Chapter 6

Rational Expressions and Equations

Exercise Set 6.1

1. $x-2=0$ when $x=2$ and $x+3=0$ when $x=-3$, so choice (e) is correct.

3. $a^2-a-12=(a-4)(a+3)$; $a-4=0$ when $a=4$ and $a+3=0$ when $a=-3$, so choice (d) is correct.

5. $2t-1=0$ when $t=\frac{1}{2}$ and $3t+4=0$ when $t=-\frac{4}{3}$, so choice (c) is correct.

7. $\frac{18}{-11x}$

We find the real number(s) that make the denominator 0. To do so we set the denominator equal to 0 and solve for x:

$$-11x=0$$
$$x=0$$

The expression is undefined for $x=0$.

9. $\frac{y-3}{y+5}$

Set the denominator equal to 0 and solve for y:

$$y+5=0$$
$$y=-5$$

The expression is undefined for $y=-5$.

11. $\frac{t-5}{3t-15}$

Set the denominator equal to 0 and solve for t:

$$3t-15=0$$
$$3t=15$$
$$t=5$$

The expression is undefined for $t=5$.

13. $\frac{x^2-25}{x^2-3x-28}$

Set the denominator equal to 0 and solve for x:

$$x^2-3x-28=0$$
$$(x-7)(x+4)=0$$
$$x-7=0 \quad or \quad x+4=0$$
$$x=7 \quad or \quad x=-4$$

The expression is undefined for $x=7$ and $x=-4$.

15. $\frac{t^2+t-20}{2t^2+11t-6}$

Set the denominator equal to 0 and solve for t:

$$2t^2+11t-6=0$$
$$(2t-1)(t+6)=0$$
$$2t-1=0 \quad or \quad t+6=0$$
$$2t=1 \quad or \quad t=-6$$
$$t=\frac{1}{2} \quad or \quad t=-6$$

The expression is undefined for $t=\frac{1}{2}$ and $t=-6$.

17. $\frac{50a^2b}{40ab^3}$

$=\frac{5a\cdot 10ab}{4b^2\cdot 10ab}$ Factoring the numerator and denominator. Note the common factor of $10ab$.

$=\frac{5a}{4b^2}\cdot\frac{10ab}{10ab}$ Rewriting as a product of two rational expressions

$=\frac{5a}{4b^2}\cdot 1$ $\qquad \frac{10ab}{10ab}=1$

$=\frac{5a}{4b^2}$ Removing the factor 1

19. $\frac{6t+12}{6t-18}=\frac{\not{6}(t+2)}{\not{6}(t-3)}=\frac{(t+2)}{(t-3)}$

21. $\frac{21t-7}{24t-8}=\frac{7\cancel{(3t-1)}}{8\cancel{(3t-1)}}=\frac{7}{8}$

23.
$$\frac{a^2-9}{a^2+4a+3}=\frac{(a+3)(a-3)}{(a+3)(a+1)}$$
$$=\frac{a+3}{a+3}\cdot\frac{a-3}{a+1}$$
$$=1\cdot\frac{a-3}{a+1}$$
$$=\frac{a-3}{a+1}$$

25.
$$\frac{-36x^8}{54x^5}=\frac{-2x^3\cdot 18x^5}{3\cdot 18x^5}$$
$$=\frac{-2x^3}{3}\cdot\frac{18x^5}{18x^5}$$
$$=\frac{-2x^3}{3}$$

Check: Let $x=1$.

$$\frac{-36x^8}{54x^5}=\frac{-36\cdot 1^8}{54\cdot 1^5}=\frac{-36}{54}=\frac{-2}{3}$$
$$\frac{-2x^3}{3}=\frac{-2\cdot 1^3}{3}=\frac{-2}{3}$$

The answer is probably correct.

27. $\dfrac{-2y+6}{-8y} = \dfrac{-2(y-3)}{-2\cdot 4y}$

$= \dfrac{-2}{-2}\cdot\dfrac{y-3}{4y}$

$= 1\cdot\dfrac{y-3}{4y}$

$= \dfrac{y-3}{4y}$

Check: Let $x = 2$.

