Chapter 5

Polynomials and Factoring

Exercise Set 5.1

1. Since $7a \cdot 5ab = 35a^2b$, choice (h) is most appropriate.

3. $5x + 10 = 5(x + 2)$ and $4x + 8 = 4(x + 2)$, so $x + 2$ is a common factor of $5x+10$ and $4x+8$ and choice (b) is most appropriate.

5. $3x^2(3x^2-1) = 9x^4-3x^2$, so choice (c) is most appropriate.

7. $3a+6a^2 = 3a(1+2a)$, so $1+2a$ is a factor of $3a+6a^2$ and choice (d) is most appropriate.

9. Answers may vary. $14x^3 = (14x)(x^2) = (7x^2)(2x) = (-2)(-7x^3)$

11. Answers may vary. $-15a^4 = (-15)(a^4) = (-5a)(3a^3) = (-3a^2)(5a^2)$

13. Answers may vary. $25t^5 = (5t^2)(5t^3) = (25t)(t^4) = (-5t)(-5t^4)$

15. $8x + 24 = 8 \cdot x + 8 \cdot 3$
$= 8(x + 3)$

17. $6x - 30 = 6 \cdot x - 6 \cdot 5$
$= 6(x - 5)$

19. $2x^2 + 2x - 8 = 2 \cdot x^2 + 2 \cdot x - 2 \cdot 4$
$= 2(x^2 + x - 4)$

21. $3t^2 + t = t \cdot 3t + t \cdot 1$
$= t(3t + 1)$

23. $-5y^2 - 10y = -5y \cdot y - 5y \cdot 2$
$= -5y(y + 2)$

25. $x^3 + 6x^2 = x^2 \cdot x + x^2 \cdot 6$
$= x^2(x + 6)$

27. $16a^4 - 24a^2 = 8a^2 \cdot 2a^2 - 8a^2 \cdot 3$
$= 8a^2(2a^2 - 3)$

29. $-6t^6 + 9t^4 - 4t^2 = -t^2 \cdot 6t^4 - t^2\left(-9t^2\right) - t^2 \cdot 4$
$= -t^2\left(6t^4 - 9t^2 + 4\right)$

31. $6x^8 + 12x^6 - 24x^4 + 30x^2$
$= 6x^2 \cdot x^6 + 6x^2 \cdot 2x^4 - 6x^2 \cdot 4x^2 + 6x^2 \cdot 5$
$= 6x^2(x^6 + 2x^4 - 4x^2 + 5)$

33. $x^5y^5 + x^4y^3 + x^3y^3 - x^2y^2$
$= x^2y^2 \cdot x^3y^3 + x^2y^2 \cdot x^2y + x^2y^2 \cdot xy - x^2y^2 \cdot 1$
$= x^2y^2(x^3y^3 + x^2y + xy - 1)$

35. $-35a^3b^4 + 10a^2b^3 - 15a^3b^2$
$= -5a^2b^2 \cdot 7ab^2 - 5a^2b^2(-2b) - 5a^2b^2 \cdot 3a$
$= -5a^2b^2\left(7ab^2 - 2b + 3a\right)$

37. $n(n-6) + 3(n-6)$
$= (n-6)(n+3)$ Factoring out the common binomial factor $n - 6$

39. $x^2(x + 3) - 7(x + 3)$
$= (x + 3)(x^2 - 7)$ Factoring out the common binomial factor $x + 3$

41. $y^2(2y-9) + (2y-9)$
$= y^2(2y-9) + 1(2y-9)$
$= (2y-9)\left(y^2+1\right)$ Factoring out the common factor $2y - 9$

43. $x^3 + 2x^2 + 5x + 10$
$= \left(x^3 + 2x^2\right) + (5x + 10)$
$= x^2(x+2) + 5(x+2)$ Factoring each binomial
$= (x+2)\left(x^2+5\right)$ Factoring out the common factor $x + 2$

45. $5a^3 + 15a^2 + 2a + 6$
$= (5a^3 + 15a^2) + (2a + 6)$
$= 5a^2(a + 3) + 2(a + 3)$ Factoring each binomial
$= (a + 3)(5a^2 + 2)$ Factoring out the common factor $a + 3$

47. $9n^3 - 6n^2 + 3n - 2$
$= 3n^2(3n-2) + 1(3n-2)$
$= (3n-2)\left(3n^2+1\right)$

49. $4t^3 - 20t^2 + 3t - 15$
$= 4t^2(t - 5) + 3(t - 5)$
$= (t - 5)(4t^2 + 3)$

51. $7x^3 + 5x^2 - 21x - 15 = x^2(7x+5) - 3(7x+5)$
$= (7x+5)\left(x^2-3\right)$

53. $6a^3 + 7a^2 + 6a + 7 = a^2(6a+7) + 1(6a+7)$
$= (6a+7)\left(a^2+1\right)$

55. $2x^3 + 12x^2 - 5x - 30 = 2x^2(x+6) - 5(x+6)$
$= (x+6)\left(2x^2-5\right)$

57. We try factoring by grouping.
$p^3 + p^2 - 3p + 10 = p^2(p + 1) - (3p - 10)$, or
$p^3 - 3p + p^2 + 10 = p(p^2 - 3) + p^2 + 10$
Because we cannot find a common binomial factor, this polynomial cannot be factored using factoring by grouping.

59. $y^3+8y^2-2y-16=y^2(y+8)-2(y+8)=(y+8)(y^2-2)$

61. $2x^3-8x^2-9x+36=2x^2(x-4)-9(x-4)$
$=(x-4)(2x^2-9)$

63. *Writing Exercise.* Yes; the opposite of a factor is also a factor so both can be correct.

65. $(x+2)(x+7)$
F O I L
$=x\cdot x+x\cdot 7+2\cdot x+2\cdot 7$
$=x^2+7x+2x+14$
$=x^2+9x+14$

67. $(x+2)(x-7)$
F O I L
$=x\cdot x+x\cdot(-7)+2\cdot x-2\cdot 7$
$=x^2-7x+2x+14$
$=x^2-5x-14$

69. $(a-1)(a-3)$
F O I L
$=a\cdot a-a\cdot 3-1\cdot a-1(-3)$
$=a^2-3a-a+3$
$=a^2-4a+3$

71. $(t-5)(t+10)$
F O I L
$=t\cdot t+t\cdot 10-5\cdot t-5\cdot 10$
$=t^2+10t-5t-50$
$=t^2+5t-50$

73. *Writing Exercise.* This is a good idea, because it is unlikely that Azrah will choose two replacement values that give the same value for non-equivalent expressions.

75. $4x^5+6x^2+6x^3+9=2x^2(2x^3+3)+3(2x^3+3)$
$=(2x^3+3)(2x^2+3)$

77. $2x^4+2x^3-4x^2-4x=2x(x^3+x^2-2x-2)$
$=2x\left(x^2(x+1)-2(x+1)\right)$
$=2x(x+1)(x^2-2)$

79. $5x^5-5x^4+x^3-x^2+3x-3$
$=5x^4(x-1)+x^2(x-1)+3(x-1)$
$=(x-1)(5x^4+x^2+3)$

We could also do this exercise as follows:
$5x^5-5x^4+x^3-x^2+3x-3$
$=(5x^5+x^3+3x)-(5x^4+x^2+3)$
$=x(5x^4+x^2+3)-1(5x^4+x^2+3)$
$=(5x^4+x^2+3)(x-1)$

81. Answers may vary. $8x^4y^3-24x^3y^3+16x^2y^4$

Exercise Set 5.2

1. If c is positive, then p and q have the same sign. If both are negative, then b is negative; if both are positive then c is positive. Thus we replace each blank with "positive."

3. If p is negative and q is negative, then b is negative because it is the sum of two negative numbers and c is positive because it is the product of two negative numbers.

5. Since c is negative, it is the product of a negative and a positive number. Then because c is the product of p and q and we know that p is negative, q must be positive.

7. $x^2+8x+16$

Since the constant term and the coefficient of the middle term are both positive, we look for a factorization of 16 in which both factors are positive. Their sum must be 8.

Pairs of factors	Sums of factors
1,16	17
2, 8	10
4, 4	8

The numbers we want are 4 and 4.

$x^2+8x+16=(x+4)(x+4)$

9. $x^2+11x+10$

Since the constant term and the coefficient of the middle term are both positive, we look for a factorization of 10 in which both factors are positive. Their sum must be 11.

Pairs of factors	Sums of factors
1,10	11
2, 5	10

The numbers we want are 1 and 10.

$x^2+11x+10=(x+1)(x+10)$

11. $x^2+10x+21$

Since the constant term and the coefficient of the middle term are both positive, we look for a factorization of 21 in which both factors are positive. Their sum must be 10.

Pairs of factors	Sums of factors
1, 21	22
3, 7	10

The numbers we want are 3 and 7.

$x^2+10x+21=(x+3)(x+7)$

13. $t^2-9t+14$

Since the constant term is positive and the coefficient of the middle term is negative, we look for a factorization of 14 in which both factors are negative. Their sum must be −9.

Pairs of factors	Sums of factors
−1,−14	−15
−2, −7	−9

The numbers we want are −2 and −7.

$t^2-9t+14=(t-2)(t-7)$

15. $b^2 - 5b + 4$

Since the constant term is positive and the coefficient of the middle term is negative, we look for a factorization of 4 in which both factors are negative. Their sum must be -5.

Pairs of factors	Sums of factors
$-1, -4$	-5
$-2, -2$	-4

The numbers we want are -1 and -4.

$b^2 - 5b + 4 = (b - 1)(b - 4)$.

17. $a^2 - 7a + 12$

Since the constant term is positive and the coefficient of the middle term is negative, we look for a factorization of 12 in which both factors are negative. Their sum must be -7.

Pairs of factors	Sums of factors
$-1, -12$	-13
$-2, -6$	-8
$-3, -4$	-7

The numbers we need are -3 and -4.

$a^2 - 7a + 12 = (a - 3)(a - 4)$.

19. $d^2 - 7d + 10$

Since the constant term is positive and the coefficient of the middle term is negative, we look for a factorization of 10 in which both factors are negative. Their sum must be -7.

Pairs of factors	Sums of factors
$-1, -10$	-11
$-2, -5$	-7

The numbers we want are -2 and -5.

$d^2 - 7d + 10 = (d - 2)(d - 5)$.

21. $x^2 - 2x - 15$

The constant term, -15, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be negative, the negative number must have the greater absolute value.

Pairs of factors	Sums of factors
$1, -15$	-14
$3, -5$	-2

The numbers we need are 3 and -5.

$x^2 - 2x - 15 = (x + 3)(x - 5)$.

23. $x^2 + 2x - 15$

The constant term, -15, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be positive, the positive number must have the greater absolute value.

Pairs of factors	Sums of factors
$-1, 15$	14
$-3, 5$	2

The numbers we need are -3 and 5.

$x^2 + 2x - 15 = (x - 3)(x + 5)$.

25. $2x^2 - 14x - 36 = 2(x^2 - 7x - 18)$

After factoring out the common factor, 2, we consider $x^2 - 7x - 18$. The constant term, -18, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be negative, the negative number must have the greater absolute value.

Pairs of factors	Sums of factors
$1, -18$	-17
$2, -9$	-7
$3, -6$	-3

The numbers we need are 2 and -9. The factorization of $x^2 - 7x - 18$ is $(x - 9)(x + 2)$. We must not forget the common factor, 2. Thus, $2x^2 - 14x - 36 = 2(x^2 - 7x - 18) = 2(x - 9)(x + 2)$.

27. $-x^3 + 6x^2 + 16x = -x(x^2 - 6x - 16)$

After factoring out the common factor, $-x$, we consider $x^2 - 6x - 16$. The constant term, -16, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be negative, the negative number must have the greater absolute value.

Pairs of factors	Sums of factors
$1, -16$	-15
$2, -8$	-6
$4, -4$	0

The numbers we need are 2 and -8. The factorization of $x^2 - 6x - 16$ is $(x + 2)(x - 8)$. We must not forget the common factor, $-x$. Thus, $-x^3 + 6x^2 + 16x = -x(x^2 - 6x - 16) = -x(x + 2)(x - 8)$.

29. $4y - 45 + y^2 = y^2 + 4y - 45$

The constant term, -45, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be positive, the positive number must have the greater absolute value.

Pairs of factors	Sums of factors
$-1, 45$	44
$-3, 15$	12
$-5, 9$	4

The numbers we need are -5 and 9.

$4y - 45 + y^2 = (y - 5)(y + 9)$

31. $x^2 - 72 + 6x = x^2 + 6x - 72$

The constant term, -72, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be positive, the positive number must have the greater absolute value.

Pairs of factors	Sums of factors
−1, 72	71
−2, 36	34
−3, 24	21
−4, 18	14
−6, 12	6

The numbers we need are −6 and 12.

$x^2 - 72 + 6x = (x-6)(x+12)$

33. $-5b^2 - 35b + 150 = -5(b^2 + 7b - 30)$

After factoring out the common factor, −5, we consider $b^2 + 7b - 30$. The constant term, −30, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be positive, the positive number must have the greater absolute value.

Pairs of factors	Sums of factors
−1, 30	29
−2, 15	13
−3, 10	7
−5, 6	1

The numbers we need are −3 and 10. The factorization of $b^2 + 7b - 30$ is $(b-3)(b+10)$. We must not forget the common factor. Thus, $-5b^2 - 35b + 150 = -5(b^2 + 7b - 30) = -5(b-3)(b+10)$.

35. $x^5 - x^4 - 2x^3 = x^3(x^2 - x - 2)$

After factoring out the common factor, x^3, we consider $x^2 - x - 2$. The constant term, −2, must be expressed as the product of a negative number and a positive number. Since the sum of those two numbers must be negative, the negative number must have the greater absolute value. The only possible factors that fill these requirements are 1 and −2. These are the numbers we need. The factorization of $x^2 - x - 2$ is $(x+1)(x-2)$. We must not forget the common factor, x^3. Thus, $x^5 - x^4 - 2x^3 = x^3(x^2 - x - 2) = x^3(x+1)(x-2)$.

37. $x^2 + 5x + 10$

Since the constant term and the coefficient of the middle term are both positive, we look for a factorization of 10 in which both factors are positive. Their sum must be 5. The only possible pairs of positive factors are 1 and 10, and 2 and 5 but neither sum is 5. Thus, this polynomial is not factorable into polynomials with integer coefficients. It is prime.

39. $32 + 12t + t^2 = t^2 + 12t + 32$

Since the constant term is positive and the coefficient of the middle term is positive, we look for a factorization of 32 in which both terms are positive. Their sum must be 12.

Pairs of factors	Sums of factors
1,32	33
2,16	18
4, 8	12

The numbers we want are 4 and 8.

$32 + 12t + t^2 = (t+4)(t+8)$.

41. $x^2 + 20x + 99$

We look for two factors, both positive, whose product is 99 and whose sum is 20.

They are 9 and 11: $9 \cdot 11 = 99$ and $9 + 11 = 20$.

$x^2 + 20 + 99 = (x+9)(x+11)$

43. $3x^3 - 63x^2 - 300x = 3x(x^2 - 21x - 100)$

After factoring out the common factor, $3x$, we consider $x^2 - 21x - 100$. We look for two factors, one positive, one negative, whose product is −100 and whose sum is −21.

They are −25 and 4: $-25(4) = -100$ and $-25 + 4 = -21$.

$x^2 - 21x - 100 = (x-25)(x+4)$, so $3x^3 - 63x^2 - 300x = 3x(x-25)(x+4)$.

45. $-2x^2 + 42x + 144 = -2(x^2 - 21x - 72)$

After factoring out the common factor, −2, we consider $x^2 - 21x - 72$. We look for two factors, one positive, and one negative, whose product is −72 and whose sum is −21. They are −24 and 3.

$x^2 - 21x - 72 = (x-24)(x+3)$, so $-2x^2 + 42x + 144 = -2(x^2 - 21x - 72) = -2(x-24)(x+3)$.

47. $y^2 - 20y + 96$

We look for two factors, both negative, whose product is 96 and whose sum is −20. They are −8 and −12.

$y^2 - 20y + 96 = (y-8)(y-12)$

49. $-a^6 - 9a^5 + 90a^4 = -a^4(a^2 + 9a - 90)$

After factoring out the common factor, $-a^4$, we consider $a^2 + 9a - 90$. We look for two factors, one positive and one negative, whose product is −90 and whose sum is 9. They are −6 and 15.

$a^2 + 9a - 90 = (a-6)(a+15)$, so $-a^6 - 9a^5 + 90a^4 = -a^4(a-6)(a+15)$.

51. $t^2 + \frac{2}{3}t + \frac{1}{9}$

We look for two factors, both positive, whose product is $\frac{1}{9}$ and whose sum is $\frac{2}{3}$. They are $\frac{1}{3}$ and $\frac{1}{3}$.

$t^2 + \frac{2}{3}t + \frac{1}{9} = \left(t + \frac{1}{3}\right)\left(t + \frac{1}{3}\right)$, or $\left(t + \frac{1}{3}\right)^2$

53. $11 + w^2 - 4w = w^2 - 4w + 11$

Since the constant term is positive and the coefficient of the middle term is negative, we look for a factorization of 11 in which both factors are negative. Their sum must be −4. The only possible pair of factors is −1 and −11, but their sum is not −4. Thus, this polynomial is not factorable into polynomials with integer coefficients. It is prime.

55. $p^2 + 7pq + 10q^2$

Think of $-7q$ as a "coefficient" of p. Then we look for factors of $10q^2$ whose sum is $-7q$. They are $-5q$ and $-2q$.

$p^2 + 7pq + 10q^2 = (p-5q)(p-2q)$.

57. $m^2 + 5mn + 5n^2 = m^2 + 5nm + 5n^2$

We look for factors of $5n^2$ whose sum is $5n$. The only reasonable possibilities are shown below.

Pairs of factors	Sums of factors
$5n,\ \ n$	$6n$
$-5n, -n$	$-6n$

There are no factors whose sum is $5n$. Thus, the polynomial is not factorable into polynomials with integer coefficients. It is prime.

59. $s^2 - 4st - 12t^2 = s^2 - 4ts - 12t^2$

We look for factors of $-12t^2$ whose sum is $-4t$. They are $-6t$ and $2t$.

$s^2 - 4st - 12t^2 = (s - 6t)(s + 2t)$

61. $6a^{10} + 30a^9 - 84a^8 = 6a^8(a^2 + 5a - 14)$

After factoring out the common factor, $6a^8$, we consider $a^2 + 5a - 14$. We look for two factors, one positive and one negative, whose product is -14 and whose sum is 5. They are -2 and 7.

$a^2 + 5a - 14 = (a - 2)(a + 7)$, so $6a^{10} + 30a^9 - 84a^8$
$= 6a^8(a - 2)(a + 7)$.

