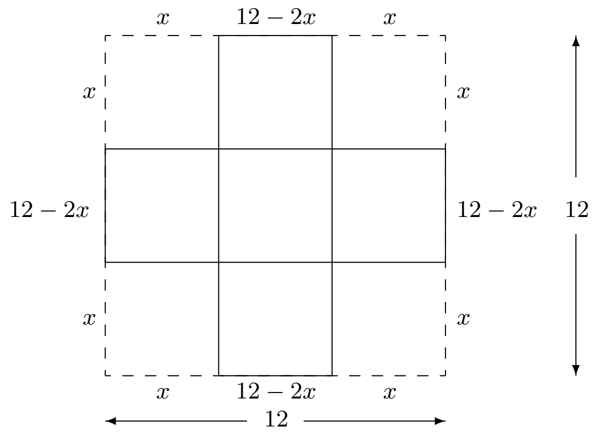
81. The shaded area is the area of the large rectangle, $6y(14y-5)$ less the area of the unshaded rectangle, $3y(3y+5)$. We have:
$6y(14y-5)-3y(3y+5)$
$= 84y^2-30y-9y^2-15y$
$= 75y^2-45y$

83. Let n = the missing number.



The area of the figure is $x^2+3x+nx+3n$. This is equivalent to $x^2+8x+15$, so we have $3x+nx=8x$ and $3n=15$. Solving either equation for n, we find that the missing number is 5.

85.



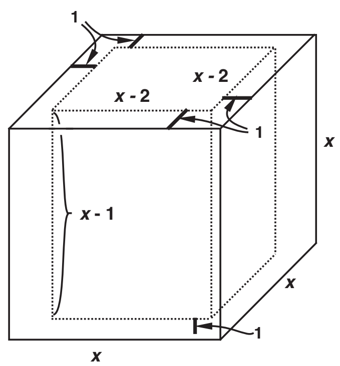
The dimensions, in inches, of the box are $12-2x$ by $12-2x$ by x. The volume is the product of the dimensions (volume = length × width × height):

$$\begin{aligned}\text{Volume} &= (12-2x)(12-2x)x\\ &= (144-48x+4x^2)x\\ &= (144x-48x^2+4x^3)\ \text{in}^3,\ \text{or}\\ &\quad\ (4x^3-48x^2+144x)\ \text{in}^3\end{aligned}$$

The outside surface area is the sum of the area of the bottom and the areas of the four sides. The dimensions, in inches, of the bottom are $12-2x$ by $12-2x$, and the dimensions, in inches, of each side are x by $12-2x$.

$$\begin{aligned}\begin{matrix}\text{Surface}\\ \text{area}\end{matrix} &= \text{Area of bottom} + \\ &\qquad 4\cdot\text{Area of each side}\\ &= (12-2x)(12-2x)+4\cdot x(12-2x)\\ &= 144-24x-24x+4x^2+48x-8x^2\\ &= 144-48x+4x^2+48x-8x^2\\ &= (144-4x^2)\ \text{in}^2,\ \text{or}\ (-4x^2+144)\ \text{in}^2\end{aligned}$$

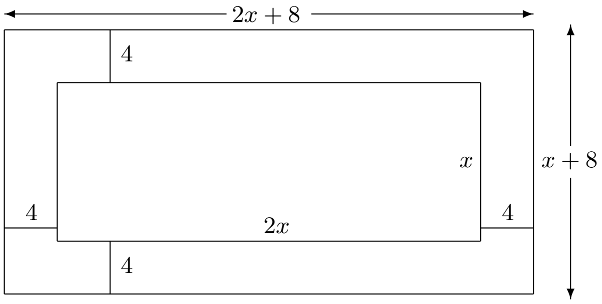
87.



The interior dimensions of the open box are $x-2$ cm by $x-2$ cm by $x-1$ cm.

$$\begin{aligned}\text{Interior volume} &= (x-2)(x-2)(x-1)\\ &= (x^2-4x+4)(x-1)\\ &= (x^3-5x^2+8x-4)\ \text{cm}^3\end{aligned}$$

89. Let x = the width of the garden. Then $2x$ = the length of the garden.



$$\underbrace{\text{Area of garden and sidewalk together}}_{(2x+8)(x+8)} \ \underset{=}{\text{is}} \ \underbrace{\text{Area of garden alone}}_{2x\cdot x} \ + \ \underbrace{256\text{ ft}^2}_{256}$$

$$(2x+8)(x+8) = 2x\cdot x + 256$$

$$2x^2+24x+64 = 2x^2+256$$
$$24x = 192$$
$$x = 8$$

The dimensions are 8 ft by 16 ft.

91. $(x-2)(x-7)-(x-7)(x-2)$

First observe that, by the commutative law of multiplication, $(x-2)(x-7)$ and $(x-7)(x-2)$ are equivalent expressions. Then when we subtract $(x-7)(x-2)$ from $(x-2)(x-7)$, the result is 0.

93. $(x+2)(x+4)(x-5)$
$= (x^2+2x+4x+8)(x-5)$
$= (x^2+6x+8)(x-5)$
$= x^3+6x^2+8x-5x^2-30x-40$
$= x^3+x^2-22x-40$

95. $(x-a)(x-b)\cdots(x-x)(x-y)(x-z)$
$= (x-a)(x-b)\cdots 0\cdot(x-y)(x-z)$
$= 0$

Exercise Set 4.6

1. It is true that FOIL is simply a memory device for finding the product of two binomials.

3. This statement is false. See Example 2(d).

5. $(x^2+2)(x+3)$
F O I L
$= x^2\cdot x + x^2\cdot 3 + 2\cdot x + 2\cdot 3$
$= x^3+2x+3x^2+6$, or x^3+3x^2+2x+6

7. $(t^4-2)(t+7)$
F O I L
$= t^4\cdot t + t^4\cdot 7 - 2\cdot t - 2\cdot 7$
$= t^5+7t^4-2t-14$

9. $(y+2)(y-3)$
F O I L
$= y\cdot y + y\cdot(-3) + 2\cdot y + 2\cdot(-3)$
$= y^2-3y+2y-6$
$= y^2-y-6$

11. $(3x+2)(3x+5)$
F O I L
$= 3x\cdot 3x + 3x\cdot 5 + 2\cdot 3x + 2\cdot 5$
$= 9x^2+15x+6x+10$
$= 9x^2+21x+10$

13. $(5x-3)(x+4)$
F O I L
$= 5x\cdot x + 5x\cdot 4 - 3\cdot x - 3\cdot 4$
$= 5x^2+20x-3x-12$
$= 5x^2+17x-12$

15. $(3-2t)(5-t)$
F O I L
$= 3\cdot 5 + 3\cdot t - 2t\cdot 5 + (-2t)(-t)$
$= 15-3t-10t+2t^2$
$= 15-13t+2t^2$

17. $(x^2+3)(x^2-7)$
F O I L
$= x^2\cdot x^2 - x^2\cdot 7 + 3\cdot x^2 - 3\cdot 7$
$= x^4-7x^2+3x^2-21$
$= x^4-4x^2-21$

19. $\left(p-\frac{1}{4}\right)\left(p+\frac{1}{4}\right)$
F O I L
$= p\cdot p + p\cdot\frac{1}{4} + \left(-\frac{1}{4}\right)\cdot p + \left(-\frac{1}{4}\right)\cdot\frac{1}{4}$
$= p^2+\frac{1}{4}p-\frac{1}{4}p-\frac{1}{16}$
$= p^2-\frac{1}{16}$

21. $(x-0.3)(x-0.3)$
F O I L
$= x\cdot x - x\cdot 0.3 + -0.3\cdot x + (-0.3)(-0.3)$
$= x^2-0.3x-0.3x+0.09$
$= x^2-0.6x+0.09$

23. $(-3n+2)(n+7)$
F O I L
$= -3n\cdot n - 3n\cdot 7 + 2\cdot n + 2\cdot 7$
$= -3n^2-21n+2n+14$
$= -3n^2-19n+14$

25. $(x+10)(x+10)$
F O I L
$= x^2 + 10x + 10x + 100$
$= x^2+10x+10x+100$
$= x^2+20x+100$