$\dfrac{-2y+6}{-8y} = \dfrac{-2\cdot 2+6}{-8\cdot 2} = \dfrac{2}{-16} = -\dfrac{1}{8}$

$\dfrac{y-3}{4y} = \dfrac{2-3}{4\cdot 2} = \dfrac{-1}{8} = -\dfrac{1}{8}$

The answer is probably correct.

29. $\dfrac{t^2-16}{t^2-t-20} = \dfrac{(t-4)(t+4)}{(t-5)(t+4)}$

$= \dfrac{t-4}{t-5}\cdot\dfrac{t+4}{t+4}$

$= \dfrac{t-4}{t-5}\cdot 1$

$= \dfrac{t-4}{t-5}$

Check: Let $t = 1$.

$\dfrac{t^2-16}{t^2-t-20} = \dfrac{1^2-16}{1^2-1-20} = \dfrac{-15}{-20} = \dfrac{3}{4}$

$\dfrac{t-4}{t-5} = \dfrac{1-4}{1-5} = \dfrac{3}{4}$

The answer is probably correct.

31. $\dfrac{3a^2+9a-12}{6a^2-30a+24} = \dfrac{3(a^2+3a-4)}{6(a^2-5a+4)}$

$= \dfrac{3(a+4)(a-1)}{3\cdot 2(a-4)(a-1)}$

$= \dfrac{3(a-1)}{3(a-1)}\cdot\dfrac{a+4}{2(a-4)}$

$= 1\cdot\dfrac{a+4}{2(a-4)}$

$= \dfrac{a+4}{2(a-4)}$

Check: Let $a = 2$.

$\dfrac{3a^2+9a-12}{6a^2-30a+24} = \dfrac{3\cdot 2^2+9\cdot 2-12}{6\cdot 2^2-30\cdot 2+24} = \dfrac{18}{-12} = -\dfrac{3}{2}$

$\dfrac{a+4}{2(a-4)} = \dfrac{2+4}{2(2-4)} = \dfrac{6}{-4} = -\dfrac{3}{2}$

The answer is probably correct.

33. $\dfrac{x^2-8x+16}{x^2-16} = \dfrac{(x-4)(x-4)}{(x+4)(x-4)}$

$= \dfrac{x-4}{x+4}\cdot\dfrac{x-4}{x-4}$

$= \dfrac{x-4}{x+4}\cdot 1$

$= \dfrac{x-4}{x+4}$

Check: Let $x = 1$.

$\dfrac{x^2-8x+16}{x^2-16} = \dfrac{1^2-8\cdot 1+16}{1^2-16} = \dfrac{1-8+16}{1-16} = \dfrac{9}{-15} = -\dfrac{3}{5}$

$\dfrac{x-4}{x+4} = \dfrac{1-4}{1+4} = -\dfrac{3}{5}$

The answer is probably correct.

35. $\dfrac{n-2}{n^3-8} = \dfrac{n-2}{(n-2)(n^2+2n+4)}$

$= \dfrac{n-2}{n-2}\cdot\dfrac{1}{n^2+2n+4}$

$= \dfrac{1}{n^2+2n+4}$

37. $\dfrac{t^2-1}{t+1} = \dfrac{(t+1)(t-1)}{t+1}$

$= \dfrac{t+1}{t+1}\cdot\dfrac{t-1}{1}$

$= 1\cdot\dfrac{t-1}{1}$

$= t-1$

Check: Let $t = 2$.

$\dfrac{t^2-1}{t+1} = \dfrac{2^2-1}{2+1} = \dfrac{3}{3} = 1$

$t-1 = 2-1 = 1$

The answer is probably correct.

39. $\dfrac{y^2+4}{y+2}$ cannot be simplified.

Neither the numerator nor the denominator can be factored.

41. $\dfrac{5x^2+20}{10x^2+40} = \dfrac{5(x^2+4)}{10(x^2+4)}$

$= \dfrac{1\cdot \cancel{5}\cdot(\cancel{x^2+4})}{2\cdot\cancel{5}\cdot(\cancel{x^2+4})}$

$= \dfrac{1}{2}$

Check: Let $x = 1$.