63. *Writing Exercise*. Since both constants are negative, the middle term will be negative so $(x - 17)(x - 18)$ cannot be a factorization of $x^2 + 35x + 306$.

65. $(2x + 3)(3x + 4)$

F O I L

$= 2x \cdot 3x + 2x \cdot 4 + 3 \cdot 3x + 3 \cdot 4$

$= 6x^2 + 8x + 9x + 12$

$= 6x^2 + 17x + 12$

67. $(2x - 3)(3x + 4)$

F O I L

$= 2x \cdot 3x + 2x \cdot 4 - 3 \cdot 3x - 3 \cdot 4$

$= 6x^2 + 8x - 9x - 12$

$= 6x^2 - x - 12$

69. $(5x - 1)(x - 7)$

F O I L

$= 5x \cdot x - 5x \cdot 7 - 1 \cdot x - 1(-7)$

$= 5x^2 - 35x - x + 7$

$= 5x^2 - 36x + 7$

71. *Writing Exercise.* There is a finite number of pairs of numbers with the correct product, but there are infinitely many pairs with the correct sum.

73. $a^2 + ba - 50$

We look for all pairs of integer factors whose product is -50. The sum of each pair is represented by b.

Pairs of factors whose product is -50	Sums of factors
$-1,\ 50$	49
$1, -50$	-49
$-2,\ 25$	23
$2, -25$	-23
$-5,\ 10$	5
$5, -10$	-5

The polynomial $a^2 + ba - 50$ can be factored if b is 49, -49, 23, -23, 5, or -5.

75. $y^2 - 0.2y - 0.08$

We look for two factors, one positive and one negative, whose product is -0.08 and whose sum is -0.2. They are -0.4 and 0.2.

$y^2 - 0.2y - 0.08 = (y - 0.4)(y + 0.2)$

77. $-\frac{1}{3}a^3 + \frac{1}{3}a^2 + 2a = -\frac{1}{3}a(a^2 - a - 6)$

After factoring out the common factor, $-\frac{1}{3}a$, we consider $a^2 - a - 6$. We look for two factors, one positive and one negative, whose product is -6 and whose sum is -1. They are 2 and -3.

$a^2 - a - 6 = (a + 2)(a - 3)$, so
$-\frac{1}{3}a^3 + \frac{1}{3}a^2 + 2a = -\frac{1}{3}a(a + 2)(a - 3)$.

79. $x^{2m} + 11x^m + 28 = (x^m)^2 + 11x^m + 28$

We look for numbers p and q such that $x^{2m} + 11x^m + 28 = (x^m + p)(x^m + q)$. We find two factors, both positive, whose product is 28 and whose sum is 11. They are 4 and 7.

$x^{2m} + 11x^m + 28 = (x^m + 4)(x^m + 7)$

81. $(a + 1)x^2 + (a + 1)3x + (a + 1)2$

$= (a + 1)(x^2 + 3x + 2)$

After factoring out the common factor $a + 1$, we consider $x^2 + 3x + 2$. We look for two factors, whose product is 2 and whose sum is 3. They are 1 and 2.

$x^2 + 3x + 2 = (x + 1)(x + 2)$, so
$(a + 1)x^2 + (a + 1)3x + (a + 1)2$
$= (a + 1)(x + 1)(x + 2)$.

83. $6x^2 + 36x + 54 = 6(x^2 + 6x + 9) = 6(x+3)(x+3) = 6(x+3)^2$

Since the surface area of a cube with sides is given by $6s^2$, we know that this cube has side $x + 3$. The volume of a cube with side s is given by s^3, so the volume of this cube is $(x + 3)^3$, or $x^3 + 9x^2 + 27x + 27$.

85. The shaded area consists of the area of a rectangle with sides x and $x + x$, or $2x$, and $\frac{3}{4}$ of the area of a circle with radius x. It can be expressed as follows:

$x \cdot 2x + \frac{3}{4}\pi x^2 = 2x^2 + \frac{3}{4}\pi x^2 = x^2\left(2 + \frac{3}{4}\pi\right)$, or

$\frac{1}{4}x^2(8 + 3\pi)$

87. The shaded area consists of the area of a square with side $x + x + x$, or $3x$, less the area of a semicircle with radius x. It can be expressed as follows:

$$3x \cdot 3x - \frac{1}{2}\pi x^2 = 9x^2 - \frac{1}{2}\pi x^2 = x^2\left(9 - \frac{1}{2}\pi\right)$$

89. $x^2 + 4x + 5x + 20 = x^2 + 9x + 20 = (x + 4)(x + 5)$

Exercise Set 5.3

1. Since $(6x-1)(2x+3) = 12x^2+16x-3$, choice (c) is correct.

3. Since $(7x+1)(2x-3) = 14x^2-19x-3$, choice (d) is correct.

5. $2x^2 + 7x - 4$

(1) There is no common factor (other than 1 or -1).

(2) Because $2x^2$ can be factored as $2x \cdot x$, we have this possibility:

$$(2x + \quad)(x + \quad)$$

(3) There are 3 pairs of factors of -4 and they can be listed two ways:

	$-4, 1$	$4, -1$	$2, -2$
and	$1, -4$	$-1, 4$	$-2, 2$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $7x$. We can immediately reject all possibilities in which a factor has a common factor, such as $(2x - 4)$ or $(2x + 2)$, because we determined at the outset that there is no common factor other than 1 and -1. We try some possibilities:

$$(2x + 1)(x - 4) = 2x^2 - 7x - 4$$
$$(2x - 1)(x + 4) = 2x^2 + 7x - 4$$

The factorization is $(2x - 1)(x + 4)$.

7. $3x^2 - 17x - 6$

(1) There is no common factor (other than 1 or -1).

(2) Because $3x^2$ can be factored as $3x \cdot x$, we have this possibility:

$$(3x + \quad)(x + \quad)$$

(3) There are 4 pairs of factors of -6 and they can be listed two ways:

	$-6, 1$	$6, -1$	$-3, 2$	$3, -2$
and	$1, -6$	$-1, 6$	$2, -3$	$-2, 3$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $-17x$. We can immediately reject all possibilities in which either factor has a common factor, such as $(3x - 6)$ or $(3x + 3)$, because at the outset we determined that there is no common factor other than 1 or -1. We try some possibilities:

$$(3x + 2)(x - 3) = 3x^2 - 7x - 6$$
$$(3x + 1)(x - 6) = 3x^2 - 17x - 6$$

The factorization is $(3x + 1)(x - 6)$.

9. $4t^2 + 12t + 5$

(1) There is no common factor (other than 1 or -1).

(2) Because $4t^2$ can be factored as $4t \cdot t$ or $2t \cdot 2t$, we have these possibilities:

$$(4t + \quad)(t + \quad) \text{ and } (2t + \quad)(2t + \quad)$$

(3) There are 2 pairs of factors of 5 and they can be listed two ways:

	$5, 1$	$-5, -1$
and	$1, 5$	$-1, -5$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $12t$. We try some possibilities:

$$(4t + 1)(t + 5) = 4t^2 + 21t + 5$$
$$(2t + 1)(2t + 5) = 4t^2 + 12t + 5$$

The factorization is $(2t + 1)(2t + 5)$.

11. $15a^2 - 14a + 3$

(1) There is no common factor (other than 1 or -1).

(2) Because $15a^2$ can be factored as $15a \cdot a$ or $5a \cdot 3a$, we have these possibilities:

$$(15a + \quad)(a + \quad) \text{ and } (5a + \quad)(3a + \quad)$$

(3) There are 2 pairs of factors of 3 and they can be listed two ways:

	$3, 1$	$-3, -1$
and	$1, 3$	$-1, -3$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $-14a$. We can immediately reject all possibilities in which either factor has a common factor, such as $(15a + 3)$ or $(3a - 3)$, because at the outset we determined that there is no common factor other than 1 or -1. Since the sign of the middle term is negative and the sign of the last term is positive, the factors of 3 must both be negative. We try some possibilities:

$$(15a - 1)(a - 3) = 15a^2 - 46a + 3$$
$$(5a - 3)(3a - 1) = 15a^2 - 14a + 3$$

The factorization is $(5a - 3)(3a - 1)$.

13. $6x^2 + 17x + 12$

(1) There is no common factor (other than 1 or -1).

(2) Because $6x^2$ can be factored as $6x \cdot x$ and $3x \cdot 2x$, we have these possibilities:

$$(6x + \quad)(x + \quad) \text{ and } (3x + \quad)(2x + \quad)$$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 12. There are 3 pairs and they can be listed two ways:

	$1, 12$	$2, 6$	$3, 4$
and	$12, 1$	$6, 2$	$4, 3$

(4) We can immediately reject all possibilities in which either factor has a common factor, such as $(6x + 12)$ or $(3x + 3)$, because at the outset we determined that there is no common factor other than 1 or -1. We try some possibilities:

$(6x+1)(x+12) = 6x^2+73x+12$

$(3x+4)(2x+3) = 6x^2+17x+12$

The factorization is $(3x+4)(2x+3)$.

15. $6x^2 - 10x - 4$

(1) We factor out the largest common factor, 2:

$2(3x^2 - 5x - 2)$.

Then we factor the trinomial $3x^2 - 5x - 2$.

(2) Because $3x^2$ can be factored as $3x \cdot x$, we have this possibility:

$(3x+ \quad)(x+ \quad)$

(3) There are 2 pairs of factors of -2 and they can be listed two ways:

$-2, 1 \quad 2, -1$

and $\quad 1, -2 \quad -1, 2$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $-5x$. We try some possibilities:

$(3x-2)(x+1) = 3x^2+x-2$

$(3x+2)(x-1) = 3x^2-x-2$

$(3x+1)(x-2) = 3x^2-5x-2$

The factorization of $3x^2-5x-2$ is $(3x+1)(x-2)$. We must include the common factor in order to get a factorization of the original trinomial.

$6x^2-10x-4 = 2(3x+1)(x-2)$

17. $7t^3 + 15t^2 + 2t$

(1) We factor out the common factor, t:

$t(7t^2+15t+2)$.

Then we factor the trinomial $7t^2+15t+2$.

(2) Because $7t^2$ can be factored as $7t \cdot t$, we have this possibility:

$(7t+ \quad)(t+ \quad)$

(3) Since all coefficients are positive, we need consider only positive factors of 2. There is only 1 such pair and it can be listed two ways:

$2, 1 \quad 1, 2$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $15t$. We try some possibilities:

$(7t+2)(t+1) = 7t^2+9t+2$

$(7t+1)(t+2) = 7t^2+15t+2$

The factorization of $7t^2+15t+2$ is $(7t+1)(t+2)$. We must include the common factor in order to get a factorization of the original trinomial.

$7t^3+15t^2+2t = t(7t+1)(t+2)$

19. $10 - 23x + 12x^2 = 12x^2 - 23x + 10$

(1) There is no common factor (other than 1 or -1).

(2) Because $12x^2$ can be factored as $12x \cdot x$, $6x \cdot 2x$, and $4x \cdot 3x$, we have these possibilities:

$(12x+ \quad)(x+ \quad)$, $(6x+ \quad)(2x+ \quad)$, and $(4x+ \quad)(3x+ \quad)$

(3) Since the sign of the middle term is negative but the sign of the last term is positive, we need consider only negative factors of 10.

$-10, -1 \quad -5, -2$

and $\quad -1, -10 \quad -2, -5$

(4) We can immediately reject all possibilities in which either factor has a common factor, such as $(2x-10)$ or $(4x-2)$, because we determined at the outset that there is no common factor other than 1 or -1. We try some possibilities:

$(12x-5)(x-2) = 12x^2-29x+10$

$(4x-1)(3x-10) = 12x^2-43x+10$

$(4x-5)(3x-2) = 12x^2-23x+10$

The factorization is $(4x-5)(3x-2)$.

21. $-35x^2 - 34x - 8$

(1) We factor out -1 in order to have a trinomial with a positive leading coefficient.

$-35x^2-34x-8 = -1(35x^2+34x+8)$

Now we factor $35x^2+34x+8$.

(2) Because $35x^2$ can be factored as $35x \cdot x$ or $7x \cdot 5x$, we have these possibilities:

$(35x+ \quad)(x+ \quad)$ and $(7x+ \quad)(5x+ \quad)$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 8. There are 2 such pairs and they can be listed two ways:

$8, 1 \quad 4, 2$

and $\quad 1, 8 \quad 2, 4$

(4) We try some possibilities:

$(35x+8)(x+1) = 35x^2+43x+8$

$(7x+8)(5x+1) = 35x^2+47x+8$

$(7x+4)(5x+2) = 35x^2+34x+8$

The factorization of $35x^2+34x+8$ is $(7x+4)(5x+2)$.

We must include the factor of -1 in order to get a factorization of the original trinomial.

$-35x^2-34x-8 = -1(7x+4)(5x+2)$, or $-(7x+4)(5x+2)$.

23. $4 + 6t^2 - 13t = 6t^2 - 13t + 4$

(1) There is no common factor (other than 1 or -1).

(2) Because $6t^2$ can be factored as $6t \cdot t$ or $3t \cdot 2t$, we have these possibilities:

$(6t+ \quad)(t+ \quad)$ and $(3t+ \quad)(2t+ \quad)$

(3) Since the sign of the middle term is negative but the sign of the last term is positive, we need to consider only negative factors of 4. There is only 1 such pair and it can be listed two ways:

$-4, -1$ and $-1, -4$

(4) We can immediately reject all possibilities in which either factor has a common factor, such as $(6t-4)$ or $(2t-4)$, because we determined at the outset that there is no common factor other than 1 or -1. We try some possibilities:

$$(6t-1)(t-4) = 6t^2 - 25t + 4$$
$$(3t-4)(2t-1) = 6t^2 - 11t + 4$$

These are the only possibilities that do not contain a common factor. Since neither is the desired factorization, we must conclude that $4 + 6t^2 - 13t$ is prime.

25. $25x^2 + 40x + 16$

(1) There is no common factor (other than 1 or -1).

(2) Because $25x^2$ can be factored as $25x \cdot x$ or $5x \cdot 5x$, we have these possibilities:

$(25x + \quad)(x + \quad)$ and $(5x + \quad)(5x + \quad)$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 16. There are 3 such pairs and two of them can be listed two ways:

	16, 1	8, 2	4, 4
and	1, 16	2, 8	

(4) We try some possibilities:

$$(25x+16)(x+1) = 25x^2 + 41x + 16$$
$$(5x+8)(5x+2) = 25x^2 + 50x + 16$$
$$(5x+4)(5x+4) = 25x^2 + 40x + 16$$

The factorization is $(5x+4)(5x+4)$, or $(5x+4)^2$.

27. $20y^2 + 59y - 3$

(1) There is no common factor (other than 1 or -1).

(2) Because $20y^2$ can be factored as $20y \cdot y$, $10y \cdot 2y$, or $5y \cdot 4y$, we have these possibilities:

$(20y + \quad)(y + \quad)$ and $(10y + \quad)(2y + \quad)$ and $(5y + \quad)(4y + \quad)$

(3) There are 2 such pairs of factors of -3, which can be listed two ways:

	$-3, 1$	$3, -1$
and	$1, -3$	$-1, 3$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is 59y. We try some possibilities:

$$(20y-3)(y+1) = 20y^2 + 17y - 3$$
$$(10y-3)(2y+1) = 20y^2 + 4y - 3$$
$$(5y-1)(4y+3) = 20y^2 + 11y - 3$$
$$(20y-1)(y+3) = 20y^2 + 59y - 3$$

The factorization is $(20y-1)(y+3)$.

29. $14x^2 + 73x + 45$

(1) There is no common factor (other than 1 or -1).

(2) Because $14x^2$ can be factored as $14x \cdot x$, and $7x \cdot 2x$, we have two possibilities:

$(14x + \quad)(x + \quad)$, and $(7x + \quad)(2x + \quad)$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 45. There are 3 such pairs and they can be listed two ways.

	45, 1	15, 3	9, 5
and	1, 45	3, 15	5, 9

(4) Look for Outer and Inner products from steps (2) and (3) for which the sum is $73x$. We try some possibilities:

$$(14x+45)(x+1) = 14x^2 + 59x + 45$$
$$(14x+15)(x+3) = 14x^2 + 57x + 45$$
$$(7x+1)(2x+45) = 14x^2 + 317x + 45$$
$$(7x+5)(2x+9) = 14x^2 + 73x + 45$$

The factorization is $(7x+5)(2x+9)$.

31. $-2x^2 + 15 + x = -2x^2 + x + 15$

(1) We factor out -1 in order to have a trinomial with a positive leading coefficient.

$$-2x^2 + x + 15 = -1(2x^2 - x - 15)$$

Now we factor $2x^2 - x - 15$.

(2) Because $2x^2$ can be factored as $2x \cdot x$ we have this possibility:

$(2x + \quad)(x + \quad)$

(3) There are 4 pairs of factors of -15 and they can be listed two ways:

	$-15, 1$	$15, -1$	$-5, 3$	$5, -3$
and	$1, -15$	$-1, 15$	$3, -5$	$-3, 5$

(4) We try some possibilities:

$$(2x-15)(x+1) = 2x^2 - 13x - 15$$
$$(2x-5)(x+3) = 2x^2 + x - 15$$
$$(2x+5)(x-3) = 2x^2 - x - 15$$

The factorization of $2x^2 - x - 15$ is $(2x+5)(x-3)$. We must include the factor of -1 in order to get a factorization of the original trinomial.

$-2x^2 + 15 + x = -1(2x+5)(x-3)$, or $-(2x+5)(x-3)$

33. $-6x^2 - 33x - 15$

(1) Factor out -3. This not only removes the largest common factor, 3. It also produces a trinomial with a positive leading coefficient.

$$-3(2x^2 + 11x + 5)$$

Then we factor the trinomial $2x^2 + 11x + 5$.

(2) Because $2x^2$ can be factored as $2x \cdot x$ we have this possibility:

$(2x + \quad)(x + \quad)$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 5. There is one such pair and it can be listed two ways:

5, 1 and 1, 5

(4) We try some possibilities:

$$(2x+5)(x+1) = 2x^2 + 7x + 5$$
$$(2x+1)(x+5) = 2x^2 + 11x + 5$$

The factorization of $2x^2+11x+5$ is $(2x+1)(x+5)$. We must include the common factor in order to get a factorization of the original trinomial.

$$-6x^2 - 33x - 15 = -3(2x+1)(x+5)$$

35. $10a^2 - 8a - 18$

(1) Factor out the common factor, 2:

$2(5a^2 - 4a - 9)$

Then we factor the trinomial $5a^2 - 4a - 9$.

(2) Because $5a^2$ can be factored as $5a \cdot a$, we have this possibility:

$(5a + \quad)(a + \quad)$

(3) There are 3 pairs of factors of -9, and they can be listed two ways.