27. $(1-3t)(1+5t^2)$
F O I L
$= 1 + 5t^2 - 3t - 15t^3$
$= 1-3t-5t^2-15t^3$

29. $(x^2+3)(x^3-1)$
F O I L
$= x^5 - x^2 + 3x^3 - 3$, or $x^5+3x^3-x^2-3$

31. $(3x^2-2)(x^4-2)$

$\quad$ F $\quad$ O $\quad$ I $\quad$ L

$= 3x^6 - 6x^2 - 2x^4 + 4$, or $3x^6-2x^4-6x^2+4$

33. $(2t^3+5)(2t^3+5)$

$\quad$ F $\quad$ O $\quad$ I $\quad$ L

$= 4t^6 + 10t^3 + 10t^3 + 25$

$= 4t^6+20t^3+25$

35. $(8x^3+5)(x^2+2)$

$\quad$ F $\quad$ O $\quad$ I $\quad$ L

$= 8x^5 + 16x^3 + 5x^2 + 10$

37. $(10x^2+3)(10x^2-3)$

$\quad$ F $\quad$ O $\quad$ I $\quad$ L

$= 100x^4 - 30x^2 + 30x^2 - 9$

$= 100x^4-9$

39. $(x+8)(x-8)$ Product of sum and difference of the same two terms

$= x^2-8^2$

$= x^2-64$

41. $(2x+1)(2x-1)$ Product of sum and difference of the same two terms

$= (2x)^2-1^2$

$= 4x^2-1$

43. $(5m^2+4)(5m^2-4)$ Product of sum and difference of the same two terms

$= (5m)^2-4^2$

$= 25m^4-16$

45. $(9a^3+1)(9a^3-1)$

$= (9a^3)^2-1^2$

$= 81a^6-1$

47. $(x^4+0.1)(x^4-.01)$

$= (x^4)^2-0.1^2$

$= x^8-0.01$

49. $\left(t-\dfrac{3}{4}\right)\left(t+\dfrac{3}{4}\right)$

$= t^2-\left(\dfrac{3}{4}\right)^2$

$= t^2-\dfrac{9}{16}$

51. $(x+3)^2$

$= x^2+2\cdot x\cdot 3+3^2$ Square of a binomial

$= x^2+6x+9$

53. $(7x^3-1)^2$ Square of a binomial

$= (7x^3)^2-2\cdot 7x^3\cdot 1+(-1)^2$

$= 49x^6-14x^3+1$

55. $\left(a-\dfrac{2}{5}\right)^2$ Square of a binomial

$= a^2-2\cdot a\cdot\dfrac{2}{5}+\left(\dfrac{2}{5}\right)^2$

$= a^2-\dfrac{4}{5}a+\dfrac{4}{25}$

57. $(t^4+3)^2$ Square of a binomial

$= (t^4)^2+2\cdot t^4\cdot 3+3^2$

$= t^8+6t^4+9$

59. $(2-3x^4)^2 = 2^2-2\cdot 2\cdot 3x^4+(3x^4)^2$

$= 4-12x^4+9x^8$

61. $(5+6t^2)^2 = 5^2+2\cdot 5\cdot 6t^2+(6t^2)^2$

$= 25+60t^2+36t^4$

63. $(7x-0.3)^2 = (7x)^2-2(7x)(0.3)+(0.3)^2$

$= 49x^2-4.2x+0.09$

65. $7n^3(2n^2-1)$

$= 7n^3\cdot 2n^2-7n^3\cdot 1$ Multiplying each term of the binomial by the monomial

$= 14n^5-7n^3$

67. $(a-3)(a^2+2a-4)$

$= a^3 + 2a^2 - 4a$ Multiplying horizontally

$\quad - 3a^2 - 6a + 12$ and aligning like terms

$= a^3 - a^2 - 10a + 12$

69. $(7-3x^4)(7-3x^4)$

$= 7^2-2\cdot 7\cdot 3x^4+(-3x^4)^2$ Squaring a binomial

$= 49-42x^4+9x^8$

71. $5x(x^2+6x-2)$

$= 5x\cdot x^2+5x\cdot 6x+5x(-2)$ Multiplying each term of the trinomial by the monomial

$= 5x^3+30x^2-10x$

73. $(q^5+1)(q^5-1)$

$= (q^5)^2-1^2$

$= q^{10}-1$

75. $3t^2(5t^3-t^2+t)$

$= 3t^2\cdot 5t^3+3t^2(-t^2)+3t^2\cdot t$ Multiplying each term of the trinomial by the monomial

$= 15t^5-3t^4+3t^3$

77. $(6x^4-3x)^2$ Squaring a binomial

$= (6x^4)^2-2\cdot 6x^4\cdot 3x+(-3x^2)$

$= 36x^8-36x^5+9x^2$

79. $(9a+0.4)(2a^3+0.5)$ Product of two binomials; use FOIL

$= 9a\cdot 2a^3+9a\cdot 0.5+0.4\cdot 2a^3+0.4\cdot 0.5$

$= 18a^4+4.5a+0.8a^3+0.2$, or

$18a^4+0.8a^3+4.5a+0.2$

81. $\left(\dfrac{1}{5}-6x^4\right)\left(\dfrac{1}{5}+6x^4\right)$

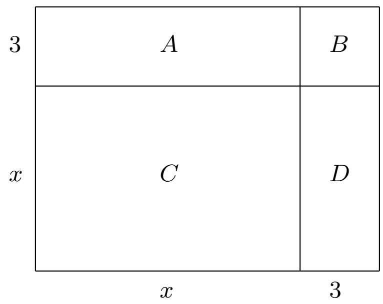
$= \left(\dfrac{1}{5}\right)^2-(6x^4)^2$

$= \dfrac{1}{25}-36x^8$

83. $(a+1)(a^2-a+1)$

$$\begin{array}{ll} = a^3 - a^2 + a & \text{Multiplying horizontally} \\ \quad\;\; a^2 - a + 1 & \text{and aligning like terms} \\ = a^3 \qquad\quad + 1 & \end{array}$$

85.



We can find the shaded area in two ways.

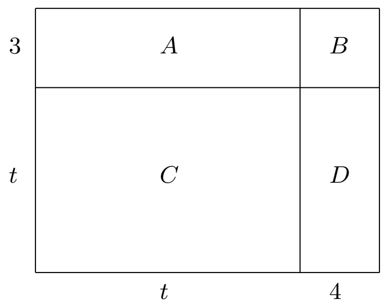
Method 1: The figure is a square with side $x+3$, so the area is $(x+3)^2 = x^2+6x+9$.

Method 2: We add the areas of A, B, C, and D.

$3 \cdot x + 3 \cdot 3 + x \cdot x + x \cdot 3 = 3x + 9 + x^2 + 3x$
$= x^2 + 6x + 9$.

Either way we find that the total shaded area is $x^2 + 6x + 9$.

87.



We can find the shaded area in two ways.

Method 1: The figure is a rectangle with dimensions $t+3$ by $t+4$, so the area is

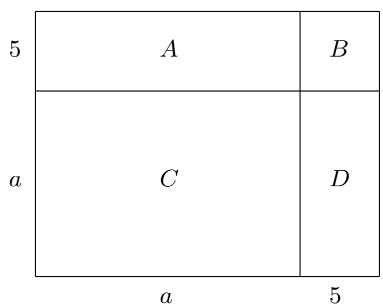
$(t+3)(t+4) = t^2 + 4t + 3t + 12 = t^2 + 7t + 12$.

Method 2: We add the areas of A, B, C, and D.

$3 \cdot t + 3 \cdot 4 + t \cdot t + t \cdot 4 = 3t + 12 + t^2 + 4t = t^2 + 7t + 12$.

Either way, we find that the area is $t^2 + 7t + 12$.

89.



We can find the shaded area in two ways.

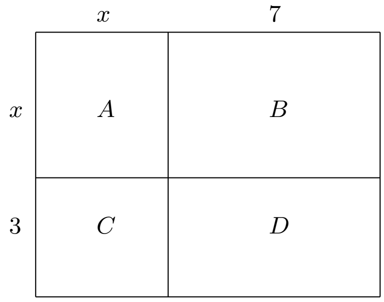
Method 1: The figure is a square with side $a+5$, so the area is $(a+5)^2 = a^2 + 10a + 25$.

Method 2: We add the areas of A, B, C, and D.

$5 \cdot a + 5 \cdot 5 + a \cdot a + 5 \cdot a = 5a + 25 + a^2 + 5a = a^2 + 10a + 25$.

Either way, we find that the total shaded area is $a^2 + 10a + 25$.

91.



We can find the shaded area in two ways.

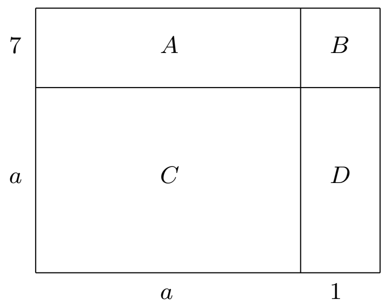
Method 1: The figure is a rectangle with dimensions $x+7$ by $x+3$, so the area is $(x+7)(x+3)$
$= x^2 + 3x + 7x + 21 = x^2 + 10x + 21$.

Method 2: We add the areas of A, B, C, and D.

$x \cdot x + x \cdot 7 + 3 \cdot x + 3 \cdot 7 = x^2 + 10x + 21$.

Either way, we find that the total shaded area is $x^2 + 10x + 21$.