$\dfrac{5x^2+20}{10x^2+40} = \dfrac{5\cdot 1^2+20}{10\cdot 1^2+40} = \dfrac{25}{50} = \dfrac{1}{2}$

$\dfrac{1}{2} = \dfrac{1}{2}$

The answer is probably correct.

43. $\dfrac{y^2+6y}{2y^2+13y+6} = \dfrac{y(y+6)}{(2y+1)(y+6)}$
$= \dfrac{y}{2y+1}\cdot\dfrac{y+6}{y+6}$
$= \dfrac{y}{2y+1}\cdot 1$
$= \dfrac{y}{2y+1}$

Check: Let $y=1$.
$\dfrac{y^2+6y}{2y^2+13y+6} = \dfrac{1^2+6\cdot 1}{2\cdot 1^2+13\cdot 1+6} = \dfrac{7}{21} = \dfrac{1}{3}$
$\dfrac{y}{2y+1} = \dfrac{1}{2\cdot 1+1} = \dfrac{1}{3}$
The answer is probably correct.

45. $\dfrac{4x^2-12x+9}{10x^2-11x-6} = \dfrac{(2x-3)(2x-3)}{(2x-3)(5x+2)}$
$= \dfrac{2x-3}{2x-3}\cdot\dfrac{2x-3}{5x+2}$
$= 1\cdot\dfrac{2x-3}{5x+2}$
$= \dfrac{2x-3}{5x+2}$

Check: Let $t=1$.
$\dfrac{4x^2-12x+9}{10x^2-11x-6} = \dfrac{4\cdot 1^2-12\cdot 1+9}{10\cdot 1^2-11\cdot 1-6} = \dfrac{1}{-7} = -\dfrac{1}{7}$
$\dfrac{2x-3}{5x+2} = \dfrac{2\cdot 1-3}{5\cdot 1+2} = \dfrac{-1}{7} = -\dfrac{1}{7}$
The answer is probably correct.

47. $\dfrac{10-x}{x-10} = \dfrac{-(x-10)}{x-10}$
$= \dfrac{-1}{1}\cdot\dfrac{x-10}{x-10} = -1\cdot 1 = -1$

Check: Let $x=1$.
$\dfrac{10-x}{x-10} = \dfrac{10-1}{1-10} = \dfrac{9}{-9} = -1$

The answer is probably correct.

49. $\dfrac{7t-14}{2-t} = \dfrac{7(t-2)}{-(t-2)}$
$= \dfrac{7}{-1}\cdot\dfrac{t-2}{t-2}$
$= \dfrac{7}{-1}\cdot 1$
$= -7$

Check: Let $t=1$.
$\dfrac{7t-14}{2-t} = \dfrac{7\cdot 1-14}{2-1} = \dfrac{-7}{1} = -7$
The answer is probably correct.

51. $\dfrac{a-b}{4b-4a} = \dfrac{a-b}{-4(a-b)}$
$= \dfrac{1}{-4}\cdot\dfrac{a-b}{a-b}$
$= -\dfrac{1}{4}\cdot 1$
$= -\dfrac{1}{4}$

Check: Let $a=2$ and $b=1$.
$\dfrac{a-b}{4b-4a} = \dfrac{2-1}{4\cdot 1-4\cdot 2} = \dfrac{1}{4-8} = -\dfrac{1}{-4}$
The answer is probably correct.

53. $\dfrac{3x^2-3y^2}{2y^2-2x^2} = \dfrac{3(x^2-y^2)}{2(y^2-x^2)}$
$= \dfrac{3(x^2-y^2)}{2(-1)(x^2-y^2)}$
$= \dfrac{3}{2(-1)}\cdot\dfrac{x^2-y^2}{x^2-y^2}$
$= \dfrac{3}{2(-1)}\cdot 1$
$= -\dfrac{3}{2}$

Check: Let $x=1$ and $y=2$.
$\dfrac{3x^2-3y^2}{2y^2-2x^2} = \dfrac{3\cdot 1^2-3\cdot 2^2}{2\cdot 2^2-2\cdot 1^2} = \dfrac{-9}{6} = -\dfrac{3}{2}$

The answer is probably correct.