	$-9, 1$	$9, -1$	$3, -3$
and	$1, -9$	$-1, 9$	$-3, 3$

(4) Look for Outer and Inner products resulting from steps (2) and (3) for which the sum is $-4a$. We try some possibilities:

$(5a - 3)(a + 3) = 5a^2 + 12a - 9$
$(5a + 9)(a - 1) = 5a^2 + 4a - 9$
$(a + 1)(5a - 9) = 5a^2 - 4a - 9$

The factorization of $5a^2 - 4a - 9$ is $(a+1)(5a-9)$. We must include the common factor in order to get a factorization of the original trinomial.

$2(a + 1)(5a - 9)$

37. $12x^2 + 68x - 24$

(1) Factor out the common factor, 4:

$4(3x^2 + 17x - 6)$

Then we factor the trinomial $3x^2 + 17x - 6$.

(2) Because $3x^2$ can be factored as $3x \cdot x$ we have this possibility:

$(3x + \quad)(x + \quad)$

(3) There are 4 pairs of factors of -6 and they can be listed two ways:

	$6, -1$	$-6, 1$	$3, -2$	$-3, 2$
and	$-1, 6$	$1, -6$	$-2, 3$	$2, -3$

(4) We can immediately reject all possibilities in which either factor has a common factor, such as $(3x + 6)$ or $(3x - 3)$, because we determined at the outset that there is no common factor other than 1 or -1. We try some possibilities:

$(3x - 1)(x + 6) = 3x^2 + 17x - 6$

The factorization of $3x^2+17x-6$ is $(3x-1)(x+6)$. We must include the common factor in order to get a factorization of the original trinomial.

$12x^2 + 68x - 24 = 4(3x - 1)(x + 6)$

39. $4x + 1 + 3x^2 = 3x^2 + 4x + 1$

(1) There is no common factor (other than 1 or -1).

(2) Because $3x^2$ can be factored as $3x \cdot x$ we have this possibility:

$(3x + \quad)(x + \quad)$

(3) Since all coefficients are positive, we need consider only positive pairs of factors of 1. There is one such pair: 1,1.

(4) We try the possible factorization:

$(3x + 1)(x + 1) = 3x^2 + 4x + 1$

The factorization is $(3x + 1)(x + 1)$.

41. $x^2 + 3x - 2x - 6 = x(x + 3) - 2(x + 3)$
$= (x + 3)(x - 2)$

43. $8t^2 - 6t - 28t + 21 = 2t(4t - 3) - 7(4t - 3)$
$= (4t - 3)(2t - 7)$

45. $6x^2 + 4x + 15x + 10 = 2x(3x + 2) + 5(3x + 2)$
$= (3x + 2)(2x + 5)$

47. $2y^2 + 8y - y - 4 = 2y(y + 4) - 1(y + 4)$
$= (y + 4)(2y - 1)$

49. $6a^2 - 8a - 3a + 4 = 2a(3a - 4) - 1(3a - 4)$
$= (3a - 4)(2a - 1)$

51. $16t^2 + 23t + 7$

(1) First note that there is no common factor (other than 1 or -1).

(2) Multiply the leading coefficient, 16, and the constant, 7:

$16 \cdot 7 = 112$

(3) We look for factors of 112 that add to 23. Since all coefficients are positive, we need to consider only positive factors.

Pairs of factors	Sums of factors
1, 112	113
2, 56	58
4, 28	32
8, 14	22
16, 7	23

The numbers we need are 16 and 7.

(4) Rewrite the middle term:

$23t = 16t + 7t$

(5) Factor by grouping:

$$16t^2 + 23t + 7 = 16t^2 + 16t + 7t + 7$$
$$= 16t(t + 1) + 7(t + 1)$$
$$= (t + 1)(16t + 7)$$

53. $-9x^2 - 18x - 5$

(1) We factor out -1 in order to have a trinomial with a positive leading coefficient.

$-9x^2 - 18x - 5 = -1(9x^2 + 18x + 5)$

Now we factor $9x^2 + 18x + 5$.

(2) Multiply the leading coefficient, 9, and the constant, 5:

$9 \cdot 5 = 45$

(3) We look for factors of 45 that add to 18. Since all coefficients are positive, we need to consider only positive factors.

Pairs of factors	Sums of factors
1, 45	46
3, 15	18
5, 9	14

The numbers we need are 3 and 15.

(4) Rewrite the middle term:

$18x = 3x + 15x$

(5) Factor by grouping:

$$\begin{aligned} 9x^2 + 18x + 5 &= 9x^2 + 3x + 15x + 5 \\ &= 3x(3x+1) + 5(3x+1) \\ &= (3x+1)(3x+5) \end{aligned}$$

We must include the factor of -1 in order to get a factorization of the original trinomial.

$-9x^2 - 18x - 5 = -1(3x+1)(3x+5)$, or
$-(3x+1)(3x+5)$

55. $10x^2 + 30x - 70$

(1) Factor out the largest common factor, 10:

$10x^2 + 30x - 70 = 10(x^2 + 3x - 7)$

Since x^2+3x-7 is prime, this trinomial cannot be factored further.

57. $18x^3 + 21x^2 - 9x$

(1) Factor out the largest common factor, $3x$:

$18x^3 + 21x^2 - 9x = 3x(6x^2 + 7x - 3)$

(2) To factor $6x^2 + 7x - 3$ by grouping we first multiply the leading coefficient, 6, and the constant, -3:

$6(-3) = -18$

(3) We look for factors of -18 that add to 7.

Pairs of factors	Sums of factors
-1, 18	17
1, -18	-17
-2, 9	7
2, -9	-7
-3, 6	3
3, -6	-3

The numbers we need are -2 and 9.

(4) Rewrite the middle term:

$7x = -2x + 9x$

(5) Factor by grouping:

$$\begin{aligned} 6x^2 + 7x - 3 &= 6x^2 - 2x + 9x - 3 \\ &= 2x(3x-1) + 3(3x-1) \\ &= (3x-1)(2x+3) \end{aligned}$$

The factorization of $6x^2+7x-3$ is $(3x-1)(2x+3)$. We must include the common factor in order to get a factorization of the original trinomial:

$18x^3 + 21x^2 - 9x = 3x(3x-1)(2x+3)$

59. $89x + 64 + 25x^2 = 25x^2 + 89x + 64$

(1) First note that there is no common factor (other than 1 or -1).

(2) Multiply the leading coefficient, 25, and the constant, 64:

$25 \cdot 64 = 1600$

(3) We look for factors of 1600 that add to 89. Since all coefficients are positive, we need to consider only positive factors. The numbers we need are 25 and 64.

(4) Rewrite the middle term:

$89x = 25x + 64x$

(5) Factor by grouping:

$$\begin{aligned} 25x^2 + 89x + 64 &= 25x^2 + 25x + 64x + 64 \\ &= 25x(x+1) + 64(x+1) \\ &= (x+1)(25x+64) \end{aligned}$$

61. $168x^3 + 45x^2 + 3x$

(1) Factor out the largest common factor, $3x$:

$168x^3 + 45x^2 + 3x = 3x(56x^2 + 15x + 1)$

(2) To factor $56x^2 + 15x + 1$ we first multiply the leading coefficient, 56, and the constant, 1:

$56 \cdot 1 = 56$

(3) We look for factors of 56 that add to 15. Since all coefficients are positive, we need to consider only positive factors. The numbers we need are 7 and 8.

(4) Rewrite the middle term:

$15x = 7x + 8x$

(5) Factor by grouping:

$$\begin{aligned} 56x^2 + 15x + 1 &= 56x^2 + 7x + 8x + 1 \\ &= 7x(8x+1) + 1(8x+1) \\ &= (8x+1)(7x+1) \end{aligned}$$

The factorization of $56x^2+15x+1$ is $(8x+1)(7x+1)$. We must include the common factor in order to get a factorization of the original trinomial:

$168x^3 + 45x^2 + 3x = 3x(8x+1)(7x+1)$

63. $-14t^4 + 19t^3 + 3t^2$

(1) Factor out $-t^2$. This not only removes the largest common factor, t^2. It also produces a trinomial with a positive leading coefficient.

$-14t^4 + 19t^3 + 3t^2 = -t^2(14t^2 - 19t - 3)$

(2) To factor $14t^2 - 19t - 3$ we first multiply the leading coefficient, 14, and the constant, -3:

$14(-3) = -42$

(3) We look for factors of -42 that add to -19. The numbers we need are -21 and 2.

(4) Rewrite the middle term:

$-19t = -21t + 2t$

(5) Factor by grouping:

$$\begin{aligned} 14t^2 - 19t - 3 &= 14t^2 - 21t + 2t - 3 \\ &= 7t(2t-3) + 1(2t-3) \\ &= (2t-3)(7t+1) \end{aligned}$$

The factorization of $14t^2-19t-3$ is $(2t-3)(7t+1)$. We must include the common factor in order to get a factorization of the original trinomial:

$-14t^4+19t^3+3t^2=-t^2(2t-3)(7t+1)$

65. $132y+32y^2-54=32y^2+132y-54$

(1) Factor out the largest common factor, 2:

$$32y^2+132y-54=2(16y^2+66y-27)$$

(2) To factor $16y^2+66y-27$ we first multiply the leading coefficient, 16, and the constant, -27:

$$16(-27)=-432$$

(3) We look for factors of -432 that add to 66. The numbers we need are 72 and -6.

(4) Rewrite the middle term:

$$66y=72y-6y$$

(5) Factor by grouping:

$$\begin{aligned}16y^2+66y-27&=16y^2+72y-6y-27\\&=8y(2y+9)-3(2y+9)\\&=(2y+9)(8y-3)\end{aligned}$$

The factorization of $16y^2+66y-27$ is $(2y+9)(8y-3)$. We must include the common factor in order to get a factorization of the original trinomial:

$132y+32y^2-54=2(2y+9)(8y-3)$

67. $2a^2-5ab+2b^2$

(1) There is no common factor (other than 1 or -1).

(2) Multiply the leading coefficient, 2, and the constant, 2:

$$2\cdot 2=4$$

(3) We look for factors of 4 that add to -5. The numbers we need are -1 and -4.

(4) Rewrite the middle term:

$$-5ab=-ab-4ab$$

(5) Factor by grouping:

$$\begin{aligned}2a^2-5ab+2b^2&=2a^2-ab-4ab+2b^2\\&=a(2a-b)-2b(2a-b)\\&=(2a-b)(a-2b)\end{aligned}$$

69. $8s^2+22st+14t^2$

(1) Factor out the largest common factor, 2:

$$8s^2+22st+14t^2=2(4s^2+11st+7t^2)$$

(2) Multiply the leading coefficient, 4, and the constant, 7:

$$4\cdot 7=28$$

(3) We look for factors of 28 that add to 11. The numbers we need are 4 and 7.

(4) Rewrite the middle term:

$$11st=4st+7st$$

(5) Factor by grouping:

$$\begin{aligned}4s^2+11st+7t^2&=4s^2+4st+7st+7t^2\\&=4s(s+t)+7t(s+t)\\&=(s+t)(4s+7t)\end{aligned}$$

The factorization of $4s^2+11st+7t^2$ is $(s+t)(4s+7t)$. We must include the common factor in order to get a factorization of the original trinomial:

$8s^2+22st+14t^2=2(s+t)(4s+7t)$

71. $27x^2-72xy+48y^2$

(1) Factor out the largest common factor, 3:

$$27x^2-72xy+48y^2=3(9x^2-24xy+16y^2)$$

(2) To factor $9x^2-24xy+16y^2$, we first multiply the leading coefficient, 9, and the constant, 16:

$$9\cdot 16=144$$

(3) We look for factors of 144 that add to -24. The numbers we need are -12 and -12.

(4) Rewrite the middle term:

$$-24xy=-12xy-12xy$$

(5) Factor by grouping:

$$\begin{aligned}9x^2-24xy+16y^2&=9x^2-12xy-12xy+16y^2\\&=3x(3x-4y)-4y(3x-4y)\\&=(3x-4y)(3x-4y)\end{aligned}$$

The factorization of $9x^2-24xy+16y^2$ is $(3x-4y)(3x-4y)$. We must include the common factor in order to get a factorization of the original trinomial:

$27x^2-72xy+48y^2=3(3x-4y)(3x-4y)$ or, $3(3x-4y)^2$

73. $-24a^2+34ab-12b^2$

(1) Factor out -2. This not only removes the largest common factor, 2. It also produces a trinomial with a positive leading coefficient.

$$-24a^2+34ab-12b^2=-2(12a^2-17ab+6b^2)$$

(2) To factor $12a^2-17ab+6b^2$, we first multiply the leading coefficient, 12, and the constant, 6:

$$12\cdot 6=72$$

(3) We look for factors of 72 that add to -17. The numbers we need are -8 and -9.

(4) Rewrite the middle term:

$$-17ab=-8ab-9ab$$

(5) Factor by grouping:

$$\begin{aligned}12a^2-17ab+6b^2&=12a^2-8ab-9ab+6b^2\\&=4a(3a-2b)-3b(3a-2b)\\&=(3a-2b)(4a-3b)\end{aligned}$$

The factorization of $12a^2-17ab+6b^2$ is $(3a-2b)(4a-3b)$. We must include the common factor in order to get a factorization of the original trinomial:

$-24a^2+34ab-12b^2=-2(3a-2b)(4a-3b)$

75. $19x^3-3x^2+14x^4=14x^4+19x^3-3x^2$

(1) Factor out the largest common factor, x^2:

$$x^2(14x^2+19x-3)$$

(2) To factor $14x^2+19x-3$ by grouping we first multiply the leading coefficient, 14, and the constant, -3:

$$14(-3)=-42$$

(3) We look for factors of -42 that add to 19. The numbers we need are 21 and -2.

(4) Rewrite the middle term:

$19x = 21x - 2x$

(5) Factor by grouping:

$$\begin{aligned}14x^2 + 19x - 3 &= 14x^2 + 21x - 2x - 3\\ &= 7x(2x+3) - 1(2x+3)\\ &= (2x+3)(7x-1)\end{aligned}$$

The factorization of $14x^2 + 19x - 3$ is $(2x+3)(7x-1)$. We must include the common factor in order to get a factorization of the original trinomial:

$19x^3 - 3x^2 + 14x^4 = x^2(2x+3)(7x-1)$

77. $18a^7 + 8a^6 + 9a^8 = 9a^8 + 18a^7 + 8a^6$

(1) Factor out the largest common factor, a^6:

$9a^8 + 18a^7 + 8a^6 = a^6(9a^2 + 18a + 8)$

(2) To factor $9a^2 + 18a + 8$ we first multiply the leading coefficient, 9, and the constant, 8:

$9 \cdot 8 = 72$

(3) Look for factors of 72 that add to 18. The numbers we need are 6 and 12.

(4) Rewrite the middle term:

$18a = 6a + 12a$

(5) Factor by grouping:

$$\begin{aligned}9a^2 + 18a + 8 &= 9a^2 + 6a + 12a + 8\\ &= 3a(3a+2) + 4(3a+2)\\ &= (3a+2)(3a+4)\end{aligned}$$

The factorization of $9a^2 + 18a + 8$ is $(3a+2)(3a+4)$. We must include the common factor in order to get a factorization of the original trinomial:

$18a^7 + 8a^6 + 9a^8 = a^6(3a+2)(3a+4)$

79. *Writing Exercise*. Kay has incorrectly changed the sign of the middle term when factoring out the largest common factor. Thus, the signs in both terms of the final factorization are wrong. The number of points that should be deducted will vary.

81. $$\begin{aligned}(x-2)^2 &= (x)^2 + 2\cdot x(-2) + (-2)^2\\ &\qquad [(A-B)^2 = A^2 - 2AB + B^2]\\ &= x^2 - 4x + 4\end{aligned}$$

83. $$\begin{aligned}(x+2)(x-2) &= x^2 - 2^2\\ &\qquad [(A+B)(A-B) = A^2 - B^2]\\ &= x^2 - 4\end{aligned}$$

85. $$\begin{aligned}(4a+1)^2 &= (4a)^2 + 2(4a)(1) + 1^2\\ &\qquad [(A+B)^2 = A^2 + 2AB + B^2]\\ &= 16a^2 + 8a + 1\end{aligned}$$

87. $$\begin{aligned}(3c-10)^2 &= (3c)^2 - 2(3c)(10) + 10^2\\ &\qquad [(A-B)^2 = A^2 - 2AB + B^2]\\ &= 9c^2 - 60c + 100\end{aligned}$$

89. $$\begin{aligned}(8n+3)(8n-3) &= (8n)^2 - 3^2\\ &\qquad [(A+B)(A-B) = A^2 - B^2]\\ &= 64n^2 - 9\end{aligned}$$

91. For the trinomial $ax^2 + bx + c$ to be prime:

(1) Show that there is no common factor (other than 1 or -1)

(2) Multiply the leading coefficient, a, and the constant c.

$a \cdot c = ac$

(3) Show there are no factors of ac that add to b.

(4) The trinomial is prime.

93. $18x^2y^2 - 3xy - 10$

We will factor by grouping.

(1) There is no common factor (other than 1 or -1).

(2) Multiply the leading coefficient, 18, and the constant, -10:

$18(-10) = -180$

(3) We look for factors of -180 that add to -3. The numbers we want are -15 and 12.

(4) Rewrite the middle term:

$-3xy = -15xy + 12xy$

(5) Factor by grouping:

$$\begin{aligned}18x^2y^2 - 3xy - 10 &= 18x^2y^2 - 15xy + 12xy - 10\\ &= 3xy(6xy-5) + 2(6xy-5)\\ &= (6xy-5)(3xy+2)\end{aligned}$$

95. $9a^2b^3 + 25ab^2 + 16$

We cannot factor the leading term, $9a^2b^3$, in a way that will produce a middle term with variable factors ab^2, so this trinomial is prime.

97. $16t^{10} - 8t^5 + 1 = 16(t^5)^2 - 8t^5 + 1$

(1) There is no common factor (other than 1 or -1).

(2) Because $16t^{10}$ can be factored as $16t^5 \cdot t^5$ or $8t^5 \cdot 2t^5$ or $4t^5 \cdot 4t^5$, we have these possibilities:

$(16t^5 + \quad)(t^5 + \quad)$ and $(8t^5 + \quad)(2t^5 + \quad)$

and $(4t^5 + \quad)(4t^5 + \quad)$

(3) Since the last term is positive and the middle term is negative we need consider only negative factors of 1. The only negative pair of factors is $-1, -1$.

(4) We try some possibilities:

$(16t^5 - 1)(t^5 - 1) = 16t^{10} - 17t^5 + 1$

$(8t^5 - 1)(2t^5 - 1) = 16t^{10} - 10t^5 + 1$

$(4t^5 - 1)(4t^5 - 1) = 16t^{10} - 8t^5 + 1$

The factorization is $(4t^5 - 1)(4t^5 - 1)$, or $(4t^5 - 1)^2$.