93.



We can find the shaded area in two ways.

Method 1: The figure is a rectangle with dimensions $a+1$ by $a+7$, so the area is

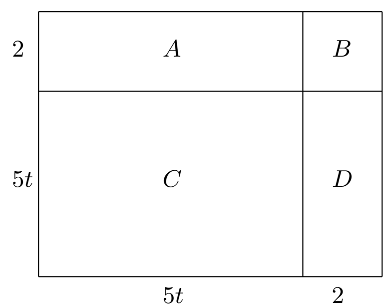
$(a+1)(a+7) = a^2 + 7a + a + 7 = a^2 + 8a + 7$

Method 2: We add the areas of A, B, C, and D.

$a \cdot a + a \cdot 1 + 7 \cdot a + 7 \cdot 1 = a^2 + a + 7a + 7 = a^2 + 8a + 7$.

Either way, we find that the total shaded area is $a^2 + 8a + 7$.

95.



We can find the shaded area in two ways.

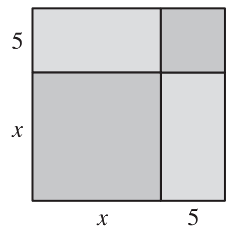
Method 1: The figure is a square with side $5t+2$, so the area is $(5t+2)^2 = 25t^2+20t+4$.

Method 2: We add the areas of A, B, C, and D.

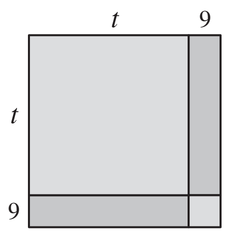
$5t\cdot 5t+5t\cdot 2+2\cdot 5t+2\cdot 2 = 25t^2+10t+10t+4 = 25t^2+20t+4$.

Either way, we find that the total shaded area is $25t^2+20t+4$.

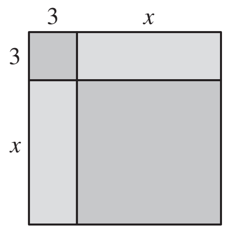
97. We draw a square with side $x+5$.



99. We draw a square with side $t+9$.



101. We draw a square with side $3+x$.



103. *Writing Exercise.* It's a good idea to study the other special products, because they allow for faster computations than the FOIL method.

105. ***Familiarize***. Let w = the energy, in kilowatt-hours per month, used by the washing machine. Then $21w$ = the amount of energy used by the refrigerator, and $11w$ = the amount of energy used by the freezer.

Translate.

$$\begin{array}{ccccccc} \underbrace{\text{Washing Machine}} & \text{and} & \underbrace{\text{refrigerator}} & \text{and} & \underbrace{\text{freezer}} & \text{is} & \underbrace{\text{Total energy}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ w & + & 21w & + & 11w & = & 189 \end{array}$$

Solve. We solve the equation.

$$w+21t+11w = 297$$
$$33w = 297$$
$$w = 9$$

Then $21w = 21\cdot 9 = 189$

and $11w = 11\cdot 9 = 99$

Check. The energy used by the refrigerator, 189 kWh, is 21 times the energy used by the washing machine. The energy used by the freezer, 99 kWh is 11 times the energy used by the washing machine. Also, $9+189+99 = 297$, the total energy used.

State. The washing machine used 9 kWh, the refrigerator used 189 kWh, and the freezer used 99 kWh.

107. $5xy = 8$

$y = \dfrac{8}{5x}$ Dividing both sides by $5x$

109. $ax - by = c$

$ax = by + c$ Adding by to both sides

$x = \dfrac{by+c}{a}$ Dividing both sides by a

111. *Writing Exercise.* The computation $(20-1)(20+1) = 400-1 = 399$ is easily performed mentally and is equivalent to the computation $19\cdot 21$.

113. $(4x^2+9)(2x+3)(2x-3)$
$= (4x^2+9)(4x^2-9)$
$= 16x^4 - 81$

115. $(3t-2)^2(3t+2)^2$
$= [(3t-2)(3t+2)]^2$
$= (9t^2-4)^2$
$= 81t^4 - 72t^2 + 16$

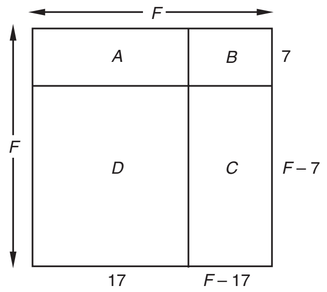
117. $(t^3-1)^4(t^3+1)^4$
$= [(t^3-1)(t^3+1)]^4$
$= (t^6-1)^4$
$= [(t^6-1)^2]^2$
$= (t^{12}-2t^6+1)^2$
$= (t^{12}-2t^6+1)(t^{12}-2t^6+1)$
$= t^{24}-2t^{18}+t^{12}-2t^{18}+4t^{12}-2t^6+$
$t^{12}-2t^6+1$
$= t^{24}-4t^{18}+6t^{12}-4t^6+1$

119. $18\times 22 = (20-2)(20+2) = 20^2-2^2$
$= 400-4 = 396$

121.
$$(x+2)(x-5) = (x+1)(x-3)$$
$$x^2-5x+2x-10 = x^2-3x+x-3$$
$$x^2-3x-10 = x^2-2x-3$$
$$-3x-10 = -2x-3 \quad \text{Adding } -x^2$$
$$-3x+2x = 10-3 \quad \text{Adding } 2x \text{ and } 10$$
$$-x = 7$$
$$x = -7$$

The solution is -7.

123.



The area of the entire figure is F^2. The area of the unshaded region, C, is $(F-7)(F-17)$. Then one expression for the area of the shaded region is
$F^2-(F-7)(F-17)$.

To find a second expression we add the areas of regions A, B, and D. We have:

$17\cdot 7+7(F-17)+17(F-7)$
$=119+7F-119+17F-119$
$=24F-119$

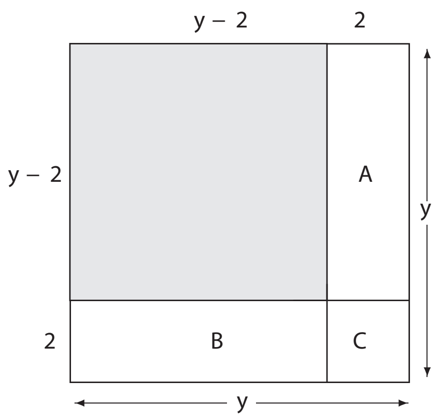
It is possible to find other equivalent expressions also.

125. The dimensions of the shaded area, regions A and D together, are $y+1$ by $y-1$ so the area is $(y+1)(y-1)$.

To find another expression we add the areas of regions A and D. The dimensions of region A are y by $y-1$, and the dimensions of region D are $y-1$ by 1, so the sum of the areas is $y(y-1)+(y-1)(1)$, or $y(y-1)+y-1$.

It is possible to find other equivalent expressions also.

127.



The shaded area is $(y-2)^2$. We find it as follows:

$$\begin{array}{ccccccccc}
\text{Shaded area} & = & \text{Area of square} & - & \text{Area of } A & - & \text{Area of } B & - & \text{Area of } C \\
(y-2)^2 & = & y^2 & - & 2(y-2) & - & 2(y-2) & - & 2\cdot 2
\end{array}$$

$(y-2)^2=y^2-2y+4-2y+4-4$
$(y-2)^2=y^2-4y+4$

129.

4.6 Connecting the Concepts

1. $(3x^2-2x+6)+(5x-3)$ Addition
$=3x^2-2x+5x+6-3$
$=3x^2+3x+3$

3. $6x^3(8x^2-7)$ Multiplication
$=48x^5-42x^3$

5. $(9x^3-7x+3)-(5x^2-10)$ Subtraction
$=9x^3-7x+3-5x^2+10$
$=9x^3-5x^2-7x+13$

7. $(9x+1)(9x-1)$ Multiplication
$=(9x)^2-1^2$
$=81x^2-1$ $(A+B)(A-B)=A^2-B^2$

9. $(4x^2-x-7)-(10x^2-3x+5)$
$=4x^2-x-7-10x^2+3x-5$
$=-6x^2+2x-12$

11. $8x^5(5x^4-6x^3+2)=40x^9-48x^8+16x^5$

13. $(2m-1)^2=(2m)^2-2(2m)(1)+1^2$
$=4m^2-4m+1$

15. $(5x^3-6x^2-2x)+(6x^2+2x+3)$
$=5x^2-6x^2-2x+6x^2+2x+3$
$=5x^3+3$

17. $(4y^3+7)^2=(4y^3)^2+2\cdot 4y^3\cdot 7+7^2$
$=16y^6+56y^3+49$

19. $(4t^2-5)(4t^2+5)$
$=(4t^2)^2-5^2$ $(A+B)(A-B)=A^2-B^2$
$=16t^4-25$

Exercise Set 4.7

1. $(3x+5y)^2$ is the square of a binomial, choice (a).