55. $\dfrac{7s^2-28t^2}{28t^2-7s^2}$

Note that the numerator and denominator are opposites. Thus, we have an expression divided by its opposite, so the result is -1.

57. *Writing Exercise.* Simplifying removes a factor equal to 1, allowing us to rewrite an expression $a\cdot 1$ as a.

59. $-\dfrac{2}{15}\cdot\dfrac{10}{7} = -\dfrac{2\cdot 10}{15\cdot 7}$
$= -\dfrac{2\cdot 2\cdot \not{5}}{3\cdot \not{5}\cdot 7}$
$= -\dfrac{4}{21}$

61. $\dfrac{5}{8}\div\left(-\dfrac{1}{6}\right) = \dfrac{5}{8}\cdot(-6)$
$= -\dfrac{5\cdot 6}{8}$
$= -\dfrac{5\cdot \not{2}\cdot 3}{\not{2}\cdot 4}$
$= -\dfrac{15}{4}$

63. $\dfrac{7}{9}-\dfrac{2}{3}\cdot\dfrac{6}{7} = \dfrac{7}{9}-\dfrac{4}{7} = \dfrac{7}{9}\cdot\dfrac{7}{7}-\dfrac{4}{7}\cdot\dfrac{9}{9}$
$= \dfrac{49}{63}-\dfrac{36}{63} = \dfrac{13}{63}$

65. *Writing Exercise.* Although a rational expression has been simplified incorrectly, it is possible that there are one or more values of the variable(s) for which the two expressions are the same. For example, $\dfrac{x^2+x-2}{x^2+3x+2}$ could be simplified incorrectly as $\dfrac{x-1}{x+2}$, but evaluating the expressions for $x=1$ gives 0 in each case. $\left(\text{The correct simplification is } \dfrac{x-1}{x+1}.\right)$

67. $\dfrac{16y^4 - x^4}{(x^2+4y^2)(x-2y)}$

$= \dfrac{(4y^2+x^2)(4y^2-x^2)}{(x^2+4y^2)(x-2y)}$

$= \dfrac{(4y^2+x^2)(2y+x)(2y-x)}{(x^2+4y^2)(x-2y)}$

$= \dfrac{(x^2+4y^2)(2y+x)(-1)(x-2y)}{(x^2+4y^2)(x-2y)}$

$= \dfrac{(x^2+4y^2)(x-2y)}{(x^2+4y^2)(x-2y)} \cdot \dfrac{(2y+x)(-1)}{1}$

$= -2y-x$, or $-x-2y$, or $-(2y+x)$

69. $\dfrac{x^5-2x^3+4x^2-8}{x^7+2x^4-4x^3-8}$

$= \dfrac{x^3(x^2-2)+4(x^2-2)}{x^4(x^3+2)-4(x^3+2)}$

$= \dfrac{(x^2-2)(x^3+4)}{(x^3+2)(x^4-4)}$

$= \dfrac{(x^2-2)(x^3+4)}{(x^3+2)(x^2+2)(x^2-2)}$

$= \dfrac{\cancel{(x^2-2)}(x^3+4)}{(x^3+2)(x^2+2)\cancel{(x^2-2)}}$

$= \dfrac{x^3+4}{(x^3+2)(x^2+2)}$

71. $\dfrac{(t^4-1)(t^2-9)(t-9)^2}{(t^4-81)(t^2+1)(t+1)^2}$

$= \dfrac{(t^2+1)(t+1)(t-1)(t+3)(t-3)(t-9)(t-9)}{(t^2+9)(t+3)(t-3)(t^2+1)(t+1)(t+1)}$

$= \dfrac{\cancel{(t^2+1)}\cancel{(t+1)}(t-1)\cancel{(t+3)}\cancel{(t-3)}(t-9)(t-9)}{(t^2+9)\cancel{(t+3)}\cancel{(t-3)}\cancel{(t^2+1)}\cancel{(t+1)}(t+1)}$