99. $-15x^{2m} + 26x^m - 8 = -15(x^m)^2 + 26x^m - 8$

(1) Factor out -1 in order to have a trinomial with a positive leading coefficient.

$-15x^{2m} + 26x^m - 8 = -1(15x^{2m} - 26x^m + 8)$

(2) Because $15x^{2m}$ can be factored as $15x^m \cdot x^m$, or $5x^m \cdot 3x^m$, we have these possibilities:

$(15x^m + \quad)(x^m + \quad)$ and $(5x^m + \quad)(3x^m + \quad)$

(3) Since the last term is positive and the middle term is negative we need consider only negative factors of 8. There are 2 such pairs and they can be listed in two ways:

$-8, -1 \quad -4, -2$

and $\quad -1, -8 \quad -2, -4$

(4) We try some possibilities:

$(15x^m - 8)(x^m - 1) = 15x^{2m} - 9x^m + 8$

$(5x^m - 8)(3x^m - 1) = 15x^{2m} - 29x^m + 8$

$(5x^m - 2)(3x^m - 4) = 15x^{2m} - 26x^m + 8$

The factorization of $15x^{2m} - 26x^m + 8$ is $(5x^m - 2)(3x^m - 4)$. We must include the common factor to get a factorization of the original trinomial.

$-15x^{2m} + 26x^m - 8 = -1(5x^m - 2)(3x^m - 4)$, or $-(5x^m - 2)(3x^m - 4)$

101. $3a^{6n} - 2a^{3n} - 1 = 3(a^{3n})^2 - 2a^{3n} - 1$

(1) There is no common factor (other than 1 or -1).

(2) Because $3a^{6n}$ can be factored as $3a^{3n} \cdot a^{3n}$, we have this possibility:

$(3a^{3n} + \quad)(a^{3n} + \quad)$

(3) The only one pair of factors of -1: 1, -1.

(4) We try some possibilities:

$(3a^{3n} - 1)(a^{3n} + 1) = 3a^{6n} + 2a^{3n} - 1$

$(3a^{3n} + 1)(a^{3n} - 1) = 3a^{6n} - 2a^{3n} - 1$

The factorization of $3a^{6n} - 2a^{3n} - 1$ is $(3a^{3n} + 1)(a^{3n} - 1)$.

103. $7(t-3)^{2n} + 5(t-3)^n - 2 = 7[(t-3)^n]^2 + 5(t-3)^n - 2$

(1) There is no common factor (other than 1 or -1).

(2) Multiply the leading coefficient, 7, and the constant, -2:

$7(-2) = -14$

(3) Look for factors of -14 that add to 5. The numbers we want are 7 and -2.

(4) Rewrite the middle term:

$5(t-3)^n = 7(t-3)^n - 2(t-3)^n$

(5) Factor by grouping:

$7(t-3)^{2n} + 5(t-3)^n - 2$

$= 7(t-3)^{2n} + 7(t-3)^n - 2(t-3)^n - 2$

$= 7(t-3)^n [(t-3)^n + 1] - 2[(t-3)^n + 1]$

$= [(t-3)^n + 1][7(t-3)^n - 2]$

The factorization of $7(t-3)^{2n} + 5(t-3)^n - 2$ is

$[(t-3)^n + 1][7(t-3)^n - 2]$

5.3 Connecting the Concepts

1. $6x^5 - 18x^2$ Common factor: $6x^2$

$= 6x^2 \cdot x^3 - 6x^2 \cdot 3$

$= 6x^2(x^3 - 3)$

3. $2x^2 + 13x - 7$ No common factor; factor with FOIL.

$= (x+7)(2x-1)$

5. $5x^2 + 40x - 100$ Common factor: 5

$= 5(x^2 + 8x - 20)$ Factor with FOIL

$= 5(x-2)(x+10)$

7. $7x^2y - 21xy - 28y$ Common factor: $7y$

$= 7y(x^2 - 3x - 4)$ Factor with FOIL

$= 7y(x-4)(x+1)$

9. $b^2 - 14b + 49$ Perfect-square trinomial

$= b^2 - 2 \cdot 7 \cdot b + 7^2$

$= (b-7)^2$

11. $c^3 + c^2 - 4c - 4$ No common factor; factor with grouping.

$= c^2(c+1) - 4(c+1)$

$= (c+1)(c^2 - 4)$

$= (c+1)(c+2)(c-2)$

13. $t^2 + t - 10$ No common factor

The trinomial is prime.

15. $15p^2 + 16pq + 4q^2$ No common factor; factor with FOIL.

$= (3p + 2q)(5p + 2q)$

17. $x^2 + 4x - 77$ No common factor; factor with FOIL.

$= (x+11)(x-7)$

19. $5 + 3x - 2x^2$ Common factor: -1; write in descending order

$= -1(2x^2 - 3x - 5)$ Factor with FOIL

$= -1(2x-5)(x+1)$

Exercise Set 5.4

1. $4x^2 + 49$ is not a trinomial. It is not a difference of squares because the terms do not have different signs. There is no common factor, so $4x^2 + 49$ is a prime polynomial.

3. $t^2 - 100 = t^2 - 10^2$, so $t^2 - 100$ is a difference of squares.

5. $9x^2 + 6x + 1 = (3x)^2 + 2 \cdot 3x \cdot 1 + 1^2$, so this is a perfect-square trinomial.

7. $2t^2 + 10t + 6$ does not contain a term that is a square so it is neither a perfect-square trinomial nor a difference of squares. (We could also say that it is not a difference of squares because it is not a binomial.) There is a common factor, 2, so this is not a prime polynomial. Thus it is none of the given possibilities.

9. $16t^2 - 25 = (4t)^2 - 5^2$, so $16t^2 - 25$ is a difference of squares.

11. $x^2 + 18x + 81$

(1) Two terms, x^2 and 81, are squares.

(2) Neither x^2 nor 81 is being subtracted.

(3) Twice the product of the square roots, $2 \cdot x \cdot 9$, is $18x$, the remaining term.

Thus, $x^2 + 18x + 81$ is a perfect-square trinomial.

13. $x^2 - 10x - 25$

(1) Two terms, x^2 and 25, are squares.

(2) There is a minus sign before 25, so $x^2 - 10x - 25$ is not a perfect-square trinomial.

15. $x^2 - 3x + 9$

(1) Two terms, x^2 and 9, are squares.

(2) There is no minus sign before x^2 or 9.

(3) Twice the product of the square roots, $2 \cdot x \cdot 3$, is $6x$. This is neither the remaining term nor its opposite, so $x^2 - 3x + 9$ is not a perfect-square trinomial.

17. $9x^2 + 25 - 30x$

(1) Two terms $9x^2$, and 25, are squares.

(2) Neither $9x^2$ nor 25 is being subtracted.

(3) Twice the product of the square roots, $2 \cdot 3x \cdot 5$, is $30x$, the opposite of the remaining term, $-30x$.

Thus, $9x^2 + 25 - 30x$ is a perfect-square trinomial.

19.
$$\begin{array}{l} x^2 + 16x + 64 \\ = x^2 + 2 \cdot x \cdot 8 + 8^2 = (x+8)^2 \\ \quad\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ = A^2 + 2 \;\; A \;\; B + B^2 = (A+B)^2 \end{array}$$

21.
$$\begin{array}{l} x^2 - 10x + 25 \\ = x^2 - 2 \cdot x \cdot 5 + 5^2 = (x-5)^2 \\ \quad\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ = A^2 - 2 \;\; A \;\; B + B^2 = (A-B)^2 \end{array}$$

23.
$$\begin{aligned} 5p^2 + 20p + 20 &= 5(p^2 + 4p + 4) \\ &= 5(p^2 + 2 \cdot p \cdot 2 + 2^2) \\ &= 5(p+2)^2 \end{aligned}$$

25.
$$\begin{aligned} 1 - 2t + t^2 &= 1^2 - 2 \cdot 1 \cdot t + t^2 \\ &= (1-t)^2 \end{aligned}$$

We could also factor as follows:
$$\begin{aligned} 1 - 2t + t^2 &= t^2 - 2t + 1 \\ &= t^2 - 2 \cdot t \cdot 1 + 1^2 \\ &= (t-1)^2 \end{aligned}$$

27.
$$\begin{aligned} 18x^2 + 12x + 2 &= 2(9x^2 + 6x + 1) \\ &= 2[(3x)^2 + 2 \cdot 3x \cdot 1 + 1^2] \\ &= 2(3x+1)^2 \end{aligned}$$

29.
$$\begin{aligned} 49 - 56y + 16y^2 &= 16y^2 - 56y + 49 \\ &= (4y)^2 - 2 \cdot 4y \cdot 7 + 7^2 \\ &= (4y-7)^2 \end{aligned}$$

We could also factor as follows:
$$\begin{aligned} 49 - 56y + 16y^2 &= 7^2 - 2 \cdot 7 \cdot 4y + (4y)^2 \\ &= (7-4y)^2 \end{aligned}$$

31.
$$\begin{aligned} -x^5 + 18x^4 - 81x^3 &= -x^3(x^2 - 18x + 81) \\ &= -x^3(x^2 - 2 \cdot x \cdot 9 + 9^2) \\ &= -x^3(x-9)^2 \end{aligned}$$

33.
$$\begin{aligned} 2n^3 + 40n^2 + 200n &= 2n(n^2 + 20n + 100) \\ &= 2n(n^2 + 2 \cdot n \cdot 10 + 10^2) \\ &= 2n(n+10)^2 \end{aligned}$$

35.
$$\begin{aligned} 20x^2 + 100x + 125 &= 5(4x^2 + 20x + 25) \\ &= 5[(2x)^2 + 2 \cdot 2x \cdot 5 + 5^2] \\ &= 5(2x+5)^2 \end{aligned}$$

37. $49 - 42x + 9x^2 = 7^2 - 2 \cdot 7 \cdot 3x + (3x)^2 = (7-3x)^2$, or $(3x-7)^2$

39.
$$\begin{aligned} &16x^2 + 24x + 9 = (4x)^2 + 2 \cdot 4x \cdot 3 + 3^2 \\ &= (4x+3)^2 \end{aligned}$$

41.
$$\begin{aligned} 2 + 20x + 50x^2 &= 2(1 + 10x + 25x^2) \\ &= 2[1^2 + 2 \cdot 1 \cdot 5x + (5x)^2] \\ &= 2(1+5x)^2, \text{ or } 2(5x+1)^2 \end{aligned}$$

43.
$$\begin{aligned} 9p^2 + 12pq + 4q^2 &= (3p)^2 + 2 \cdot 3p \cdot 2q + (2q)^2 \\ &= (3p+2q)^2 \end{aligned}$$

45. $a^2 - 12ab + 49b^2$

This is not a perfect square trinomial because $-2 \cdot a \cdot 7b = -14ab \neq -12ab$. Nor can it be factored using the methods of Sections 5.2 and 5.3. Thus, it is prime.

47.
$$\begin{aligned} -64m^2 - 16mn - n^2 &= -1(64m^2 + 16mn + n^2) \\ &= -1[(8m)^2 + 2 \cdot 8m \cdot n + n^2] \\ &= -1(8m+n)^2, \text{ or } -(8m+n)^2 \end{aligned}$$

49.
$$\begin{aligned} -32s^2 + 80st - 50t^2 &= -2(16s^2 - 40st + 25t^2) \\ &= -2[(4s)^2 - 2 \cdot 4s \cdot 5t + (5t)^2] \\ &= -2(4s-5t)^2 \end{aligned}$$

51. $x^2 - 100$

(1) The first expression is a square: x^2

The second expression is a square: $100 = 10^2$

(2) The terms have different signs.

Thus, $x^2 - 100$ is a difference of squares $x^2 - 10^2$.

53. $n^4 + 1$

(1) The first expression is a square: $n^4 = (n^2)^2$

The second expression is a square: $1 = 1^2$

(2) The terms do not have different signs.

Thus, $n^4 + 1$ is not a difference of squares.

55. $-1 + 64t^2$ or $64t^2 - 1$

(1) The first term is a square: $1 = 1^2$.

The second term is a square: $64t^2 = (8t)^2$.

(2) The terms have different signs.

Thus, $-1 + 64t^2$ is a difference of squares, $(8t)^2 - 1^2$.

57. $x^2 - 25 = x^2 - 5^2 = (x+5)(x-5)$

59. $p^2 - 9 = p^2 - 3^2 = (p+3)(p-3)$

61. $-49+t^2 = t^2-49 = t^2-7^2 = (t+7)(t-7)$, or $(7+t)(-7+t)$

63. $6a^2 - 24 = 6(a^2-4) = 6(a^2-2^2) = 6(a+2)(a-2)$

65. $49x^2 - 14x + 1 = (7x)^2 - 2 \cdot 7x \cdot 1 + 1^2 = (7x-1)^2$

67. $200 - 2t^2 = 2(100-t^2) = 2(10^2 - t^2)$
$= 2(10+t)(10-t)$

69. $-80a^2 + 45 = -5(16a^2 - 9) = -5[(4a^2) - 3^2]$
$= -5(4a+3)(4a-3)$

71. $5t^2 - 80 = 5(t^2-16) = 5(t^2-4^2)$
$= 5(t+4)(t-4)$

73. $8x^2 - 162 = 2(4x^2 - 81) = 2[(2x)^2 - 9^2]$
$= 2(2x+9)(2x-9)$

75. $36x - 49x^3 = x(36 - 49x^2) = x[6^2 - (7x)^2]$
$= x(6+7x)(6-7x)$

77. $49a^4 - 20$

There is no common factor (other than 1 or −1). Since 20 is not a square, this is not a difference of squares. Thus, the polynomial is prime.

79.
$$\begin{aligned} &t^4 - 1 \\ &= (t^2)^2 - 1^2 \\ &= (t^2+1)(t^2-1) \\ &= (t^2+1)(t+1)(t-1) \quad \text{Factoring further; } t^2-1 \text{ is a difference of squares} \end{aligned}$$

81.
$$\begin{aligned} -3x^3 + 24x^2 - 48x &= -3x(x^2 - 8x + 16) \\ &= -3x(x^2 - 2\cdot x \cdot 4 + 4^2) \\ &= -3x(x-4)^2 \end{aligned}$$

83.
$$\begin{aligned} 75t^3 - 27t &= 3t(25t^2 - 9) \\ &= 3[(5t)^2 - 3^2] \\ &= 3(5t+3)(5t-3) \end{aligned}$$

85.
$$\begin{aligned} a^8 - 2a^7 + a^6 &= a^6(a^2 - 2a + 1) \\ &= a^6(a^2 - 2\cdot a\cdot 1 + 1^2) \\ &= a^6(a-1)^2 \end{aligned}$$

87.
$$\begin{aligned} 10a^2 - 10b^2 &= 10(a^2 - b^2) \\ &= 10(a+b)(a-b) \end{aligned}$$

89.
$$\begin{aligned} 16x^4 - y^4 &= (4x^2)^2 - (y^2)^2 \\ &= (4x^2+y^2)(4x^2-y^2) \\ &= (4x^2+y^2)(2x+y)(2x-y) \end{aligned}$$

91.
$$\begin{aligned} 18t^2 - 8s^2 &= 2(9t^2 - 4s^2) \\ &= 2[(3t)^2 - (2s)^2] \\ &= 2(3t+2s)(3t-2s) \end{aligned}$$

$= (a^2+9b^2)(a+3b)(a-3b)$

93. *Writing Exercise.* Two terms must be squares. There must be no minus sign before either square. The remaining term must be twice the product of the square roots of the squares or must be the opposite of that product.

95. $\left(2x^2y^4\right)^3 = 2^3\left(x^2\right)^3\left(y^4\right)^3 = 8x^6y^{12}$

97.
$$\begin{aligned} (x+1)(x+1)(x+1) &= \left(x^2+x+x+1\right)(x+1) \quad \text{FOIL} \\ &= \left(x^2+2x+1\right)(x+1) \\ &= \left(x^2+2x+1\right)\cdot x + \left(x^2+2x+1\right)\cdot 1 \\ &= x^3+2x^2+x+x^2+2x+1 \\ &= x^3+3x^2+3x+1 \end{aligned}$$

99.
$$\begin{aligned} &(p+q)^3 = (p+q)(p+q)^2 \\ &= (p+q)\left(p^2+2pq+q^2\right) \\ &= p\left(p^2+2pq+q^2\right) + q\left(p^2+2pq+q^2\right) \\ &= p^3+2p^2q+pq^2+p^2q+2pq^2+q^3 \\ &= p^3+3p^2q+3pq^2+q^3 \end{aligned}$$

101. *Writing Exercise.*
$$\begin{aligned} &(x+3)(x-3) \\ &= (x-3)(x+3) \quad \text{Using a commutative law} \\ &= x^2 - 9 \end{aligned}$$

Since x^2-9 and x^2+9 are not equivalent the student's factorization of x^2+9 is incorrect. (Also it can be easily shown by multiplying that $(x+3)(x-3) \neq x^2+9$.) The student should recall that, if the greatest common factor has been removed, a sum of squares cannot be factored further.

103.
$$\begin{aligned} x^8 - 2^8 &= (x^4+2^4)(x^4-2^4) \\ &= (x^4+2^4)(x^2+2^2)(x^2-2^2) \\ &= (x^4+2^4)(x^2+2^2)(x+2)(x-2), \text{ or} \\ &\quad (x^4+16)(x^2+4)(x+2)(x-2) \end{aligned}$$

105.
$$\begin{aligned} &18x^3 - \frac{8}{25}x = 2x\left(9x^2 - \frac{4}{25}\right) \\ &= 2x\left(3x+\frac{2}{5}\right)\left(3x-\frac{2}{5}\right) \end{aligned}$$

107.
$$\begin{aligned} &(y-5)^4 - z^8 \\ &= [(y-5)^2+z^4][(y-5)^2-z^4] \\ &= [(y-5)^2+z^4][y-5+z^2][y-5-z^2] \\ &= (y^2-10y+25+z^4)(y-5+z^2)(y-5-z^2) \end{aligned}$$

109.
$$\begin{aligned} -x^4 + 8x^2 + 9 &= -1(x^4 - 8x^2 - 9) \\ &= -1(x^2-9)(x^2+1) \\ &= -1(x+3)(x-3)(x^2+1) \end{aligned}$$

111.
$$\begin{aligned} &(y+3)^2 + 2(y+3) + 1 \\ &= (y+3)^2 + 2\cdot(y+3)\cdot 1 + 1^2 \\ &= [(y+3)+1]^2 \\ &= (y+4)^2 \end{aligned}$$

113. $27p^3 - 45p^2 - 75p + 125$
$= 9p^2(3p-5) - 25(3p-5)$
$= (3p-5)(9p^2-25)$
$= (3p-5)(3p+5)(3p-5)$, or
$(3p-5)^2(3p+5)$

115. $81 - b^{4k} = 9^2 - (b^{2k})^2$
$= (9+b^{2k})(9-b^{2k})$
$= (9+b^{2k})[3^2-(b^k)^2]$
$= (9+b^{2k})(3+b^k)(3-b^k)$

117. $x^2(x+1)^2 - (x^2+1)^2$
$= x^2(x^2+2x+1) - (x^4+2x^2+1)$
$= x^4 + 2x^3 + x^2 - x^4 - 2x^2 - 1$
$= 2x^3 + x^2 - 2x^2 - 1$
$= 2x^3 - x^2 - 1$

119. $y^2 + 6y + 9 - x^2 - 8x - 16$
$= (y^2+6y+9) - (x^2+8x+16)$
$= (y+3)^2 - (x+4)^2$
$= [(y+3)+(x+4)][(y+3)-(x+4)]$
$= (y+3+x+4)(y+3-x-4)$
$= (y+x+7)(y-x-1)$

121. For $c = a^2$, $2 \cdot a \cdot 3 = 24$. Then $a = 4$, so $c = 4^2 = 16$.

123. $(x+1)^2 - x^2$
$= [(x+1)+x][(x+1)-x]$
$= 2x+1$
$= (x+1)+x$

Exercise Set 5.5

1. $x^3 - 1 = (x)^3 - 1^3$
This is a difference of two cubes.