3. $(5a+6b)(-6b+5a)$, or $(5a+6b)(5a-6b)$ is the product of the sum and difference of the same two terms, choice (b).

5. $(r-3s)(5r+3s)$ is neither the square of a binomial nor the product of the sum and difference of the same two terms, so choice (c) is appropriate.

7. $(4x-9y)(4x-9y)$, or $(4x-9y)^2$ is the square of a binomial, choice (a).

9. We replace x by 5 and y by -2.
$x^2-2y^2+3xy=5^2-2(-2)^2+3\cdot 5(-2)$
$=25-8-30.$
$=-13.$

11. We replace x by 2, y by -3, and z by -4.

$xy^2z - z = 2(-3)^2(-4) - (-4) = -72 + 4 = -68$

13. Evaluate the polynomial for $h = 160$ and $A = 20$.

$$\begin{aligned}&0.041h - 0.018A - 2.69\\&= 0.041(160) - 0.018(20) - 2.69\\&= 6.56 - 0.36 - 2.69\\&= 3.51\end{aligned}$$

The woman's lung capacity is 3.51 liters.

15. Evaluate the polynomial for $w = 125$, $h = 64$, and $a = 27$.

$$\begin{aligned}&917 + 6w + 6h - 6a\\&= 917 + 6(125) + 6(64) - 6(27)\\&= 917 + 750 + 384 - 162\\&= 1889\end{aligned}$$

The daily caloric needs are 1889 calories.

17. Evaluate the polynomial for $h = 7$, $r = 1\frac{1}{2} = \frac{3}{2}$, and $\pi \approx 3.14$.

$$\begin{aligned}2\pi rh + \pi r^2 &\approx 2(3.14)\left(\frac{3}{2}\right)(7) + 3.14\left(\frac{3}{2}\right)^2\\&\approx 65.94 + 7.065\\&\approx 73.005\end{aligned}$$

The surface area is about 73.005 in^2.

19. Evaluate the polynomial for $h = 50$, $v = 18$, and $t = 2$.

$$\begin{aligned}&h + vt - 4.9t^2\\&= 50 + 18 \cdot 2 - 4.9(2)^2\\&= 50 + 36 - 19.6\\&= 66.4\end{aligned}$$

The ball will be 66.4 m above the ground 2 seconds after it is thrown.

21. $3x^2y + 5xy + 2y^2 - 11$

Term	Coefficient	Degree	
$3x^2y$	3	3	
$-5xy$	-5	2	
$2y^2$	2	2	
-11	-11	0	(Think: $-11 = -11x^0$)

The degree of the polynomial is the degree of the term of highest degree. The term of highest degree is $3x^2y$. Its degree is 3, so the degree of the polynomial is 3.

23. $7 - abc + a^2b + 9ab^2$

Term	Coefficient	Degree
7	7	0
$-abc$	-1	3
a^2b	1	3
$9ab^2$	9	3

The terms of highest degree are $-abc$, a^2b and $9ab^2$. Each has degree 3. The degree of the polynomial is 3.

25. $3r + s - r - 7s = (3-1)r + (1-7)s = 2r - 6s$

27. $5xy^2 - 2x^2y + x + 3x^2$

There are no like terms, so none of the terms can be combined.

29.
$$\begin{aligned}&6u^2v - 9uv^2 + 3vu^2 - 2v^2u + 11u^2\\&= (6+3)u^2v + (-9-2)uv^2 + 11u^2\\&= 9u^2v - 11uv^2 + 11u^2\end{aligned}$$

31.
$$\begin{aligned}&5a^2c - 2ab^2 + a^2b - 3ab^2 + a^2c - 2ab^2\\&= (5+1)a^2c + (-2-3-2)ab^2 + a^2b\\&= 6a^2c - 7ab^2 + a^2b\end{aligned}$$

33.
$$\begin{aligned}&(6x^2 - 2xy + y^2) + (5x^2 - 8xy - 2y^2)\\&= (6+5)x^2 + (-2-8)xy + (1-2)y^2\\&= 11x^2 - 10xy - y^2\end{aligned}$$

35.
$$\begin{aligned}&(3a^4 - 5ab + 6ab^2) - (9a^4 + 3ab - ab^2)\\&= 3a^4 - 5ab + 6ab^2 - 9a^4 - 3ab + ab^2 \quad \text{Adding the opposite}\\&= (3-9)a^4 + (-5-3)ab + (6+1)ab^2\\&= -6a^4 - 8ab + 7ab^2\end{aligned}$$

37. $(5r^2 - 4rt + t^2) + (-6r^2 - 5rt - t^2) + (-5r^2 + 4rt - t^2)$

Observe that the polynomials $5r^2 - 4rt + t^2$ and $-5r^2 + 4rt - t^2$ are opposites. Thus, their sum is 0 and the sum in the exercise is the remaining polynomial, $-6r^2 - 5rt - t^2$.

39.
$$\begin{aligned}&(x^3 - y^3) - (-2x^3 + x^2y - xy^2 + 2y^3)\\&= x^3 - y^3 + 2x^3 - x^2y + xy^2 - 2y^3\\&= 3x^3 - 3y^3 - x^2y + xy^2, \text{ or}\\&\quad 3x^3 - x^2y + xy^2 - 3y^3\end{aligned}$$

41.
$$\begin{aligned}&(2y^4x^3 - 3y^3x) + (5y^4x^3 - y^3x) - (9y^4x^3 - y^3x)\\&= (2+5-9)y^4x^3 + (-3-1+1)y^3x\\&= -2y^4x^3 - 3y^3x\end{aligned}$$

43.
$$\begin{aligned}&(4x + 5y) + (-5x + 6y) - (7x + 3y)\\&= 4x + 5y - 5x + 6y - 7x - 3y\\&= (4-5-7)x + (5+6-3)y\\&= -8x + 8y\end{aligned}$$

45.
$$\begin{aligned}(4c-d)(3c+2d) &= \overset{F}{12c^2} + \overset{O}{8cd} - \overset{I}{3cd} - \overset{L}{2d^2}\\&= 12c^2 + 5cd - 2d^2\end{aligned}$$

47.
$$\begin{aligned}(xy-1)(xy+5) &= \overset{F}{x^2y^2} + \overset{O}{5xy} - \overset{I}{xy} - \overset{L}{5}\\&= x^2y^2 + 4xy - 5\end{aligned}$$

49. $(2a-b)(2a+b) \quad [(A+B)(A-B) = A^2 - B^2]$
$= 4a^2 - b^2$

51.
$$\begin{aligned}(5rt-2)(4rt-3) &= \overset{F}{20r^2t^2} - \overset{O}{15rt} - \overset{I}{8rt} + \overset{L}{6}\\&= 20r^2t^2 - 23rt + 6\end{aligned}$$

53. $(m^3n + 8)(m^3n - 6)$
$$\begin{aligned}&= \overset{F}{m^6n^2} - \overset{O}{6m^3n} + \overset{I}{8m^3n} - \overset{L}{48}\\&= m^6n^2 + 2m^3n - 48\end{aligned}$$

55. $(6x - 2y)(5x - 3y)$
$$\begin{aligned}&= \overset{F}{30x^2} - \overset{O}{18xy} - \overset{I}{10xy} + \overset{L}{6y^2}\\&= 30x^2 - 28xy + 6y^2\end{aligned}$$

57. $(pq + 0.1)(-pq + 0.1)$
$= (0.1 + pq)(0.1 - pq)$ $[(A + B)(A - B) = A^2 - B^2]$
$= 0.01 - p^2q^2$

59. $(x + h)^2$
$= x^2 + 2xh + h^2$ $[(A + B)^2 = A^2 + 2AB + B^2]$

61. $(4a - 5b)^2$
$= 16a^2 - 40ab + 25b^2$ $[(A-B)^2 = A^2 - 2AB + B^2]$

63. $(ab + cd^2)(ab - cd^2) = (ab)^2 - (cd^2)^2$
$= a^2b^2 - c^2d^4$

65. $(2xy + x^2y + 3)(xy + y^2)$
$= (2xy + x^2y + 3)(xy) + (2xy + x^2y + 3)(y^2)$
$= 2x^2y^2 + x^3y^2 + 3xy + 2xy^3 + x^2y^3 + 3y^2$

67. $(a + b - c)(a + b + c)$
$= [(a + b) - c][(a + b) + c]$
$= (a + b)^2 - c^2$
$= a^2 + 2ab + b^2 - c^2$

69. $[a + b + c][a - (b + c)]$
$= [a + (b + c)][a - (b + c)]$
$= a^2 - (b + c)^2$
$= a^2 - (b^2 + 2bc + c^2)$
$= a^2 - b^2 - 2bc - c^2$

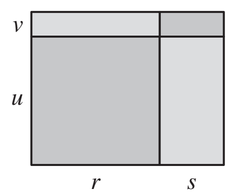
71. The figure is a square with side $x + y$. Thus the area is $(x + y)^2 = x^2 + 2xy + y^2$.