3. $9x^4 - 25 = \left(3x^2\right)^2 - 5^2$
This is a difference of two squares.

5. $1000t^3 + 1 = (10t)^3 + 1^3$
This is a sum of two cubes.

7. $25x^2 + 8x$ has a common factor of x so it is not prime, but it does not fall into any of the other categories. It is classified as none of these.

9. $s^{21} - t^{15} = \left(s^7\right)^3 - \left(t^5\right)^3$
This is a difference of two cubes .

11. $x^3 - 64 = x^3 - 4^3$
$= (x-4)(x^2+4x+16)$
$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

13. $z^3 + 1 = z^3 + 1^3$
$= (z+1)(z^2-z+1)$
$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

15. $t^3 - 1000 = t^3 - 10^3$
$= (t-10)(t^2+10t+100)$
$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

17. $27x^3 + 1 = (3x)^3 - 1^3$
$= (3x+1)(9x^2-3x+1)$
$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

19. $64 - 125x^3 = 4^3 - (5x)^3 = (4-5x)\left(16+20x+25x^2\right)$

21. $x^3 - y^3 = (x-y)\left(x^2+xy+y^2\right)$

23. $a^3 + \frac{1}{8} = a^3 + \left(\frac{1}{2}\right)^3 = \left(a+\frac{1}{2}\right)\left(a^2 - \frac{1}{2}a + \frac{1}{4}\right)$

25. $8t^3 - 8 = 8\left(t^3-1\right) = 8\left(t^3-1^3\right) = 8(t-1)\left(t^2+t+1\right)$

27. $$\begin{aligned} 54x^3+2 &= x\left(27x^3+1\right) = 2\left[(3x)^3+1^3\right] \\ &= 2(3x+1)\left(9x^2-3x+1\right) \end{aligned}$$

29. $$\begin{aligned} rs^4+64rs &= rs\left(s^3+64\right) \\ &= rs\left(s^3+4^3\right) \\ &= rs(s+4)\left(s^2-4s+16\right) \end{aligned}$$

31. $$\begin{aligned} 5x^3-40z^3 &= 5\left(x^3-8z^3\right) \\ &= 5\left[x^3-(2z)^3\right] \\ &= 5(x-2z)\left(x^2+2xz+4z^2\right) \end{aligned}$$

33. $y^3 - \frac{1}{1000} = y^3 - \left(\frac{1}{10}\right)^3 = \left(y-\frac{1}{10}\right)\left(y^2+\frac{1}{10}y+\frac{1}{100}\right)$

35. $x^3 + 0.001 = x^3 + (0.1)^3 = (x+0.1)\left(x^2-0.1x+0.01\right)$

37. $$\begin{aligned} 64x^6-8t^6 &= 8\left(8x^6-t^6\right) \\ &= 8\left[\left(2x^2\right)^3-\left(t^2\right)^3\right] \\ &= 8\left(2x^2-t^2\right)\left(4x^4+2x^2t^2+t^4\right) \end{aligned}$$

39. $$\begin{aligned} 54y^4-128y &= 2y\left(27y^3-64\right) \\ &= 2y\left[(3y)^3-4^3\right] \\ &= 2y(3y-4)\left(9y^2+12y+16\right) \end{aligned}$$

41. $z^6 - 1$

$= \left(z^3\right)^2 - 1^2$	Writing as a c ference of squa
$= \left(z^3+1\right)\left(z^3-1\right)$	Factoring a diff ence of squares
$= (z+1)\left(z^2-z+1\right)(z-1)\left(z^2+z+1\right)$	Factoring a su and a difference cubes

43. $t^6 + 64y^6 = \left(t^2\right)^3 + \left(4y^2\right)^3 = \left(t^2+4y^2\right)\left(t^4-4t^2y^2+16y^4\right)$

45. $x^{12} - y^3z^{12} = (x^4)^3 - (yz^4)^3$
$= (x^4-yz^4)(x^8+x^4yz^4+y^2z^8)$

47. *Writing Exercise.*
$(x+y)^3 = (x+y)(x+y)(x+y)$
$= (x^2+2xy+y^2)(x+y)$
$= x^3+3x^2y+3xy^2+y^3$
$\neq x^3+y^3$

49. Two points on the line are $(-2,-5)$ and $(3,-6)$.

$$m = \frac{-6-(-5)}{3-(-2)} = \frac{-1}{5}$$

51. $2x - 5y = 10$

Find the y-intercept:

$$-5y = 10$$
$$y = -2$$

The y-intercept is $(0, -2)$.

Find the x-intercept:

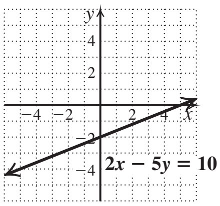
$$2x = 10$$
$$x = 5$$

The x-intercept is $(5, 0)$.

Find find a third point, we replace y with 2 and solve for x.

$$2x - 5 \cdot 2 = 10$$
$$2x - 10 = 10$$
$$2x = 20$$
$$x = 10$$

The point is $(10, 2)$.



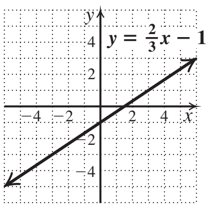
53. Graph: $y = \frac{2}{3}x - 1$

Because the equation is in the form $y = mx + b$, we know the y-intercept is $(0, -1)$. We find two other points on the line, substituting multiples of 3 for x to avoid fractions.

When $x = -3$, $y = \frac{2}{3}(-3) - 1 = -2 - 1 = -3$.

When $x = 6$, $y = \frac{2}{3} \cdot 6 - 1 = 4 - 1 = 3$.

x	y
0	-1
-3	-3
6	3



55. *Writing Exercise.* The model shows a cube with volume a^3 from which a portion whose volume is b^3 has been removed. This leaves a remaining volume which can be expressed as $a^2(a-b) + ab(a-b) + b^2(a-b)$, or $(a-b)(a^2+ab+b^2)$. Thus, $a^3 - b^3 = (a-b)(a^2+ab+b^2)$.

57. $x^{6a} - y^{3b} \quad = \quad (x^{2a})^3 - (y^b)^3$

59. $(x+5)^3 + (x-5)^3$ Sum of cubes

$$= [(x+5)+(x-5)][(x+5)^2 - (x+5)(x-5) + (x-5)^2]$$
$$= 2x[(x^2+10x+25) - (x^2-25) + (x^2-10x+25)]$$
$$= 2x(x^2+10x+25-x^2+25+x^2-10x+25)$$
$$= 2x(x^2+75)$$

61. $5x^3y^6 - \frac{5}{8}$

63. $x^{6a} - (x^{2a}+1)^3$

$$= \left[x^{2a} - (x^{2a}+1)\right]\left[x^{4a} + x^{2a}(x^{2a}+1) + (x^{2a}+1)^2\right]$$
$$= (x^{2a} - x^{2a} - 1)(x^{4a} + x^{4a} + x^{2a} + x^{4a} + 2x^{2a} + 1)$$
$$= -(3x^{4a} + 3x^{2a} + 1)$$

65. $t^4 - 8t^3 - t + 8$

$$= t^3(t-8) - (t-8)$$
$$= (t-8)(t^3-1)$$
$$= (t-8)(t-1)(t^2+t+1)$$

Exercise Set 5.6

1. common factor

3. grouping

5. $5a^2 - 125$

$= 5(a^2 - 25)$ 5 is a common factor.

$= 5(a+5)(a-5)$ Factoring the difference of squares

7. $y^2 + 49 - 14y$

$= y^2 - 14y + 49$ Perfect-square trinomial

$= (y-7)^2$

9. $3t^2 + 16t + 21$

There is no common factor (other than 1 or -1). This trinomial has three terms, but it is not a perfect-square trinomial. Multiply the leading coefficient and the constant, 3 and 21: $3 \cdot 21 = 63$. Try to factor 63 so that the sum of the factors is 16. The numbers we want are 7 and 9: $7 \cdot 9 = 63$ and $7 + 9 = 16$. Split the middle term and factor by grouping.

$$3t^2 + 16t + 21 = 3t^2 + 7t + 9t + 21$$
$$= t(3t+7) + 3(3t+7)$$
$$= (3t+7)(t+3)$$

11. $x^3 + 18x^2 + 81x$

$= x(x^2 + 18x + 81)$ x is a common factor.

$= x(x^2 + 2 \cdot x \cdot 9 + 9^2)$ Perfect-square trinomial

$= x(x+9)^2$

13. $x^3 - 5x^2 - 25x + 125$

$= x^2(x-5) - 25(x-5)$ Factoring by grouping

$= (x-5)(x^2-25)$

$= (x-5)(x+5)(x-5)$ Factoring the difference of squares

$= (x-5)^2(x+5)$

15. $27t^3 - 3t$
$= 3t(9t^2 - 1)$ 3t is a common factor.
$= 3t[(3t)^2 - 1^2]$ Difference of squares
$= 3t(3t + 1)(3t - 1)$

17. $9x^3 + 12x^2 - 45x$
$= 3x(3x^2 + 4x - 15)$ $3x$ is a common factor.
$= 3x(x + 3)(3x - 5)$ Factoring the trinomial

19. $t^2 + 25$

The polynomial has no common factor and is not a difference of squares. It is prime.

21. $6y^2 + 18y - 240$
$= 6(y^2 + 3y - 40)$ 6 is a common factor.
$= 6(y + 8)(y - 5)$ Factoring the trinomial

23. $-2a^6 + 8a^5 - 8a^4$
$= -2a^4(a^2 - 4a + 4)$ Factoring out $-2a^4$
$= -2a^4(a - 2)^2$ Factoring the perfect-square trinomial

25. $5x^5 - 80x$
$= 5x(x^4 - 16)$ $5x$ is a common factor.
$= 5x[(x^2)^2 - 4^2]$ Difference of squares
$= 5x(x^2 + 4)(x^2 - 4)$ Difference of squares
$= 5x(x^2 + 4)(x + 2)(x - 2)$

27. $t^4 - 9$ Difference of squares
$= (t^2 + 3)(t^2 - 3)$

29. $-x^6 + 2x^5 - 7x^4$
$= -x^4(x^2 - 2x + 7)$

The trinomial is prime, so this is the complete factorization.

31. $p^2 - q^2$ Difference of squares
$= (p + q)(p - q)$

33. $ax^2 + ay^2 = a(x^2 + y^2)$

35. $= 2\pi rh + 2\pi r^2$ $2\pi r$ is a common factor
$= 2\pi r(h + r)$

37. $(a + b)(5a) + (a + b)(3b)$
$= (a + b)(5a + 3b)$ $(a + b)$ is a common factor.

39. $x^2 + x + xy + y$
$= x(x + 1) + y(x + 1)$ Factoring by grouping
$= (x + 1)(x + y)$

41. $a^2 - 2a - ay + 2y$
$= a(a - 2) - y(a - 2)$ Factoring by grouping
$= (a - 2)(a - y)$

43. $3x^2 + 13xy - 10y^2 = (3x - 2y)(x + 5y)$

45. $8m^3n - 32m^2n^2 + 24mn$
$= 8mn(m^2 - 4mn + 3)$ $8mn$ is a common factor

47. $4b^2 + a^2 - 4ab$
$= 4b^2 - 4ab + a^2$
$= (2b)^2 - 2 \cdot 2b \cdot a + a^2$ Perfect-square trinomial
$= (2b - a)^2$

This result can also be expressed as $(a - 2b)^2$.

49. $16x^2 + 24xy + 9y^2$
$= (4x)^2 + 2 \cdot 4x \cdot 3y + (3y)^2$ Perfect-square trinomial
$= (4x + 3y)^2$

51. $m^2 - 5m + 8$

We cannot find a pair of factors whose product is 8 and whose sum is -5, so $m^2 - 5m + 8$ is prime.

53. $a^4b^4 - 16$
$= (a^2b^2)^2 - 4^2$ Difference of squares
$= (a^2b^2 + 4)(a^2b^2 - 4)$ Difference of squares
$= (a^2b^2 + 4)(ab + 2)(ab - 2)$

55. $80cd^2 - 36c^2d + 4c^3$
$= 4c(20d^2 - 9cd + c^2)$ $4c$ is a common factor
$= 4c(4d - c)(5d - c)$ Factoring the trinomial

57. $3b^2 + 17ab - 6a^2 = (3b - a)(b + 6a)$

59. $-12 - x^2y^2 - 8xy$
$= -x^2y^2 - 8xy - 12$
$= -1(x^2y^2 + 8xy + 12)$
$= -1(xy + 2)(xy + 6)$, or $-(xy + 2)(xy + 6)$

61. $5p^2q^2 + 25pq - 30$
$= 5(p^2q^2 + 5pq - 6)$ 5 is a common factor
$= 5(pq + 6)(pq - 1)$ Factoring the trinomial

63. $4ab^5 - 32b^4 + a^2b^6$
$= b^4(4ab - 32 + a^2b^2)$ b^4 is a common factor.
$= b^4(a^2b^2 + 4ab - 32)$
$= b^4(ab + 8)(ab - 4)$ Factoring the trinomial

65. $x^6 + x^5y - 2x^4y^2$
$= x^4(x^2 + xy - 2y^2)$ x^4 is a common factor.
$= x^4(x + 2y)(x - y)$ Factoring the trinomial

67. $36a^2 - 15a + \frac{25}{16}$
$= (6a)^2 - 2 \cdot 6a \cdot \frac{5}{4} + \left(\frac{5}{4}\right)^2$ Perfect-square trinomial
$= \left(6a - \frac{5}{4}\right)^2$

69. $\frac{1}{81}x^2 - \frac{8}{27}x + \frac{16}{9}$
$= \left(\frac{1}{9}x\right)^2 - 2 \cdot \frac{1}{9}x \cdot \frac{4}{3} + \left(\frac{4}{3}\right)^2$ Perfect-square trinomial
$= \left(\frac{1}{9}x - \frac{4}{3}\right)^2$

If we had factored out $\frac{1}{9}$ at the outset, the final result would have been $\frac{1}{9}\left(\frac{1}{3}x-4\right)^2$.

71. $1-16x^{12}y^{12}$
$= (1+4x^6y^6)(1-4x^6y^6)$ Difference of squares
$= (1+4x^6y^6)(1+2x^3y^3)(1-2x^3y^3)$ Difference of squares

73. $4a^2b^2+12ab+9$
$= (2ab)^2+2\cdot 2ab\cdot 3+3^2$ Perfect-square trinomial
$= (2ab+3)^2$

75. $z^4+6z^3-6z^2-36z$
$= z(z^3+6z^2-6z-36)$ z is a common factor
$= z\left[z^2(z+6)-6(z+6)\right]$ Factoring by grouping
$= z(z+6)(z^2-6)$

77. *Writing Exercise.* Both are correct; $(x-4)^2 = x^2-8x+16 = 16-8x+x^2 = (4-x)^2$

79. $8x-9=0$
$8x=9$ Adding 9 to both sides
$x=\frac{9}{8}$ Dividing both sides by 8

The solution is $\frac{9}{8}$.

81. $2x+7=0$
$2x=-7$ Subtracting 7 from both sides
$x=-\frac{7}{2}$ Dividing both sides by 2

The solution is $-\frac{7}{2}$.

83. $3-x=0$
$3=x$ Adding x to both sides

The solution is 3.

85. $2x-5=8x+1$
$-5=6x+1$ Subtracting $2x$ from both sides
$-6=6x$ Subtracting 1 from both sides
$-1=x$ Dividing both sides by 6

The solution is -1.

87. *Writing Exercise.* Find the product of a binomial and a trinomial. One example is found as follows:

$$(x+1)(2x^2-3x+4)$$
$$= 2x^3-3x^2+4x+2x^2-3x+4$$
$$= 2x^3-x^2+x+4$$

89. $-(x^5+7x^3-18x)$
$= -x(x^4+7x^2-18)$
$= -x(x^2+9)(x^2-2)$

91. $-x^4+7x^2+18$
$= -1(x^4-7x^2-18)$
$= -1(x^2+2)(x^2-9)$
$= -1(x^2+2)(x+3)(x-3)$, or
$-(x^2+2)(x+3)(x-3)$

93. $y^2(y+1)-4y(y+1)-21(y+1)$
$= (y+1)(y^2-4y-21)$
$= (y+1)(y-7)(y+3)$

95. $(y+4)^2+2x(y+4)+x^2$
$= (y+4)^2+2\cdot(y+4)\cdot x+x^2$ Perfect-square trinomial
$= [(y+4)+x]^2$
$= (y+4+x)^2$

97. $2(a+3)^4-(a+3)^3(b-2)-(a+3)^2(b-2)^2$
$= (a+3)^2[2(a+3)^2-(a+3)(b-2)-(b-2)^2]$
$= (a+3)^2[2(a+3)+(b-2)][(a+3)-(b-2)]$
$= (a+3)^2(2a+6+b-2)(a+3-b+2)$
$= (a+3)^2(2a+b+4)(a-b+5)$

99. $49x^4+14x^2+1-25x^6$
$= (7x^2+1)^2-25x^6$ Perfect-square trinomial
$= \left[(7x^2+1)-5x^3\right]\left[(7x^2+1+5x^3)\right]$ Difference of squares
$= (7x^2+1-5x^3)(7x^2+1+5x^3)$

Exercise Set 5.7

1. Equations of the type $ax^2+bx+c=0$, with $a\neq 0$, are quadratic, so choice (c) is correct.

3. The principle of zero products states that $A\cdot B=0$ if and only if $A=0$ or $B=0$, so choice (d) is correct.

5. $(x+2)(x+9)=0$

We use the principle of zero products.