73. The figure is a triangle with base $ab + 2$ and height $ab - 2$. Its area is $\frac{1}{2}(ab + 2)(ab - 2) = \frac{1}{2}(a^2b^2 - 4) = \frac{1}{2}a^2b^2 - 2$.

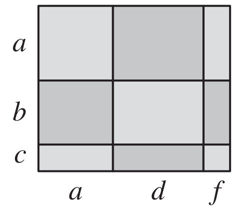
75. The figure is a rectangle with dimensions $a + b + c$ by $a + d + c$. Its area is
$(a + b + c)(a + d + c)$
$= [(a + c) + b][(a + c) + d]$
$= (a + c)^2 + (a + c)d + b(a + c) + bd$
$= a^2 + 2ac + c^2 + ad + cd + ab + bc + bd$

77. The figure is a parallelogram with base $m - n$ and height $m + n$. Its area is $(m - n)(m + n) = m^2 - n^2$.

79. We draw a rectangle with dimensions $r + s$ by $u + v$.



81. We draw a rectangle with dimensions $a+b+c$ by $a+d+f$.



83. *Writing Exercise.* Yes; consider $a + b + c + d$. This is a polynomial in 4 variables but it has degree 1.

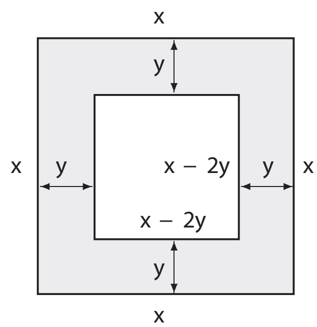
85.
$$\begin{array}{r} x^2 - 3x - 7 \\ -(\quad +5x - 3) \\ \hline x^2 - 8x - 4 \end{array}$$

87.
$$\begin{array}{r} 3x^2 + x + 5 \\ -(3x^2 + 3x\ ) \\ \hline -2x + 5 \end{array}$$

89.
$$\begin{array}{r} 5x^3 - 2x^2 + 1 \\ -(5x^3 - 15x^2\ ) \\ \hline 13x^2 + 1 \end{array}$$

91. *Writing Exercise.* The leading term of a polynomial is the term of highest degree. When a polynomial has several variables it is possible that more than one term has the highest degree as in Exercise 23.

93. It is helpful to add additional labels to the figure.



The area of the large square is $x \cdot x$, or x^2. The area of the small square is $(x - 2y)(x - 2y)$, or $(x - 2y)^2$.

Area of shaded region = Area of large square − Area of small square

Area of shaded region = $x^2 - (x - 2y)^2$
$= x^2 - (x^2 - 4xy + 4y^2)$
$= x^2 - x^2 + 4xy - 4y^2$
$= 4xy - 4y^2$

95. The unshaded region is a circle with radius $a - b$. Then the shaded area is the area of a circle with radius a less the area of a circle with radius $a - b$. Thus, we have:

$$\begin{aligned}\text{Shaded area} &= \pi a^2 - \pi(a-b)^2\\ &= \pi a^2 - \pi(a^2-2ab+b^2)\\ &= \pi a^2 - \pi a^2 + 2\pi ab - \pi b^2\\ &= 2\pi ab - \pi b^2\end{aligned}$$

97. The figure can be thought of as a cube with side x, a rectangular solid with dimensions x by x by y, a rectangular solid with dimensions x by y by y, and a rectangular solid with dimensions y by y by $2y$. Thus the volume is

$x^3 + x\cdot x\cdot y + x\cdot y\cdot y + y\cdot y\cdot 2y$, or

$x^3 + x^2y + xy^2 + 2y^3$.

99. The surface area of the solid consists of the surface area of a rectangular solid with dimensions x by x by h less the areas of 2 circles with radius r plus the lateral surface area of a right circular cylinder with radius r and height h. Thus, we have

$2x^2 + 2xh + 2xh - 2\pi r^2 + 2\pi rh$, or

$2x^2 + 4xh - 2\pi r^2 + 2\pi rh$.

101. *Writing Exercise.* The height of the observatory is 40 ft and its radius is 30/2, or 15 ft, so the surface area is $2\pi rh + \pi r^2 \approx 2(3.14)(15)(40) + (3.14)(15)^2 \approx 4474.5\text{ ft}^2$. Since $4474.5\text{ ft}^2/250\text{ ft}^2 = 17.898$, 18 gallons of paint should be purchased.

103. For the formula $= 2*A4 + 3*B4$, we substitute 5 for $A4$ and 10 for $B4$.

$$\begin{aligned}= 2*A4 + 3*B4 &= 2\cdot 5 + 3\cdot 10\\ &= 10 + 30\\ &= 40\end{aligned}$$

The value of $D4$ is 40.

105. Replace t with 2 and multiply.

$$\begin{aligned}&P(1+r)^2\\ &= P(1+2r+r^2)\\ &= P + 2Pr + Pr^2\end{aligned}$$

107. Substitute \$10,400 for P, 8.5%, or 0.085 for r, and 5 for t.

$$\begin{aligned}&P(1+r)^t\\ &= \$10,400(1+0.085)^5\\ &\approx \$15,638.03\end{aligned}$$

Exercise Set 4.8

1. $$\begin{aligned}\frac{40x^6-25x^3}{5} &= \frac{40x^6}{5} - \frac{25x^3}{5}\\ &= \frac{40}{5}x^6 - \frac{25}{5}x^3 \quad \text{Dividing coefficients}\\ &= 8x^6 - 5x^3\end{aligned}$$

To check, we multiply the quotient by 5:

$(8x^6 - 5x^3)5 = 40x^6 - 25x^3$.

The answer checks.

3. $$\begin{aligned}&\frac{u-2u^2+u^7}{u}\\ &= \frac{u}{u} - \frac{2u^2}{u} + \frac{u^7}{u}\\ &= 1 - 2u + u^6\end{aligned}$$

Check: We multiply.

$u(1-2u+u^6) = u - 2u^2 + u^7$

5. $$\begin{aligned}&(18t^3 - 24t^2 + 6t) \div (3t)\\ &= \frac{18t^3-24t^2+6t}{3t}\\ &= \frac{18t^3}{3t} - \frac{24t^2}{3t} + \frac{6t}{3t}\\ &= 6t^2 - 8t + 2\end{aligned}$$

Check: We multiply.

$3t(6t^2 - 8t + 2) = 18t^3 - 24t^2 + 6t$

7. $$\begin{aligned}&(42x^5 - 36x^3 + 9x^2) \div (6x^2)\\ &= \frac{42x^5 - 36x^3 + 9x^2}{6x^2}\\ &= \frac{42x^5}{6x^2} - \frac{36x^3}{6x^2} + \frac{9x^2}{6x^2}\\ &= 7x^3 - 6x + \frac{3}{2}\end{aligned}$$

Check: We multiply.

$$6x^2\left(7x^3 - 6x + \frac{3}{2}\right) = 42x^5 - 36x^3 + 9x^2$$

9. $$\begin{aligned}&(32t^5 + 16t^4 - 8t^3) \div (-8t^3)\\ &= \frac{32t^5 + 16t^4 - 8t^3}{-8t^3}\\ &= \frac{32t^5}{-8t^3} + \frac{16t^4}{-8t^3} - \frac{8t^3}{-8t^3}\\ &= -4t^2 - 2t + 1\end{aligned}$$

Check: We multiply.

$-8t^3(-4t^2 - 2t + 1) = 32t^5 + 16t^4 - 8t^3$

11. $$\begin{aligned}&\frac{8x^2 - 10x + 1}{2x}\\ &= \frac{8x^2}{2x} - \frac{10x}{2x} + \frac{1}{2x}\\ &= 4x - 5 + \frac{1}{2x}\end{aligned}$$

Check: We multiply.

$$2x\left(4x - 5 + \frac{1}{2x}\right) = 8x^2 - 10x + 1$$

13. $$\begin{aligned}&\frac{5x^3y + 10x^5y^2 + 15x^2y}{5x^2y}\\ &= \frac{5x^3y}{5x^2y} + \frac{10x^5y^2}{5x^2y} + \frac{15x^2y}{5x^2y}\\ &= x + 2x^3y + 3\end{aligned}$$

Check: We multiply.

$5x^2y(x + 2x^3y + 3) = 5x^3y + 10x^5y^2 + 15x^2y$

15. $\dfrac{9r^2s^2+3r^2s-6rs^2}{-3rs}$

$= \dfrac{9r^2s^2}{-3rs}+\dfrac{3r^2s}{-3rs}-\dfrac{6rs^2}{-3rs}$

$= -3rs-r+2s$

Check: We multiply.