$x+2=0$ *or* $x+9=0$
$x=-2$ *or* $x=-9$

Check:

For -2:

$(x+2)(x+9)$	$= 0$
$(-2+2)(-2+9)$	0
$0\cdot 7$	

$0 \stackrel{?}{=} 0$ TRUE

For -9:

$(x+2)(x+9)$	$= 0$
$(-9+2)(-9+9)$	0
$-7\cdot 0$	

$0 \stackrel{?}{=} 0$ TRUE

The solutions are -2 and -9.

7. $(2t-3)(t+6)=0$

$2t-3=0 \quad or \quad t+6=0$

$2t=3 \quad or \quad t=-6$

$t=\frac{3}{2} \quad or \quad t=-6$

The solutions are $\frac{3}{2}$ and -6.

9. $4(7x-1)(10x-3)=0$

$(7x-1)(10x-3)=0$ Dividing both sides by 4

$7x-1=0 \quad or \quad 10x-3=0$

$7x=1 \quad or \quad 10x=3$

$x=\frac{1}{7} \quad or \quad x=\frac{3}{10}$

The solutions are $\frac{1}{7}$ and $\frac{3}{10}$.

11. $x(x-7)=0$

$x=0 \quad or \quad x-7=0$

$x=0 \quad or \quad x=7$

The solutions are 0 and 7.

13. $\left(\frac{2}{3}x-\frac{12}{11}\right)\left(\frac{7}{4}x-\frac{1}{12}\right)=0$

$\frac{2}{3}x-\frac{12}{11}=0 \quad or \quad \frac{7}{4}x-\frac{1}{12}=0$

$\frac{2}{3}x=\frac{12}{11} \quad or \quad \frac{7}{4}x=\frac{1}{12}$

$x=\frac{3}{2}\cdot\frac{12}{11} \quad or \quad x=\frac{4}{7}\cdot\frac{1}{12}$

$x=\frac{18}{11} \quad or \quad x=\frac{1}{21}$

The solutions are $\frac{18}{11}$ and $\frac{1}{21}$.

15. $6n(3n+8)=0$

$6n=0 \quad or \quad 3n+8=0$

$n=0 \quad or \quad 3n=-8$

$n=0 \quad or \quad n=-\frac{8}{3}$

The solutions are 0 and $-\frac{8}{3}$.

17. $(20-0.4x)(7-0.1x)=0$

$20-0.4x=0 \quad or \quad 7-0.1x=0$

$-0.4x=-20 \quad or \quad -0.1x=-7$

$x=50 \quad or \quad x=70$

The solutions are 50 and 70.

19. $(3x-2)(x+5)(x-1)=0$

$3x-2=0 \quad or \quad x+5=0 \quad or \quad x-1=0$

$3x=2 \quad or \quad x=-5 \quad or \quad x=1$

$x=\frac{2}{3} \quad or \quad x=-5 \quad or \quad x=1$

The solutions are -5, $\frac{2}{3}$, and 1.

21. $x^2-7x+6=0$

$(x-6)(x-1)=0$ Factoring

$x-6=0 \quad or \quad x-1=0$ Using the principle of zero products

$x=6 \quad or \quad x=1$

The solutions are 6 and 1.

23. $x^2+4x-21=0$

$(x-3)(x+7)=0$ Factoring

$x-3=0 \quad or \quad x+7=0$ Using the principle of zero products

$x=3 \quad or \quad x=-7$

The solutions are 3 and -7.

25. $n^2+11n+18=0$

$(n+9)(n+2)=0$

$n+9=0 \quad or \quad n+2=0$

$n=-9 \quad or \quad n=-2$

The solutions are -9 and -2.

27. $x^2-10x=0$

$x(x-10)=0$

$x=0 \quad or \quad x-10=0$

$x=0 \quad or \quad x=10$

The solutions are 0 and 10.

29. $6t+t^2=0$

$t(6+t)=0$

$t=0 \quad or \quad 6+t=0$

$t=0 \quad or \quad t=-6$

The solutions are 0 and -6.

31. $x^2-36=0$

$(x+6)(x-6)=0$

$x+6=0 \quad or \quad x-6=0$

$x=-6 \quad or \quad x=6$

The solutions are -6 and 6.

33. $4t^2=49$

$4t^2-49=0$

$(2t+7)(2t-7)=0$

$2t+7=0 \quad or \quad 2t-7=0$

$2t=-7 \quad or \quad 2t=7$

$t=\frac{-7}{2} \quad or \quad t=\frac{7}{2}$

The solutions are $\frac{-7}{2}$ and $\frac{7}{2}$.

35. $0=25+x^2+10x$

$0=x^2+10x+25$ Writing in descending order

$0=(x+5)(x+5)$

$x+5=0 \quad or \quad x+5=0$

$x=-5 \quad or \quad x=-5$

The solution is -5.

37. $64 + x^2 = 16x$
$x^2 - 16x + 64 = 0$
$(x-8)(x-8) = 0$
$x - 8 = 0$ *or* $x - 8 = 0$
$x = 8$ *or* $x = 8$
The solution is 8.

39. $4t^2 = 8t$
$4t^2 - 8t = 0$
$4t(t-2) = 0$
$t = 0$ *or* $t - 2 = 0$
$t = 0$ *or* $t = 2$
The solutions are 0 and 2.

41. $4y^2 = 7y + 15$
$4y^2 - 7y - 15 = 0$
$(4y+5)(y-3) = 0$
$4y + 5 = 0$ *or* $y - 3 = 0$
$4y = -5$ *or* $y = 3$
$y = \frac{-5}{4}$ *or* $y = 3$
The solutions are $\frac{-5}{4}$ and 3.

43. $(x-7)(x+1) = -16$
$x^2 - 6x - 7 = -16$
$x^2 - 6x + 9 = 0$
$(x-3)(x-3) = 0$
$x - 3 = 0$ *or* $x - 3 = 0$
$x = 3$ *or* $x = 3$
The solution is 3.

45. $15z^2 + 7 = 20z + 7$
$15z^2 - 20z + 7 = 7$
$15z^2 - 20z = 0$
$5z(3z-4) = 0$
$5z = 0$ *or* $3z - 4 = 0$
$z = 0$ *or* $3z = 4$
$z = 0$ *or* $z = \frac{4}{3}$
The solutions are 0 and $\frac{4}{3}$.

47. $36m^2 - 9 = 40$
$36m^2 - 49 = 0$
$(6m+7)(6m-7) = 0$
$6m + 7 = 0$ *or* $6m - 7 = 0$
$6m = -7$ *or* $6m = 7$
$m = \frac{-7}{6}$ *or* $m = \frac{7}{6}$
The solutions are $\frac{-7}{6}$ or $\frac{7}{6}$.

49. $(x+3)(3x+5) = 7$
$3x^2 + 14x + 15 = 7$
$3x^2 + 14x + 8 = 0$
$(3x+2)(x+4) = 0$
$3x + 2 = 0$ *or* $x + 4 = 0$
$3x = -2$ *or* $x = -4$
$x = \frac{-2}{3}$ *or* $x = -4$
The solutions are $-\frac{2}{3}$ and -4.

51. $3x^2 - 2x = 9 - 8x$
$3x^2 + 6x - 9 = 0$ Adding $8x$ and -9
$3(x^2 + 2x - 3) = 0$
$3(x+3)(x-1) = 0$
$x + 3 = 0$ *or* $x - 1 = 0$
$x = -3$ *or* $x = 1$
The solutions are -3 and 1.

53. $x^2(2x-1) = 3x$
$2x^3 - x^2 = 3x$
$2x^3 - x^2 - 3x = 0$
$x(2x^2 - x - 3) = 0$
$x(2x-3)(x+1) = 0$
$x = 0$ *or* $2x - 3 = 0$ *or* $x + 1 = 0$
$x = 0$ *or* $2x = 3$ *or* $x = \frac{3}{2}$
$x = 0$ *or* $x = \frac{3}{2}$ *or* $x = -1$
The solutions are -1, 0, and $\frac{3}{2}$.

55. $(2x-5)(3x^2 + 29x + 56) = 0$
$(2x-5)(3x+8)(x+7) = 0$
$2x - 5 = 0$ *or* $3x + 8 = 0$ *or* $x + 7 = 0$
$2x = 5$ *or* $3x = -8$ *or* $x = -7$
$x = \frac{5}{2}$ *or* $x = -\frac{8}{3}$ *or* $x = -7$
The solutions are -7, $-\frac{8}{3}$, and $\frac{5}{2}$.

57. The solutions of the equation are the first coordinates of the x-intercepts of the graph. From the graph we see that the x-intercepts are $(-1, 0)$ and $(4, 0)$, so the solutions of the equation are -1 and 4.

59. The solutions of the equation are the first coordinates of the x-intercepts of the graph. From the graph we see that the x-intercepts are $(-3, 0)$ and $(2, 0)$, so the solutions of the equation are -3 and 2.

61. We let $y = 0$ and solve for x.
$0 = x^2 - x - 6$
$0 = (x-3)(x+2)$
$x - 3 = 0$ *or* $x + 2 = 0$
$x = 3$ *or* $x = -2$
The x-intercepts are (3,0) and $(-2, 0)$.

63. We let $y = 0$ and solve for x.

$$0 = x^2 + 2x - 8$$
$$0 = (x+4)(x-2)$$
$$x + 4 = 0 \quad or \quad x - 2 = 0$$
$$x = -4 \quad or \quad x = 2$$

The x-intercepts are $(-4, 0)$ and $(2, 0)$.

65. We let $y = 0$ and solve for x.

$$0 = 2x^2 + 3x - 9$$
$$0 = (2x-3)(x+3)$$
$$2x - 3 = 0 \quad or \quad x + 3 = 0$$
$$2x = 3 \quad or \quad x = -3$$
$$x = \frac{3}{2} \quad or \quad x = -3$$

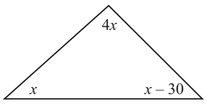
The x-intercepts are $\left(\frac{3}{2}, 0\right)$ and $(-3, 0)$.

67. *Writing Exercise.* The graph has no x-intercepts.

69. Let m and n represent the numbers. The sum of the two numbers is $m + n$. Thus the square of the sum of the two numbers is $(m+n)^2$.

71. Let x represent the first integer. Then $x + 1$ represents the second integer. Thus, the product is $x(x+1)$.

73. ***Familiarize.*** We draw a picture. We let $x =$ the measure of the second angle. Then $4x =$ the measure of the first angle, and $x - 30 =$ the measure of the third angle.



Recall that the measures of the angles of any triangle add up to 180°.

Translate.

$$\underbrace{\text{Measure of first angle}} + \underbrace{\text{Measure of 2nd angle}} + \underbrace{\text{Measure of third angle}} \text{ is } \underbrace{180^\circ}$$
$$4x \quad + \quad x \quad + \quad (x - 30) \quad = \quad 180$$

Carry out. we solve the equation

$$4x + x + (x - 30) = 180$$
$$6x - 30 = 180$$
$$6x = 210$$
$$x = 35$$

Possible answers for the angle measures are as follows:
First angle: $4x = 4(35^\circ) = 140^\circ$
Second angle: $x = 35\,\deg$
Third angle: $x - 30 = 35 - 30 = 5\,\deg$.

Check. Consider 140°, 35°and 5°. The first angle is four times the first, and the third is 30°less than the second. The sum of the angles is 180°. These numbers check.

State. The measure of the first angle is 140°, the measure of the second angle is 35°, and the measure of the third angle is 5°.

75. *Writing Exercise.* One solution of the equation is 0. Dividing both sides of the equation by x, leaving the solution $x = 3$, is equivalent to dividing by 0.

77. a)

$$x = -4 \quad or \quad x = 5$$
$$x + 4 = 0 \quad or \quad x - 5 = 0$$
$$(x+4)(x-5) = 0 \qquad \text{Principle of zero products}$$
$$x^2 - x - 20 = 0 \qquad \text{Multiplying}$$

b)

$$x = -1 \quad or \quad x = 7$$
$$x + 1 = 0 \quad or \quad x - 7 = 0$$
$$(x+1)(x-7) = 0$$
$$x^2 - 6x - 7 = 0$$

c)

$$x = \frac{1}{4} \quad or \quad x = 3$$
$$x - \frac{1}{4} = 0 \quad or \quad x - 3 = 0$$
$$\left(x - \frac{1}{4}\right)(x-3) = 0$$
$$x^2 - \frac{13}{4}x + \frac{3}{4} = 0$$
$$4\left(x^2 - \frac{13}{4}x + \frac{3}{4}\right) = 4 \cdot 0 \qquad \text{Multiplying both sides by 4}$$
$$4x^2 - 13x + 3 = 0$$

d)

$$x = \frac{1}{2} \quad or \quad x = \frac{1}{3}$$
$$x - \frac{1}{2} = 0 \quad or \quad x - \frac{1}{3} = 0$$
$$\left(x - \frac{1}{2}\right)\left(x - \frac{1}{3}\right) = 0$$
$$x^2 - \frac{5}{6}x + \frac{1}{6} = 0$$
$$6x^2 - 5x + 1 = 0 \qquad \text{Multiplying by 6}$$

e)

$$x = \frac{2}{3} \quad or \quad x = \frac{3}{4}$$
$$x - \frac{2}{3} = 0 \quad or \quad x - \frac{3}{4} = 0$$
$$\left(x - \frac{2}{3}\right)\left(x - \frac{3}{4}\right) = 0$$
$$x^2 - \frac{17}{12}x + \frac{1}{2} = 0$$
$$12x^2 - 17x + 6 = 0 \qquad \text{Multiplying by 12}$$

f)

$$x = -1 \quad or \quad x = 2 \quad or \quad x = 3$$
$$x + 1 = 0 \quad or \quad x - 2 = 0 \quad or \quad x - 3 = 0$$
$$(x+1)(x-2)(x-3) = 0$$
$$(x^2 - x - 2)(x-3) = 0$$
$$x^3 - 4x^2 + x + 6 = 0$$

79.

$$a(9+a) = 4(2a+5)$$
$$9a + a^2 = 8a + 20$$
$$a^2 + a - 20 = 0 \qquad \text{Subtracting } 8a \text{ and } 20$$
$$(a+5)(a-4) = 0$$

$a+5=0 \quad or \quad a-4=0$

$a=-5 \quad or \quad a=4$

The solutions are -5 and 4.

81. $-x^2+\frac{9}{25}=0$

$x^2-\frac{9}{25}=0$ Multiplying by -1

$\left(x-\frac{3}{5}\right)\left(x+\frac{3}{5}\right)=0$

$x-\frac{3}{5}=0 \quad or \quad x+\frac{3}{5}=0$

$x=\frac{3}{5} \quad or \quad x=-\frac{3}{5}$

The solutions are $\frac{3}{5}$ and $-\frac{3}{5}$.

83. $(t+1)^2=9$

Observe that $t+1$ is a number which yields 9 when it is squared. Thus, we have

$t+1=-3 \quad or \quad t+1=3$

$t=-4 \quad or \quad t=2$

The solutions are -4 and 2.

We could also do this exercise as follows:

$(t+1)^2=9$

$t^2+2t+1=9$

$t^2+2t-8=0$

$(t+4)(t-2)=0$

$t+4=0 \quad or \quad t-2=0$

$t=-4 \quad or \quad t=2$

Again we see that the solutions are -4 and 2.

85. a) $2(x^2+10x-2)=2\cdot 0$ Multiplying (a) by 2

$2x^2+20x-4=0$

(a) and $2x^2+20x-4=0$ are equivalent.

b) $(x-6)(x+3)=x^2-3x-18$ Multiplying

(b) and $x^2-3x-18=0$ are equivalent.

c) $5x^2-5=5(x^2-1)=5(x+1)(x-1)=$
$(x+1)5(x-1)=(x+1)(5x-5)$

(c) and $(x+1)(5x-5)=0$ are equivalent.

d) $2(2x-5)(x+4)=2\cdot 0$ Multiplying (d) by 2

$2(x+4)(2x-5)=0$

$(2x+8)(2x-5)=0$

(d) and $(2x+8)(2x-5)=0$ are equivalent.

e) $4(x^2+2x+9)=4\cdot 0$ Multiplying (e) by 4

$4x^2+8x+36=0$

(e) and $4x^2+8x+36=0$ are equivalent.

f) $3(3x^2-4x+8)=3\cdot 0$ Multiplying (f) by 3

$9x^2-12x+24=0$

(f) and $9x^2-12x+24=0$ are equivalent.

87. *Writing Exercise.* Graph $y=-x^2-x+6$ and $y=4$ on the same set of axes. The first coordinates of the points of intersection of the two graphs are the solutions of $-x^2-x+6=4$.

89. 2.33, 6.77

91. $-4.59, -9.15$

93. -3.76, 0

5.7 Connecting the Concepts

1. Expression

3. Equation

5. Expression

7. $\left(2x^3-5x+1\right)+\left(x^2-3x-1\right)$
$=2x^3+x^2+(-5-3)x+(1-1)$
$=2x^3+x^2-8x$

9. $t^2-100=0$

$(t+10)(t-10)=0$

$t+10=0 \quad or \quad t-10=0$

$t=-10 \quad or \quad t=10$

The solutions are -10 and 10.

11. $n^2-10n+9=(n-1)(n-9)$ Factor with FOIL

13. $4t^2+20t+25=0$

$(2t)^2+2\cdot 2t\cdot 5+5^2=0$

$(2t+5)(2t+5)=0$

$2t+5=0 \quad or \quad 2t+5=0$

$2t=-5 \quad or \quad 2t=-5$

$t=\frac{-5}{2} \quad or \quad t=\frac{-5}{2}$

The solutions is $\frac{-5}{2}$.

15. $16x^2-81=(4x+9)(4x-9)$ Difference of squares

17. $\left(a^2-2\right)-\left(5a^2+a+9\right)$
$=a^2-2-5a^2-a-9$
$=(1-5)a^2-a+(-2-9)$
$=-4a^2-a-11$

19. $3x^2+5x+2=0$

$(x+1)(3x+2)=0$

$x+1=0 \quad or \quad 3x+2=0$

$x=-1 \quad or \quad x=\frac{-2}{3}$

The solutions are -1 and $-\frac{2}{3}$.

Exercise Set 5.8

1. ***Familiarize***. Let $x =$ the number.

Translate. We reword the problem.

$$\underbrace{\text{The square of a number}}_{\downarrow\; x^2} \quad \underset{\downarrow\; -}{\text{minus}} \quad \underbrace{\text{the number}}_{\downarrow\; x} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 6}{6.}$$

Carry out. We solve the equation.

$$x^2 - x = 6$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \quad or \quad x + 2 = 0$$
$$x = 3 \quad or \quad x = -2$$

Check. For 3: The square of 3 is 3^2, or 9, and $9 - 3 = 6$.

For -2: The square of -2, or 4 and $4 - (-2) = 4 + 2 = 6$. Both numbers check.

State. The numbers are 3 and -2.

3. ***Familiarize***. Let $x =$ the length of the shorter leg, in m. Then $x + 2 =$ the length of the longer leg.

Translate. We use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$x^2 + (x+2)^2 = 10^2$$

Carry out. We solve the equation.

$$x^2 + (x+2)^2 = 10^2$$
$$x^2 + x^2 + 4x + 4 = 100$$
$$2x^2 + 4x + 4 = 100$$
$$2x^2 + 4x - 96 = 0$$
$$2(x^2 + 2x - 48) = 0$$
$$2(x+8)(x-6) = 0$$

$$x + 8 = 0 \quad or \quad x - 6 = 0$$
$$x = -8 \quad or \quad x = 6$$

Check. The number -8 cannot be the length of a side because it is negative. When $x = 6$, then $x + 2 = 8$, and $6^2 + 8^2 = 36 + 64 = 100 = 10^2$, so the number 6 checks.

State.The lengths of the sides are 6 m, 8 m, and 10 m.

5. ***Familiarize***. The parking spaces are consecutive integers. Let $x =$ the smaller integer. Then $x + 1 =$ the larger integer.

Translate. We reword the problem.

$$\underbrace{\text{Smaller integer}}_{\downarrow\; x} \quad \underset{\downarrow\; \cdot}{\text{times}} \quad \underbrace{\text{larger integer}}_{\downarrow\; (x+1)} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 132}{132.}$$

Carry out. We solve the equation.

$$x(x+1) = 132$$
$$x^2 + x = 132$$
$$x^2 + x - 132 = 0$$
$$(x+12)(x-11) = 0$$

$$x + 12 = 0 \quad or \quad x - 11 = 0$$
$$x = -12 \quad or \quad x = 11$$

Check. The solutions of the equation are -12 and 11. Since a parking space number cannot be negative, we only need to check 11. When $x = 11$, then $x + 1 = 12$, and $11 \cdot 12 = 132$. This checks.

State. The parking space numbers are 11 and 12.

7. ***Familiarize***. Let $x =$ the smaller even integer. Then $x + 2 =$ the larger even integer.

Translate. We reword the problem.

$$\underbrace{\text{Smaller even integer}}_{\downarrow\; x} \quad \underset{\downarrow\; \cdot}{\text{times}} \quad \underbrace{\text{larger even integer}}_{\downarrow\; (x+2)} \quad \underset{\downarrow\; =}{\text{is}} \quad \underset{\downarrow\; 168}{168.}$$

Carry out.

$$x(x+2) = 168$$
$$x^2 + 2x = 168$$
$$x^2 + 2x - 168 = 0$$
$$(x+14)(x-12) = 0$$

$$x + 14 = 0 \quad or \quad x - 12 = 0$$
$$x = -14 \quad or \quad x = 12$$

Check. The solutions of the equation are -14 and 12. When x is -14, then $x + 2$ is -12 and $-14(-12) = 168$. The numbers -14 and -12 are consecutive even integers which are solutions of the problem. When x is 12, then $x + 2 = 14$ and $12 \cdot 14 = 168$. The numbers 12 and 14 are also consecutive even integers which are solutions of the problem.

State. We have two solutions, each of which consists of a pair of numbers: -14 and -12 or 12 and 14.

9. ***Familiarize***. Let $w =$ the width of the porch, in feet. Then $5w =$ the length. Recall that the area of a rectangle is Length $\cdot$ Width.

Translate.

$$\underbrace{\text{The area of the rectangle}}_{\downarrow\; 5w \cdot w} \quad \underset{\downarrow\; =}{\text{is}} \quad \underbrace{180 \text{ ft}^2.}_{\downarrow\; 180}$$

Carry out. We solve the equation.

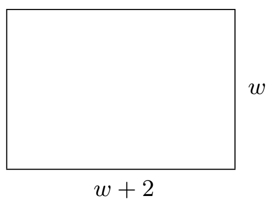
$$5w \cdot w = 180$$
$$5w^2 = 180$$
$$5w^2 - 180 = 0$$
$$5(w^2 - 36) = 0$$
$$5(w+6)(w-6) = 0$$

$$w + 6 = 0 \quad or \quad w - 6 = 0$$
$$w = -6 \quad or \quad w = 6$$

Check. Since the width must be positive, -6 cannot be a solution. If the width is 6 ft, then the length is $5 \cdot 6$ ft, or 30 ft, and the area is $6 \text{ ft} \cdot 30 \text{ ft} = 180 \text{ ft}^2$. Thus, 6 checks.

State. The porch is 30 ft long and 6 ft wide.

11. ***Familiarize***. We make a drawing. Let w = the width, in cm. Then $w+2$ = the length, in cm.



Recall that the area of a rectangle is length times width.

Translate. We reword the problem.

Length	times	width	is	$\underbrace{24 \text{ cm}^2}$.
↓	↓	↓	↓	↓
$(w+2)$	$\cdot$	w	$=$	24

Carry out. We solve the equation.

$$(w+2)w = 24$$
$$w^2 + 2w = 24$$
$$w^2 + 2w - 24 = 0$$
$$(w+6)(w-4) = 0$$
$$w + 6 = 0 \quad or \quad w - 4 = 0$$
$$w = -6 \quad or \quad w = 4$$

Check. Since the width must be positive, -6 cannot be a solution. If the width is 4 cm, then the length is $4+2$, or 6 cm, and the area is $6 \cdot 4$, or 24 cm^2. Thus, 4 checks.

State. The width is 4 cm, and the length is 6 cm.

13. ***Familiarize***. Using the labels shown on the drawing in the text, we let b = the base, in inches, and $b-3$ = the height, in inches. Recall that the formula for the area of a triangle is $\frac{1}{2} \cdot \text{(base)} \cdot \text{(height)}$.

Translate.

$\frac{1}{2}$	times	base	times	height	is	$\underbrace{54 \text{ in}^2}$.
↓	↓	↓	↓	↓	↓	↓
$\frac{1}{2}$	$\cdot$	(b)	$\cdot$	$(b-3)$	$=$	54

Carry out.

$$\frac{1}{2}b(b-3) = 54$$
$$b(b-3) = 108 \quad \text{Multiplying by 2}$$
$$b^2 - 3b = 108$$
$$b^2 - 3b - 108 = 0$$
$$(b-12)(b+9) = 0$$
$$b - 12 = 0 \quad or \quad b + 9 = 0$$
$$b = 12 \quad or \quad b = -9$$

Check. Since the height must be positive, -9 cannot be a solution. If the base is 12 in, then the height is $12-3$, or 9 in, and the area is $\frac{1}{2} \cdot 12 \cdot 9$, or 54 in^2. Thus, 12 checks.

State. The base of the triangle is 12 in, and the height is 9 in.

15. ***Familiarize***. Using the labels show on the drawing in the text, we let x = the length of the foot of the sail, in ft, and $x+5$ = the height of the sail, in ft. Recall that the formula for the area of a triangle is $\frac{1}{2} \cdot \text{(base)} \cdot \text{(height)}$.

Translate.

$\frac{1}{2}$	times	base	times	height	is	$\underbrace{42 \text{ ft}^2}$.
↓	↓	↓	↓	↓	↓	↓
$\frac{1}{2}$	$\cdot$	x	$\cdot$	$(x+5)$	$=$	42

Carry out.

$$\frac{1}{2}x(x+5) = 42$$
$$x(x+5) = 84 \quad \text{Multiplying by 2}$$
$$x^2 + 5x = 84$$
$$x^2 + 5x - 84 = 0$$
$$(x+12)(x-7) = 0$$
$$x + 12 = 0 \quad or \quad x - 7 = 0$$
$$x = -12 \quad or \quad x = 7$$

Check. The solutions of the equation are -12 and 7. The length of the base of a triangle cannot be negative, so -12 cannot be a solution. Suppose the length of the foot of the sail is 7 ft. Then the height is $7+5$, or 12 ft, and the area is $\frac{1}{2} \cdot 7 \cdot 12$, or 42 ft^2. These numbers check.

State. The length of the foot of the sail is 7 ft, and the height is 12 ft.

17. ***Familiarize and Translate***. We substitute 150 for A in the formula.

$$A = -50t^2 + 200t$$
$$150 = -50t^2 + 200t$$

Carry out. We solve the equation.

$$150 = -50t^2 + 200t$$
$$0 = -50t^2 + 200t - 150$$
$$0 = -50(t^2 - 4t + 3)$$
$$0 = -50(t-1)(t-3)$$
$$t - 1 = 0 \quad or \quad t - 3 = 0$$
$$t = 1 \quad or \quad t = 3$$

Check. Since $-50 \cdot 1^2 + 200 \cdot 1 = -50 + 200 = 150$, the number 1 checks. Since $-50 \cdot 3^2 + 200 \cdot 3 = -450 + 600 = 150$, the number 3 checks also.

State. There will be about 150 micrograms of Albuterol in the bloodstream 1 minute and 3 minutes after an inhalation.

19. ***Familiarize***. We will use the formula $N = 0.3t^2 + 0.6t$.

Translate. Substitute 36 for N.

$$36 = 0.3t^2 + 0.6t$$

Carry out.

$$\begin{aligned} 36 &= 0.3t^2 + 0.6t \\ 360 &= 3t^2 + 6t \quad \text{Multiplying by 10} \\ 0 &= 3t^2 + 6t - 360 \\ 0 &= 3(t^2 + 2t - 120) \\ 0 &= 3(t + 12)(t - 10) \end{aligned}$$

$$t + 12 = 0 \quad \text{or} \quad t - 10 = 0$$
$$t = -12 \quad \text{or} \quad t = 10$$

Check. The solutions of the equation are -12 and 10. Since the number of years cannot be negative, -12 cannot be a solution. However, 10 checks since $0.3(10)^2 + 0.6(10) = 30 + 6 = 36$.

State. 10 years after 1998 is the year 2008.

21. ***Familiarize***. We will use the formula $x^2 - x = N$.

Translate. Substitute 240 for N.

$$x^2 - x = 240$$

Carry out.

$$\begin{aligned} x^2 - x &= 240 \\ x^2 - x - 240 &= 0 \\ (x - 16)(x + 15) &= 0 \end{aligned}$$

$$x - 16 = 0 \quad \text{or} \quad x + 15 = 0$$
$$x = 16 \quad \text{or} \quad x = -15$$

Check. The solutions of the equation are 16 and -15. Since the number of teams cannot be negative, -15 cannot be a solution. But 16 checks since $16^2 - 16 = 256 - 16 = 240$.

State. There are 16 teams in the league.

23. ***Familiarize***. We will use the formula $H = \frac{1}{2}(n^2 - n)$.

Translate. Substitute 12 for n.

$$H = \frac{1}{2}(12^2 - 12)$$

Carry out. We do the computation on the right.

$$\begin{aligned} H &= \frac{1}{2}(12^2 - 12) \\ H &= \frac{1}{2}(144 - 12) \\ H &= \frac{1}{2}(132) \\ H &= 66 \end{aligned}$$

Check. We can recheck the computation, or we can solve the equation $66 = \frac{1}{2}(n^2 - n)$. The answer checks.

State. 66 handshakes are possible.

25. ***Familiarize***. We will use the formula $H = \frac{1}{2}(n^2 - n)$, since "high fives" can be substituted for handshakes.

Translate. Substitute 66 for H.

$$66 = \frac{1}{2}(n^2 - n)$$

Carry out.

$$\begin{aligned} 66 &= \frac{1}{2}(n^2 - n) \\ 132 &= n^2 - n \quad \text{Multiplying by 2} \\ 0 &= n^2 - n - 132 \\ 0 &= (n - 12)(n + 11) \end{aligned}$$

$$n - 12 = 0 \quad \textit{or} \quad n + 11 = 0$$
$$n = 12 \quad \textit{or} \quad n = -11$$

Check. The solutions of the equation are 12 and -11. Since the number of players cannot be negative, -11 cannot be a solution. However, 12 checks since $\frac{1}{2}(12^2 - 12) = \frac{1}{2}(144 - 12) = \frac{1}{2}(132) = 66$.

State. 12 players were on the team.

27. ***Familiarize***. Let h = the vertical height to which each brace reaches, in feet. We have a right triangle with hypotenuse 15 ft and legs 12 ft and h.

Translate. We use the Pythagorean theorem.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 12^2 + h^2 &= 15^2 \end{aligned}$$

Carry out. We solve the equation.

$$\begin{aligned} 12^2 + h^2 &= 15^2 \\ 144 + h^2 &= 225 \\ h^2 - 81 &= 0 \\ (h + 9)(h - 9) &= 0 \end{aligned}$$

$$h + 9 = 0 \quad \text{or} \quad h - 9 = 0$$
$$h = -9 \quad \text{or} \quad h = 9$$

Check. Since the vertical height must be positive, -9 cannot be a solution. If the height is 9 ft, then we have $12^2 + 9^2 = 144 + 81 = 225 = 15^2$. The number 9 checks.

State. Each brace reaches 9 ft vertically.

29. ***Familiarize***.Let w = the width of Main Street, in ft. We have a right triangle with hypotenuse 40 ft and legs of 24 ft and w.

Translate. We use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

Carry out.

$$\begin{aligned} 24^2 + w^2 &= 40^2 \\ 576 + w^2 &= 1600 \\ w^2 - 1024 &= 0 \\ (w - 32)(w + 32) &= 0 \end{aligned}$$

$$w - 32 = 0 \quad \textit{or} \quad w + 32 = 0$$
$$w = 32 \quad \textit{or} \quad w = -32$$

Check. Since the width must be positive, -32 cannot be a solution. If the width is 32 ft, then we have $24^2 + 32^2 = 576 + 1024 = 1600 = 40^2$. The number 32 checks.

State. The width of Main Street is 32 ft.

31. ***Familiarize***. Let l = the length of the leg, in ft. Then $l + 200$ = the length of the hypotenuse in feet.

Translate. We use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$400^2 + l^2 = (l + 200)^2$$

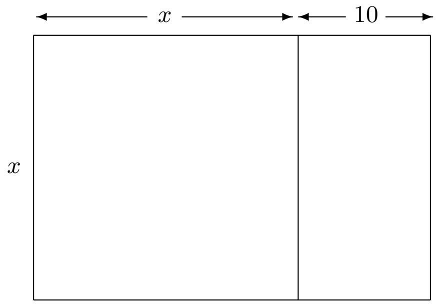
Carry out.

$$400^2 + l^2 = l^2 + 400l + 40{,}000$$
$$160{,}000 + l^2 = l^2 + 400l + 40{,}000$$
$$120{,}000 = 400l$$
$$300 = l$$

Check. When $l = 300$, then $l + 200 = 500$, and $400^2 + 300^2 = 160{,}000 + 90{,}000 = 250{,}000 = 500^2$, so the number 300 checks.

State. The dimensions of the garden are 300 ft by 400 ft by 500 ft.

33. ***Familiarize***. We label the drawing. Let x = the length of a side of the dining room, in ft. Then the dining room has dimensions x by x and the kitchen has dimensions x by 10. The entire rectangular space has dimension x by $x + 10$. Recall that we multiply these dimensions to find the area of the rectangle.



Translate.

$$\underbrace{\text{The area of the rectangular space}}_{x(x+10)} \quad \underbrace{\text{is}}_{=} \quad \underbrace{264 \text{ ft}^2.}_{264}$$

Carry out. We solve the equation.

$$x(x + 10) = 264$$
$$x^2 + 10x = 264$$
$$x^2 + 10x - 264 = 0$$
$$(x + 22)(x - 12) = 0$$
$$x + 22 = 0 \quad or \quad x - 12 = 0$$
$$x = -22 \ or \quad x = 12$$

Check. Since the length of a side of the dining room must be positive, -22 cannot be a solution. If x is 12 ft, then $x + 10$ is 22 ft, and the area of the space is $12 \cdot 22$, or 264 ft^2. The number 12 checks.

State. The dining room is 12 ft by 12 ft, and the kitchen is 12 ft by 10 ft.

35. ***Familiarize***. We will use the formula $h = 48t - 16t^2$.

Translate. Substitute $\frac{1}{2}$ for t.

$$h = 48 \cdot \frac{1}{2} - 16\left(\frac{1}{2}\right)^2$$

Carry out. We do the computation on the right.

$$h = 48 \cdot \frac{1}{2} - 16\left(\frac{1}{2}\right)^2$$
$$h = 48 \cdot \frac{1}{2} - 16 \cdot \frac{1}{4}$$
$$h = 24 - 4$$
$$h = 20$$

Check. We can recheck the computation, or we can solve the equation $20 = 48t - 16t^2$. The answer checks.

State. The rocket is 20 ft high $\frac{1}{2}$ sec after it is launched.

37. ***Familiarize***. We will use the formula $h = 48t - 16t^2$.

Translate. Substitute 32 for h.

$$32 = 48t - 16t^2$$

Carry out. We solve the equation.

$$32 = 48t - 16t^2$$
$$0 = -16t^2 + 48t - 32$$
$$0 = -16(t^2 - 3t + 2)$$
$$0 = -16(t - 1)(t - 2)$$
$$t - 1 = 0 \quad or \quad t - 2 = 0$$
$$t = 1 \quad or \quad t = 2$$

Check. When $t = 1$, $h = 48 \cdot 1 - 16 \cdot 1^2 = 48 - 16 = 32$. When $t = 2$, $h = 48 \cdot 2 - 16 \cdot 2^2 = 96 - 64 = 32$. Both numbers check.

State. The rocket will be exactly 32 ft above the ground at 1 sec and at 2 sec after it is launched.

39. *Writing Exercise.* No; if we cannot factor the quadratic expression $ax^2 + bx + c$, $a \neq 0$, then we cannot solve the quadratic equation $ax^2 + bx + c = 0$.

41. $\dfrac{-3}{5} \cdot \dfrac{4}{7} = -\dfrac{3 \cdot 4}{5 \cdot 7} = \dfrac{-12}{35}$

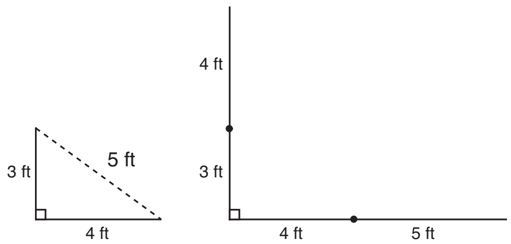
43. $\dfrac{-5}{6} - \dfrac{1}{6} = \dfrac{-5}{6} + \dfrac{-1}{6} = -1$

45. $\dfrac{-3}{8} \cdot \left(\dfrac{-10}{15}\right) = \dfrac{\cancel{3} \cdot \cancel{2} \cdot \cancel{5}}{\cancel{2} \cdot 4 \cdot \cancel{3} \cdot \cancel{5}} = \dfrac{1}{4}$

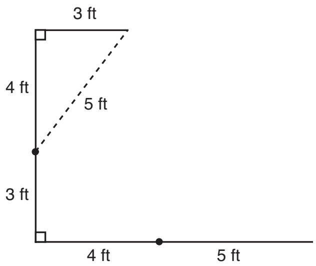
47. $\dfrac{5}{24} + \dfrac{3}{28} = \dfrac{5}{24} \cdot \dfrac{7}{7} + \dfrac{3}{28} \cdot \dfrac{6}{6}$

$= \dfrac{35}{168} + \dfrac{18}{168} = \dfrac{53}{168}$

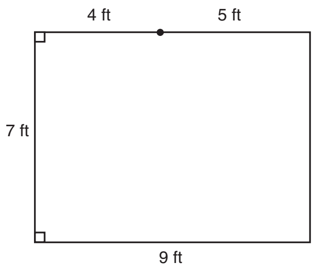
49. *Writing Exercise.* She could use the measuring sticks to draw a right angle as shown below. Then she could use the 3-ft and 4-ft sticks to extend one leg to 7 ft and the 4-ft and 5-ft sticks to extend the other leg to 9 ft.