$-3rs(-3rs-r+2s)=9r^2s^2+3r^2s-6rs^2$

17.

$$\begin{array}{r l}
x-6 & \\
x-2\,\overline{)\,x^2+8x+12} & \\
\underline{x^2-2x} & \\
-6x+12 & \leftarrow (x^2+8x)-(x^2-2x)=-6x \\
\underline{-6x+12} & \\
0 & \leftarrow (-6x+12)-(-6x+12)=0
\end{array}$$

The answer is $x-6$.

19.

$$\begin{array}{r l}
t-5 & \\
t-5\,\overline{)\,t^2-10t-20} & \\
\underline{t^2-5t} & \\
-5t-20 & \leftarrow (t^2-10t)-(t^2-5t)= \\
 & \quad -5t \\
\underline{-5t+25} & \\
-45 & \leftarrow (-5t-20)-(-5t+25)= \\
 & \quad -45
\end{array}$$

The answer is $t-5+\dfrac{-45}{t-5}$.

21.

$$\begin{array}{r l}
2x-1 & \\
x+6\,\overline{)\,2x^2+11x-5} & \\
\underline{2x^2+12x} & \\
-x-5 & \leftarrow (2x^2+11x)-(2x^2+12x)=-x \\
\underline{-x-6} & \\
1 & \leftarrow (-x-5)-(-x-6)=1
\end{array}$$

The answer is $2x-1+\dfrac{1}{x+6}$.

23.

$$\begin{array}{r l}
t^2-3t+9 & \\
t+3\,\overline{)\,t^3+0t^2+0t+27} & \leftarrow \text{Writing in the missing terms} \\
\underline{t^3+3t^2} & \\
-3t^2+0t & \leftarrow t^3-(t^3+3t^2)=-3t^2 \\
\underline{-3t^2-9t} & \\
9t+27 & \leftarrow -3t^2-(-3t^2-9t)=9t \\
\underline{9t+27} & \\
0 & \leftarrow (9t+27)=-(9t+27)=0
\end{array}$$

The answer is t^2-3t+9.

25.

$$\begin{array}{r l}
a+5 & \\
a-5\,\overline{)\,a^2+0a-21} & \leftarrow \text{Writing in the missing term} \\
\underline{a^2-5a} & \\
5a-21 & \leftarrow a^2-(a^2-5a)=5a \\
\underline{5a-25} & \\
4 & \leftarrow (5a^2-21)-(5a-25)=4
\end{array}$$

The answer is $a+5+\dfrac{4}{a-5}$.

27.

$$\begin{array}{r l}
x-3 & \\
5x-1\,\overline{)\,5x^2-16x+0} & \leftarrow \text{Writing in the missing term} \\
\underline{5x^2-x} & \\
-15x+0 & \leftarrow (5x^2-16x)-(5x^2-x)= \\
 & \quad -15x \\
\underline{-15x+3} & \\
-3 & \leftarrow (-15x+0)-(-15x+3)=-3
\end{array}$$

The answer is $x-3-\dfrac{3}{5x-1}$.

29.

$$\begin{array}{r l}
3a+1 & \\
2a+5\,\overline{)\,6a^2+17a+8} & \\
\underline{6a^2+15a} & \\
2a+8 & \leftarrow (6a^2+17a)-(6a^2+15a)=2a \\
\underline{2a+5} & \\
3 & \leftarrow (2a+8)-(2a+5)=3
\end{array}$$

The answer is $3a+1+\dfrac{3}{2a+5}$.

31.

$$\begin{array}{r l}
t^2-3t+1 & \\
2t-3\,\overline{)\,2t^3-9t^2+11t-3} & \\
\underline{2t^3-3t^2} & \\
-6t^2+11t & \leftarrow (2t^3-9t^2)-(2t^3-3t^2)= \\
 & \quad -6t^2 \\
\underline{-6t^2+9t} & \\
2t-3 & \leftarrow (-6t^2+11t)- \\
 & \quad (-6t^2+9t)=2t \\
\underline{2t-3} & \\
0 & \leftarrow (2t-3)-(2t-3)=0
\end{array}$$

The answer is t^2-3t+1.

33.

$$\begin{array}{r l}
x^2\qquad+1 & \\
x-1\,\overline{)\,x^3-x^2+x-1} & \\
\underline{x^3-x^2} & \\
x-1 & \leftarrow (x^3-x^2)-(x^3-x^2)=0 \\
\underline{x-1} & \\
0 & \leftarrow (x-1)-(x-1)=0
\end{array}$$

The answer is x^2+1.

35.

$$\begin{array}{r l}
t^2\qquad-1 & \\
t^2+5\,\overline{)\,t^4+0t^3+4t^2+3t-6} & \leftarrow \text{Writing in the missing term} \\
\underline{t^4\qquad+5t^2} & \\
-t^2+3t-6 & \leftarrow (t^4-4t^2) \\
\underline{-t^2\qquad-5} & \quad -(t^4+5t^2)=-t^2 \\
3t-1 & \leftarrow (-t^2+3t-6) \\
 & \quad -(-t^2-5)=3t-1
\end{array}$$

The answer is $t^2-1+\dfrac{3t-1}{t^2+5}$.

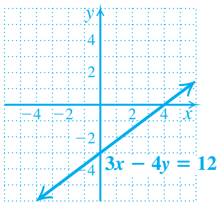
37.

$$\begin{array}{r l}
3x^2\qquad-3 & \\
2x^2+1\,\overline{)\,6x^4+0x^3-3x^2+x-4} & \leftarrow \text{Writing in the missing term} \\
\underline{6x^4\qquad+3x^2} & \\
-6x^2+x-4 & \leftarrow (6x^4-3x^2)- \\
\underline{-6x^2\qquad-3} & \quad (6x^4+3x^2)=-6x^2 \\
x-1 & \leftarrow (-6x^2+x-4) \\
 & \quad -(-6x^2-3)=x-1
\end{array}$$

The answer is $3x^2-3+\dfrac{x-1}{2x^2+1}$.

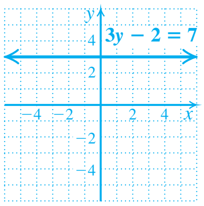
39. *Writing Exercise.* The distributive law is used in each step when a term of the quotient is multiplied by the divisor and when the subtraction is performed. The distributive law is also used when the quotient is checked.

41. $3x - 4y = 12$



43. $3y - 2 = 7$

$y = 3$



45. $m = \dfrac{5-2}{-7-3} = \dfrac{-3}{10}$

47. $y = -5x - 10$

49. *Writing Exercise.* Find the product of $x-5$ and a binomial of the form $ax+b$. Then add 3 to this result.

51.
$$\begin{aligned}
&(10x^{9k} - 32x^{6k} + 28x^{3k}) \div (2x^{3k})\\
&= \frac{10x^{9k} - 32x^{6k} + 28x^{3k}}{2x^{3k}}\\
&= \frac{10x^{9k}}{2x^{3k}} - \frac{32x^{6k}}{2x^{3k}} + \frac{28x^{3k}}{2x^{3k}}\\
&= 5x^{9k-3k} - 16x^{6k-3k} + 14x^{3k-3k}\\
&= 5x^{6k} - 16x^{3k} + 14
\end{aligned}$$

53.
$$\begin{array}{r l}
 & 3t^{2h} + 2t^h - 5\\
2t^h + 3 & \overline{\big)\, 6t^{3h} + 13t^{2h} - 4t^h - 15}\\
 & \underline{6t^{3h} + 9t^{2h}}\\
 & \qquad 4t^{2h} - 4t^h\\
 & \qquad \underline{4t^{2h} + 6t^h}\\
 & \qquad\qquad -10t^h - 15\\
 & \qquad\qquad \underline{-10t^h - 15}\\
 & \qquad\qquad\qquad 0
\end{array}$$

The answer is $3t^{2h} + 2t^h - 5$.

55.
$$\begin{array}{r l}
 & a + 3\\
5a^2 - 7a - 2 & \overline{\big)\, 5a^3 + 8a^2 - 23a - 1}\\
 & \underline{5a^3 - 7a^2 - 2a}\\
 & \qquad 15a^2 - 21a - 1\\
 & \qquad \underline{15a^2 - 21a - 6}\\
 & \qquad\qquad 5
\end{array}$$

The answer is $a + 3 + \dfrac{5}{5a^2 - 7a - 2}$.