Next she could draw another right angle with either the 7-ft side or the 9-ft side as a side.



Then she could use the sticks to extend the other side to the appropriate length. Finally she would draw the remaining side of the rectangle.



51. ***Familiarize***. First we find the length of the other leg of the right triangle. Then we find the area of the triangle, and finally we multiply by the cost per square foot of the sailcloth. Let $x =$ the length of the other leg of the right triangle, in feet.

Translate. We use the Pythagorean theorem to find x.

$$a^2 + b^2 = c^2$$

$$x^2 + 24^2 = 26^2 \quad \text{Substituting}$$

Carry out.

$$x^2 + 24^2 = 26^2$$
$$x^2 + 576 = 676$$
$$x^2 - 100 = 0$$
$$(x + 10)(x - 10) = 0$$
$$x + 10 = 0 \quad or \quad x - 10 = 0$$
$$x = -10 \quad or \quad x = 10$$

Since the length of the leg must be positive, -10 cannot be a solution. We use the number 10. Find the area of the triangle:

$$\frac{1}{2}bh = \frac{1}{2} \cdot 10 \text{ ft} \cdot 24 \text{ ft} = 120 \text{ ft}^2$$

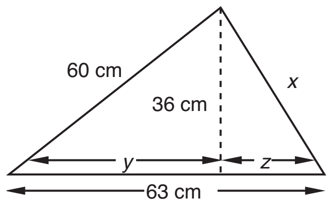
Finally, we multiply the area, 120 ft^2, by the price per square foot of the sailcloth, \$1.50:

$$120 \cdot (1.50) = 180$$

Check. Recheck the calculations. The answer checks.

State. A new main sail costs \$180.

53. ***Familiarize***. We add labels to the drawing in the text.



First we will use the Pythagorean theorem to find y. Then we will subtract to find z and, finally, we will use the Pythagorean theorem again to find x.

Translate. We use the Pythagorean theorem to find y.

$$a^2 + b^2 = c^2$$

$$y^2 + 36^2 = 60^2 \quad \text{Substituting}$$

Carry out.

$$y^2 + 36^2 = 60^2$$
$$y^2 + 1296 = 3600$$
$$y^2 - 2304 = 0$$
$$(y + 48)(y - 48) = 0$$
$$y + 48 = 0 \quad or \quad y - 48 = 0$$
$$y = -48 \quad or \quad y = 48$$

Since the length y cannot be negative, we use 48 cm. Then $z = 63 - 48 = 15$ cm.

Now we find x. We use the Pythagorean theorem again.

$$15^2 + 36^2 = x^2$$
$$225 + 1296 = x^2$$
$$1521 = x^2$$
$$0 = x^2 - 1521$$
$$0 = (x + 39)(x - 39)$$
$$x + 39 = 0 \quad or \quad x - 39 = 0$$
$$x = -39 \quad or \quad x = 39$$

Since the length x cannot be negative, we use 39 cm.

Check. We repeat all of the calculations. The answer checks.

State. The value of x is 39 cm.

55. ***Familiarize***. Let $w =$ the width of the side turned up. Then $20 - 2w =$ the length, in inches of the base.

Recall that we multiply these dimensions to find the area of the rectangle.

Translate.

$$\underbrace{\text{The area of the rectangular cross-section}} \quad \text{is} \quad \underbrace{48 \text{ in}^2}.$$
$$\downarrow \qquad\qquad \downarrow \quad \downarrow$$
$$w(20 - 2w) \quad = \quad 48$$

Carry out. We solve the equation.

$$w(20-2w) = 48$$
$$0 = 2w^2 - 20w + 48$$
$$0 = 2(w^2 - 10w + 24)$$
$$0 = 2(w-6)(w-4)$$

$w - 6 = 0$ *or* $w - 4 = 0$

$w = 6$ *or* $w = 4$

Check. If $w = 6$ in., $20 - 2(6) = 8$ in. and the area is

6 in. $\cdot$ 8 in. $= 48$ in.^{2}

If $w = 4$ in., $20 - 2(4) = 12$ in. and the area is

4 in. $\cdot$ 12 in. $= 48$ in.^{2}

State. The possible depths of the gutter are 4 in. or 6 in.

57. ***Familiarize***. First we can use the Pythagorean theorem to find x, in ft. Then the height of the telephone pole is $x + 5$.

Translate. We use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$\left(\frac{1}{2}x + 1\right)^2 + x^2 = 34^2$$

Carry out. We solve the equation.

$$\left(\frac{1}{2}x + 1\right)^2 + x^2 = 34^2$$
$$\frac{1}{4}x^2 + x + 1 + x^2 = 1156$$
$$x^2 + 4x + 4 + 4x^2 = 4624 \quad \text{Multiplying by 4}$$
$$5x^2 + 4x + 4 = 4624$$
$$5x^2 + 4x - 4620 = 0$$
$$(5x + 154)(x - 30) = 0$$

$5x + 154 = 0$ *or* $x - 30 = 0$

$5x = -154$ *or* $x = 30$

$x = -30.8$ *or* $x = 30$

Check. Since the length x must be positive, -30.8 cannot be a solution. If x is 30 ft, then $\frac{1}{2}x + 1$ is $\frac{1}{2} \cdot 30 + 1$, or 16 ft. Since $16^2 + 30^2 = 1156 = 34^2$, the number 30 checks. When x is 30 ft, then $x + 5$ is 35 ft.

State. The height of the telephone pole is 35 ft.

59. First substitute 18 for N in the given formula.

$$18 = -0.009t(t-12)^3$$

Graph $y_1 = 18$ and $y_2 = -0.009x(x-12)^3$ in the given window and use the TRACE feature to find the first coordinates of the points of intersection of the graphs. We find $x \approx 2$ hr and $x \approx 4.2$ hr.

61. Graph $y = -0.009x(x-12)^3$ and use the TRACE feature to find the first coordinate of the highest point on the graph. We find $x = 3$ hr.

Chapter 5 Review

1. False. The largest common variable factor is the <u>smallest</u> power of the variable in the polynomial.

3. True. see p. 329

5. False. Some quadratic equations have two different solutions.

7. True. see p 357

9. $20x^3 = (4 \cdot 5)\left(x \cdot x^2\right) = (-2x)\left(-10x^2\right) = (20x)\left(x^2\right)$

11. $12x^4 - 18x^3 = 6x^3 \cdot 2x - 6x^3 \cdot 3 = 6x^3\,(2x-3)$

13. $100t^2 - 1 = (10t+1)\,(10t-1)$

15. $x^2 + 14x + 49 = (x+7)\,(x+7) = (x+7)^2$

17. $6x^3 + 9x^2 + 2x + 3$
$= 3x^2 \cdot 2x + 3x^2 \cdot 3 + 2x \cdot 1 + 3 \cdot 1$
$= 3x^2\,(2x+3) + 1\,(2x+3)$
$= (2x+3)\left(3x^2+1\right)$

19. $25t^2 + 9 - 30t = 25t^2 - 30t + 9$
$= (5t-3)\,(5t-3) = (5t-3)^2$

21. $81a^4 - 1 = \left(9a^2+1\right)\left(9a^2-1\right)$
$= \left(9a^2+1\right)(3a+1)\,(3a-1)$

23. $2x^3 - 250$
$= 2(x^3 - 125)$
$= 2(x^3 - 5^3)$
$= 2(x-5)(x^2+5x+25)$

25. $a^2b^4 - 64 = \left(ab^2+8\right)\left(ab^2-8\right)$

27. $75 + 12x^2 - 60x = 12x^2 - 60x + 75$
$= 3\left(4x^2 - 20x + 25\right)$
$= 3\,(2x-5)\,(2x-5) = 3\,(2x-5)^2$

29. $-t^3 + t^2 + 42t = -t\left(t^2 - t - 42\right)$
$= -t\,(t-7)\,(t+6)$

31. $n^2 - 60 - 4n = n^2 - 4n - 60$
$= (n-10)\,(n+6)$

33. $4t^2 + 13t + 10 = (4t+5)\,(t+2)$

35. $7x^3 + 35x^2 + 28x = 7x\left(x^2 + 5x + 4\right)$
$= 7x\,(x+1)\,(x+4)$

37. $20x^2 - 20x + 5 = 5\left(4x^2 - 4x + 1\right)$
$= 5\,(2x-1)\,(2x-1)$
$= 5\,(2x-1)^2$

39. $15 - 8x + x^2 = x^2 - 8x + 15$
$= (x-3)\,(x-5)$

41. $x^2y^2 + 6xy - 16 = (xy+8)\,(xy-2)$

43. $m^2 + 5m + mt + 5t = m\,(m+5) + t\,(m+5)$
$= (m+5)\,(m+t)$

45. $6m^2 + 2mn + n^2 + 3mn = 2m(3m+n) + n(n+3m)$
$= (3m+n)(2m+n)$

47. $(x-9)(x+11) = 0$

$x - 9 = 0 \quad or \quad x + 11 = 0$
$x = 9 \quad or \quad x = -11$

49. $16x^2 = 9$
$16x^2 - 9 = 0$
$(4x+3)(4x-3) = 0$

$4x + 3 = 0 \quad or \quad 4x - 3 = 0$
$x = \frac{-3}{4} \quad or \quad x = \frac{3}{4}$

51. $2x^2 - 7x = 30$
$2x^2 - 7x - 30 = 0$
$(2x+5)(x-6) = 0$

$2x + 5 = 0 \quad or \quad x - 6 = 0$
$x = \frac{-5}{2} \quad or \quad x = 6$

53. $9t - 15t^2 = 0$
$3t(3-5t) = 0$

$3t = 0 \quad or \quad 3 - 5t = 0$
$t = 0 \quad or \quad t = \frac{3}{5}$

55. ***Familiarize*** Let $n =$ the number.

Translate.

The number squared is 12 more than the number

$n^2 = 12 + n$

Carry out. We solve the equation.

$n^2 = 12 + n$
$n^2 - n - 12 = 0$
$(n-4)(n+3) = 0$

$n - 4 = 0 \quad or \quad n + 3 = 0$
$n = 4 \quad or \quad n = -3$

Check.

$4^2 = 12 + 4$
$16 = 16$ True
$(-3)^2 = 12 + (-3)$
$9 = 9$ True

State. The solutions are 4 and -3.

57. We let $y = 0$ and solve for x

$0 = 2x^2 - 3x - 5$
$0 = (2x-5)(x+1)$

$2x - 5 = 0 \quad or \quad x + 1 = 0$
$x = \frac{5}{2} \quad or \quad x = -1$

The x-intercepts are $\left(\frac{5}{2},\ 0\right)$ and $(-1, 0)$

59. ***Familiarize***. Let $d =$ the diagonal of the brace.

Translate. We use the Pythagorean theorem.

$a^2 + b^2 = c^2$
$8^2 + 6^2 = d^2$

Carry out. We solve the equation.

$8^2 + 6^2 = c^2$
$8^2 + 6^2 = d^2$
$100 = d^2$
$0 = d^2 - 100$
$0 = (d+10)(d-10)$
$= 0$

$d + 10 = 0 \quad or \quad d - 10 = 0$
$d = -10 \quad or \quad d = 10$

Check. Since the diagonal must be positive, -10 cannot be a solution.

$8^2 + 6^2 = 10^2$
$100 = 100$ True

State. The diagonal is 10 holes long.

61. *Writing Exercise.* The equations solved in this chapter have an x^2-term (are quadratic), whereas those solved previously have no x^2-term (are linear). The principle of zero products is used to solve quadratic equations and is not used to solve linear equations.

63. ***Familiarize*** Let $n =$ the number.

Translate.

The cube of the number is twice the square of the number

$n^3 = 2\ n^2$

Carry out. We solve the equation.

$n^3 = 2n^2$
$n^3 - 2n^2 = 0$
$n^2(n-2) = 0$

$n^2 = 0 \quad or \quad n - 2 = 0$
$n = 0 \quad or \quad n = 2$

Check.

$n^3 = 2n^2$
$0^3 = 2(0)^2$
$0 = 0$ True

$n^3 = 2n^2$
$2^3 = 2(2)^2$
$8 = 8$ True

State. The number is 0 or 2.

65. ***Familiarize***. Let $s =$ the side of a square, in cm, $s + 5 =$ side of the new square. Recall $A = x^2$.

Translate. The new square is $2\frac{1}{2}$ times the area of the original square

$(s+5)^2 = 2\frac{1}{4}(s)^2$

Carry out. We solve the equation.

$$\begin{aligned} 4s^2 + 40s + 100 &= 9s^2 \\ 0 &= 5s^2 - 40s - 100 \\ 0 &= 5(s^2 - 8s - 20) \\ 0 &= 5(s - 10)(s + 2) \end{aligned}$$

$$s - 10 = 0 \quad \textit{or} \quad s + 2 = 0$$

$$s = 10 \quad \textit{or} \quad s = -2$$

Check. Since the side of the square must be positive, -2 cannot be a solution.

$$\begin{aligned} (s+5)^2 &= 2\frac{1}{4}s^2 \\ (10+5)^2 &= \frac{9}{4}(10)^2 \\ 225 &= 225 \qquad \text{True} \end{aligned}$$

State. The original square has side 10 cm and area of 100 cm^2. The new square has side 15 cm and area of 225 cm^2.

67. $x^2 + 25 = 0$

No real solution, the sum of two squares cannot be factored.

Chapter 5 Test

1. Answers may vary.
$(3x^2)(4x^2),\ (-2x)(-6x^3),\ (12x^3)(x)$

3. $$\begin{aligned} x^2 + 25 - 10x &= x^2 - 10x + 25 \\ &= (x-5)(x-5) \\ &= (x-5)^2 \end{aligned}$$

5. $$\begin{aligned} x^3 + x^2 + 2x + 2 &= x^2(x+1) + 2(x+1) \\ &= (x+1)(x^2+2) \end{aligned}$$

7. $$\begin{aligned} a^3 + 3a^2 - 4a &= a(a^2 + 3a - 4) \\ &= a(a+4)(a-1) \end{aligned}$$

9. $4t^2 - 25 = (2t+5)(2t-5)$

11. $$\begin{aligned} -6m^3 - 9m^2 - 3m &= -3m(2m^2 + 3m + 1) \\ &= -3m(2m+1)(m+1) \end{aligned}$$

13. $$\begin{aligned} 45r^2 + 60r + 20 &= 5(9r^2 + 12r + 4) \\ &= 5(3r+2)(3r+2) \\ &= 5(3r+2)^2 \end{aligned}$$

15. $$\begin{aligned} 49t^2 + 36 + 84t &= 49t^2 + 84t + 36 \\ &= (7t+6)(7t+6) \\ &= (7t+6)^2 \end{aligned}$$

17. $x^2 + 3x + 6$ is prime.

19. $$\begin{aligned} 6t^3 + 9t^2 - 15t &= 3t(2t^2 + 3t - 5) \\ &= 3t(2t+5)(t-1) \end{aligned}$$

21. $$\begin{aligned} &x^2 - 6x + 5 = 0 \\ &(x-5)(x-1) = 0 \\ &x - 5 = 0 \text{ or } x - 1 = 0 \\ &x = 5 \text{ or } x = 1 \end{aligned}$$

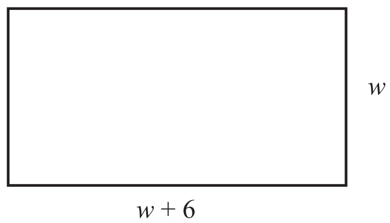
23. $$\begin{aligned} 4t - 10t^2 &= 0 \\ 2t(2-5t) &= 0 \\ 2t = 0 &\text{ or } 2 - 5t = 0 \\ t = 0 &\text{ or } -5t = -2 \\ t = 0 &\text{ or } t = \frac{2}{5} \end{aligned}$$

The solutions are 0 and $\frac{2}{5}$.

25. $$\begin{aligned} x(x-1) &= 20 \\ x^2 - x &= 20 \\ x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \\ x - 5 = 0 &\text{ or } x + 4 = 0 \\ x = 5 &\text{ or } x = -4 \end{aligned}$$

The solutions are -4 and 5.

27. ***Familiarize.*** We make a drawing, Let $w =$ the width, in m, Then $w + 6 =$ the length in m.



Recall the area of a rectangle is length times width.

Translate. We reword the problem

Length	times	width	is	40 m^2
$(w+6)$	$\cdot$	w	$=$	40

Carry out. We solve the equation.

$$\begin{aligned} (w+6)w &= 40 \\ w^2 + 6w &= 40 \\ w^2 + 6w - 40 &= 0 \\ (w+10)(w-4) &= 0 \\ w + 10 = 0 &\text{ or } w - 4 = 0 \\ w = -10 &\text{ or } w = 4 \end{aligned}$$

Check. Since the width must be positive, -10 cannot be a solution. If the width is 4m, the length is $4+6$ or 10 m, and the area is $10 \cdot 4$, or 40 m^2. Thus 4 checks.
State. The width is 4 m and the length is 10 m.

29. ***Familiarize.*** From the given drawing x is the distance in feet we are looking for.
Translate.We use the Pythagorean Theorem. $3^2+4^2 = x^2$
Carry out. We solve the equation:

$$\begin{aligned} 3^2 + 4^2 &= x^2 \\ 9 + 16 &= x^2 \\ 25 &= x^2 \\ x = 5 &\text{ or } -5 \end{aligned}$$

Check. The number -5 is not a solution because distance cannot be negative. If $x = 5$, $3^2 + 4^2 = 9 + 16 = 5^2$, so the answer checks.

State. The distance is 5 ft.

31. $$\begin{aligned}(a+3)^2 - 2(a+3) - 35 &= [(a+3) - 7]\,[(a+3) + 5] \\ &= [a + 3 - 7][a + 3 + 5] \\ &= (a-4)(a+8)\end{aligned}$$