57.
$$\begin{aligned}
&(4x^5 - 14x^3 - x^2 + 3) +\\
&\qquad (2x^5 + 3x^4 + x^3 - 3x^2 + 5x)\\
&= 6x^5 + 3x^4 - 13x^3 - 4x^2 + 5x + 3
\end{aligned}$$

$$\begin{array}{r l}
 & 2x^2 + x - 3\\
3x^3 - 2x - 1 & \overline{\big)\, 6x^5 + 3x^4 - 13x^3 - 4x^2 + 5x + 3}\\
 & \underline{6x^5 \qquad - 4x^3 - 2x^2}\\
 & \qquad 3x^4 - 9x^3 - 2x^2 + 5x\\
 & \qquad \underline{3x^4 \qquad - 2x^2 - x}\\
 & \qquad\qquad -9x^3 \qquad + 6x + 3\\
 & \qquad\qquad \underline{-9x^3 \qquad + 6x + 3}\\
 & \qquad\qquad\qquad 0
\end{array}$$

The answer is $2x^2 + x - 3$.

59.
$$\begin{array}{r l}
 & x - 3\\
x - 1 & \overline{\big)\, x^2 - 4x + c}\\
 & \underline{x^2 - x}\\
 & \quad -3x + c\\
 & \quad \underline{-3x + 3}\\
 & \qquad c - 3
\end{array}$$

We set the remainder equal to 0.

$c - 3 = 0$

$c = 3$

Thus, c must be 3.

61.
$$\begin{array}{r l}
 & c^2x + (2c + c^2)\\
x - 1 & \overline{\big)\, c^2x^2 + 2cx + 1}\\
 & \underline{c^2x^2 - c^2x}\\
 & (2c + c^2)x + 1\\
 & \underline{(2c + c^2)x - (2c + c^2)}\\
 & \qquad 1 + (2c + c^2)
\end{array}$$

We set the remainder equal to 0.

$c^2 + 2c + 1 = 0$

$(c+1)^2 = 0$

$c + 1 = 0 \quad \textit{or} \quad c + 1 = 0$

$c = -1 \quad \textit{or} \quad c = -1$

Thus, c must be -1.

Chapter 4 Review

1. True; see page 255 in the text.

3. True; see page 272 in the text.

5. False; the degree of the polynomial is the degree of the leading term.

7. True; see page 283 in the text.

9. $n^3 \cdot n^8 \cdot n = n^{3+8+1} = n^{12}$

11. $t^6 \cdot t^0 = t^{6+0} = t^6$

13. $\dfrac{(a+b)^4}{(a+b)^4} = 1$

15. $\left(-2xy^2\right)^3 = (-2)^3 x^{1\cdot 3} y^{2\cdot 3} = -8x^3y^6$

17. $\left(a^2b\right)(ab)^5 = a^2b \cdot a^5b^5 = a^{2+5}b^{1+5} = a^7b^6$

19. $8^{-6} = \dfrac{1}{8^6}$

21. $4^5 \cdot 4^{-7} = 4^{5-7} = 4^{-2} = \dfrac{1}{4^2}$ or $\dfrac{1}{16}$

23. $\left(w^3\right)^{-5} = w^{3(-5)} = w^{-15} = \dfrac{1}{w^{15}}$

25. $\left(\dfrac{2x}{y}\right)^{-3} = \dfrac{2^{-3}x^{-3}}{y^{-3}} = \dfrac{y^3}{8x^3}$

27. $0.0000109 = 1.09 \times 10^m$
To write 1.09 as 0.0000109, we move the decimal point 5 places to the left. Thus, m is -5 and
$0.0000109 = 1.09 \times 10^{-5}$

29.
$$\begin{aligned} \frac{1.28 \times 10^{-8}}{2.5 \times 10^{-4}} &= \frac{1.28}{2.5} \times \frac{10^{-8}}{10^{-4}} \\ &= \frac{1.28}{2.5} \times 10^{-8+4} \\ &= 0.512 \times 10^{-4} \\ &= 5.12 \times 10^{-5} \end{aligned}$$

31. $-4y^5 + 7y^2 - 3y - 2$
The terms are $-4y^5$, $7y^2$, $-3y$, and -2.

33. $7n^4 - \dfrac{5}{6}n^2 - 4n + 10$
The coefficients are 7, $-\dfrac{5}{6}$, -4, and 10.

35. $-2x^5 + 7 + 3x^2 + x$

a)

Term	$-2x^5$	7	$-3x^2$	x
Degree	5	0	2	1

b) the term of highest degree is $-2x^5$. This is the leading term. Then the leading coefficient is -2 since $-2x^2 = -2 \cdot x^5$.

c) Since the term of highest degree is $-2x^5$, the degree of the polynomial is 5.

37. The polynomial $4 - 9t^3 - 7t^4 + 10t^2$ has four monomials so it is a polynomial with no special name.

39. $3x - x^2 + 4x = -x^2 + (3+4)\,x$
$= -x^2 + 7x$

41. $-4t^3 + 2t + 4t^3 + 8 - t - 9$
$= (-4+4)\,t^3 + (2-1)\,t + (8-9)$
$= t - 1$

43. For $x = -2$:
$$\begin{aligned} &9x - 6 \\ &= 9(-2) - 6 \\ &= -18 - 6 \\ &= -24 \end{aligned}$$

45. $\left(8x^4 - x^3 + x - 4\right) + \left(x^5 + 7x^3 - 3x - 5\right)$
$= 8x^4 - x^3 + x - 4 + x^5 + 7x^3 - 3x - 5$
$= x^5 + 8x^4 + (-1+7)\,x^3 + (1-3)\,x + (-4-5)$
$= x^5 + 8x^4 + 6x^3 - 2x - 9$

47. $\left(y^2 + 8y - 7\right) - \left(4y^2 - 10\right)$
$= y^2 + 8y - 7 - 4y^2 + 10$
$= (1-4)\,y^2 + 8y + (-7+10)$
$= -3y^2 + 8y + 3$

49.
$$\begin{array}{rrrrr} -\frac{3}{4}x^4 & +\frac{1}{2}x^3 & & & +\frac{7}{8} \\ & -\frac{1}{4}x^3 & -\ x^2 & -\frac{7}{4}x & \\ +\frac{3}{2}x^4 & & +\frac{2}{3}x^2 & & -\frac{1}{2} \\ \hline +\frac{3}{4}x^4 & +\frac{1}{4}x^3 & -\frac{1}{3}x^2 & -\frac{7}{4}x & +\frac{3}{8} \end{array}$$

51. Let $w =$ the width, then $w + 3$ is the length, in meters.

a) Recall that the perimeter of a rectangle is the sum of all sides, so
$$\begin{aligned} \text{perimeter} &= 2w + 2(w+3) = 2w + 2w + 6 \\ &= 4w + 6. \end{aligned}$$

b) Recall that the area of a rectangle is the product of the length and the width. So,
$$\text{area} = w(w+3) = w^2 + 3w.$$

53. $(7x+1)^2 = (7x)^2 + 2 \cdot 7x \cdot 1 + 1^2 \qquad (A+B)^2 = A + 2AB + B^2$
$= 49x^2 + 14x + 1$

55. $(d-8)(d+8) = d^2 - 8^2 \qquad (A+B)(A-B) = A^2 - B^2$
$= d^2 - 64$

57. $(x-8)^2 = x^2 - 2 \cdot x \cdot 8 + 8^2 \qquad (A-B)^2 = A - 2AB + B^2$
$= x^2 - 16x + 64$

59. $(2a+9)(2a-9) = (2a)^2 - 9^2 \qquad (A+B)(A-B) = A^2 - B$
$= 4a^2 - 81$

61. $\left(x^4 - 2x + 3\right)\left(x^3 + x - 1\right)$
$= x^4\left(x^3 + x - 1\right) - 2x\left(x^3 + x - 1\right) + 3\left(x^3 + x - 1\right)$
$= x^7 + x^5 - x^4 - 2x^4 - 2x^2 + 2x + 3x^3 + 3x - 3$
$= x^7 + x^5 - 3x^4 + 3x^3 - 2x^2 + 5x - 3$

63. $(2t^2 + 3)(t^2 - 7)$
F O I L
$= 2t^4 - 14t^2 + 3t^2 - 21$
$= 2t^4 - 11t^2 - 21$

65. $(-7 + 2n)(7 + 2n) = (2n - 7)(2n + 7)$
$= (2n)^2 - 7^2 \qquad (A+B)(A-B) = A^2 - B^2$
$= 4n^2 - 49$

67. $x^5y - 7xy + 9x^2 - 8$

Term	Coefficient	Degree
x^5y	1	6
$-7xy$	-7	2
$9x^2$	9	2
-8	-8	0

The term of highest degree is x^5y. Its degree is 6, so the degree of the polynomial is 6.

69. $u + 3v - 5u + v - 7$
$= (1-5)\,u + (3+1)\,v - 7$
$= -4u + 4v - 7$

71. $\left(4a^2 - 10ab - b^2\right) + \left(-2a^2 - 6ab + b^2\right)$
$= 4a^2 - 10ab - b^2 - 2a^2 - 6ab + b^2$
$= (4-2)\,a^2 + (-10-6)\,ab + (-1+1)\,b^2$
$= 2a^2 - 16ab$

73. $(2x + 5y)(x - 3y)$
F O I L
$= 2x^2 - 6xy + 5xy - 15y^2$
$= 2x^2 - xy - 15y^2$

75. The figure is a triangle with base $x + y$ and height $x - y$. Its area is
$\frac{1}{2}\,(x+y)\,(x-y) = \frac{1}{2}\left(x^2 - y^2\right) = \frac{1}{2}x^2 - \frac{1}{2}y^2.$

77.
$$\begin{array}{r|l} & 3x^2 - 7x + 4 \\ \hline 2x+3 & 6x^3 - 5x^2 - 13x + 13 \\ & \underline{6x^3 + 9x^2} \\ & \quad -14x^2 - 13x \\ & \quad \underline{-14x^2 - 21x} \\ & \qquad\qquad 8x + 13 \\ & \qquad\qquad \underline{8x + 12} \\ & \qquad\qquad\qquad 1 \end{array}$$

The answer is $3x^2 - 7x + 4 + \dfrac{1}{2x+3}$.

79. *Writing Exercise.* In the expression $5x^3$, the exponent refers only to the x. In the expression $(5x)^3$, the entire expression $5x$ is the base.

81. a) For $\left(x^5 - 6x^2 + 3\right)\left(x^4 + 3x^3 + 7\right)$, the highest terms of each factor are x^5 and x^4. Their product is x^9, which is degree 9. Thus, the degree of the product is 9.
b) For $\left(x^7 - 4\right)^4$, the term of highest degree is x^7, which is degree 7. Taking that term to the fourth power results in a term of degree 28.

83. Let $c =$ the coefficient of x^3.
Let $2c =$ the coefficient of x^4.
Let $2c - 3 =$ the coefficient of x.
Let $2c - 3 - 7 =$ the constant $\cdot$ (the remaining term)
The coefficient of x^2 is 0.

Solve:
$$\begin{aligned} 2c + c + 0 + 2c - 3 + 2c - 3 - 7 &= 15 \\ 7c - 13 &= 15 \\ 7c &= 28 \\ c &= 4 \end{aligned}$$

Coefficient of x^3, $c = 4$
Coefficient of x^4, $2c = 2 \cdot 4 = 8$
Coefficient of x^2, 0
Coefficient of x, $2c - 3 = 2 \cdot 4 - 3 = 8 - 3 = 5$
Constant, $2c - 3 - 7 = 2 \cdot 4 - 10 = 8 - 10 = -2$
The polynomial is $8x^4 + 4x^3 + 5x - 2$.

85.
$$\begin{aligned} (x-7)(x+10) &= (x-4)(x-6) \\ x^2 + 10x - 7x - 70 &= x^2 - 6x - 4x + 24 \\ x^2 + 3x - 70 &= x^2 - 10x + 24 \\ 3x - 70 &= -10x + 24 && \text{Subtracting } x^2 \\ 3x &= -10x + 94 && \text{Adding 70} \\ 13x &= 94 && \text{Adding } 10x \\ x &= \frac{94}{13} \end{aligned}$$

The solution is $\dfrac{94}{13}$

Chapter 4 Test

1. $x^7 \cdot x \cdot x^5 = x^{7+1+5} = x^{13}$

3. $\dfrac{(3m)^4}{(3m)^4} = 1$

5.
$$\begin{aligned} (5x^4y)(-2x^5y^3)^3 &= (5x^4y)(-2)^3(x^5y^3)^3 \\ &= 5x^4y \cdot (-8)x^{5\cdot 3}y^{3\cdot 3} \\ &= 5x^4y \cdot (-8) \cdot x^{15}y^9 \\ &= 5(-8)x^{4+15}y^1 + 9 \\ &= -40x^{19}y^{10} \end{aligned}$$

7. $y^{-7} = \dfrac{1}{y^7}$

9. $t^{-4} \cdot t^{-5} = t^{-4+(-5)} = t^{-9} = \frac{1}{t^9}$

11.
$$\begin{aligned} (2a^3b^{-1})^{-4} &= 2^{-4}(a^3)^{-4}(b^{-1})^{-4} \\ &= 2^{-4} \cdot a^{-12} \cdot b^4 \\ &= \frac{1}{2^4} \cdot \frac{1}{a^{12}} \cdot b^4 \\ &= \frac{b^4}{16a^{12}} \end{aligned}$$

13. $3,060,000,000 = 3.06 \times 10^9$

15.
$$\begin{aligned} \frac{5.6 \times 10^6}{3.2 \times 10^{-11}} &= \frac{5.6}{3.2} \times \frac{10^6}{10^{-11}} \\ &= 1.75 \times 10^{6-(-11)} \\ &= 1.75 \times 10^{17} \end{aligned}$$

17. Two monomials are added so $4x^2y - 7y^3$ is a binomial.

19. $2t^3 - t + 7t^5 + 4$
The degrees of the terms are $3, 1, 5, 0$; the leading term is $7t^5$; the leading coefficient is 7; the degree of the polynomial is 5.

21. $4a^2 - 6 + a^2 = (4+1)a^2 - 6 = 5a^2 - 6$

23. $3 - x^2 + 8x + 5x^2 - 6x - 2x + 4x^3$
$= 4x^3 + (-1+5)x^2 + (8-6-2)x + 3$
$= 4x^3 + 4x^2 + 3$

25. $\left(x^4 + \dfrac{2}{3}x + 5\right) + \left(4x^4 + 5x^2 + \dfrac{1}{3}x\right)$
$= (1+4)x^4 + 5x^2 + \left(\dfrac{2}{3}x + \dfrac{1}{3}\right)x + 5$
$= 5x^4 + 5x^2 + x + 5$

27. $(t^3 - 0.3t^2 - 20) - (t^4 - 1.5t^3 + 0.3t^2 - 11)$
$= t^3 - 0.3t^2 - 20 - t^4 + 1.5t^3 - 0.3t^2 + 11$
$= -t^4 + (1 + 1.5)t^3 + (-0.3 - 0.3)t^2$
$+(-20 + 11)$
$= -t^4 + 2.5t^3 - 0.6t^2 - 9$

29. $$\left(x - \frac{1}{3}\right)^2 = x^2 - 2 \cdot x \cdot \frac{1}{3} + \frac{1}{3}^2$$
$$= x^2 - \frac{2}{3}x + \frac{1}{9}$$

31. $$(3b + 5)(2b - 1) = 6b^2 - 3b + 10b - 5$$
$$= 6b^2 - 7b - 5$$

33. $$(8 - y)(6 + 5y) = 48 + 40y - 6y - 5y^2$$
$$= 48 + 34y - 5y^2$$

35. $$(8a^3 + 3)^2 = (8a^3)^2 + 2(8a^3)(3) + 3^2$$
$$= 64a^6 + 48a^3 + 9$$

37. $2x^3y - y^3 + xy^3 + 8 - 6x^3y - x^2y^2 + 11$
$= (2 - 6)x^3y - x^2y^2 + xy^3 - y^3 + (8 + 11)$
$= -4x^3y - x^2y^2 + xy^3 - y^3 + 19$

39. $$(3x^5 - y)(3x^5 + y) = (3x^5)^2 - y^2$$
$$= 9x^{10} - y^2$$

41. $\frac{1}{5^6} = 5^{-6}$

43.
$$\begin{aligned}
x^2 + (x - 7)(x + 4) &= 2(x - 6)^2 \\
x^2 + x^2 + 4x - 7x - 28 &= 2\left[x^2 - 2 \cdot x \cdot 6 + 6^2\right] \\
x^2 + x^2 + 4x - 7x - 28 &= 2\left[x^2 - 12x + 36\right] \\
2x^2 - 3x - 28 &= 2x^2 - 24x + 72 \\
-3x - 28 &= -24x + 72 \\
21x - 28 &= 72 \\
21x &= 100 \\
x &= \frac{100}{21}
\end{aligned}$$

45. ***Familiarize.*** We need to multiply to determine the hours wasted. There are 60 sec in a minute and 60 min in an hr. 1 sec $= \frac{1}{60} \cdot \frac{1}{60} = \frac{1}{3600}$hr. So 4 sec $= \frac{4}{3600}$hr

Translate. We multiply the number of spam emails by the time wasted on each spam email. 12.4 billion $= 1.24 \times 10^{10}$.

Carry out
$s = 1.24 \times 10^{10} \cdot \frac{4}{3600} \approx 1.4 \times 10^7$hr

Check. We recalculate to check our solution. The answer checks.

State. About 1.4×10^7hr each day are wasted due to spam.