$= \dfrac{(t-1)(t-9)(t-9)}{(t^2+9)(t+1)}$, or $\dfrac{(t-1)(t-9)^2}{(t^2+9)(t+1)}$

73. $\dfrac{(x^2-y^2)(x^2-2xy+y^2)}{(x+y)^2(x^2-4xy-5y^2)}$

$= \dfrac{(x+y)(x-y)(x-y)(x-y)}{(x+y)(x+y)(x-5y)(x+y)}$

$= \dfrac{\cancel{(x+y)}(x-y)(x-y)(x-y)}{\cancel{(x+y)}(x+y)(x-5y)(x+y)}$

$= \dfrac{(x-y)^3}{(x+y)^2(x-5y)}$

75. *Writing Exercise.*

$\dfrac{5(2x+5)-25}{10} = \dfrac{10x+25-25}{10}$

$= \dfrac{10x}{10}$

$= x$

You get the same number you selected.

A person asked to select a number and then perform these operations would probably be surprised that the result is the original number.

Exercise Set 6.2

1. $\dfrac{3x}{8} \cdot \dfrac{x+2}{5x-1} = \dfrac{3x(x+2)}{8(5x-1)}$

3. $\dfrac{a-4}{a+6} \cdot \dfrac{a+2}{a+6} = \dfrac{(a-4)(a+2)}{(a+6)(a+6)}$, or $\dfrac{(a-4)(a+2)}{(a+6)^2}$

5. $\dfrac{2x+3}{4} \cdot \dfrac{x+1}{x-5} = \dfrac{(2x+3)(x+1)}{4(x-5)}$

7. $\dfrac{n-4}{n^2+4} \cdot \dfrac{n+4}{n^2-4} = \dfrac{(n-4)(n+4)}{(n^2+4)(n^2-4)}$

9. $\dfrac{y+6}{1+y} \cdot \dfrac{y-3}{y+3} = \dfrac{(y+6)(y-3)}{(1+y)(y+3)}$

11. $\dfrac{8t^3}{5t} \cdot \dfrac{3}{4t}$

$= \dfrac{8t^3 \cdot 3}{5t \cdot 4t}$ Multiplying the numerators and the denominators

$= \dfrac{2\cdot 4\cdot t\cdot t\cdot t\cdot 3}{5\cdot t\cdot 4\cdot t}$ Factoring the numerator and the denominator

$= \dfrac{2\cdot \cancel{4}\cdot t\cdot \cancel{t}\cdot \cancel{t}\cdot 3}{5\cdot \cancel{t}\cdot \cancel{4}\cdot \cancel{t}}$ Removing a factor equal to 1

$= \dfrac{6t}{5}$ Simplifying

13. $\dfrac{3c}{d^2} \cdot \dfrac{8d}{6c^3}$

$= \dfrac{3c \cdot 8d}{d^2 \cdot 6c^3}$ Multiplying the numerators and the denominators

$= \dfrac{3\cdot c\cdot 2\cdot 4\cdot d}{d\cdot d\cdot 3\cdot 2\cdot c\cdot c\cdot c}$ Factoring the numerator and the denominator

$= \dfrac{\cancel{3}\cdot \cancel{c}\cdot \cancel{2}\cdot 4\cdot \cancel{d}}{\cancel{d}\cdot d\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{c}\cdot c\cdot c}$

$= \dfrac{4}{dc^2}$

15. $\dfrac{x^2-3x-10}{(x-2)^2} \cdot (x-2) = \dfrac{(x^2-3x-10)(x-2)}{(x-2)^2}$

$= \dfrac{(x-5)(x+2)(x-2)}{(x-2)(x-2)}$

$= \dfrac{(x-5)\ (x+2)\cancel{(x-2)}}{(x-2)\ \cancel{(x-2)}}$

$= \dfrac{(x-5)(x+2)}{x-2}$

17. $\dfrac{n^2-6n+5}{n+6} \cdot \dfrac{n-6}{n^2+36} = \dfrac{(n^2-6n+5)(n-6)}{(n+6)(n^2+36)}$

$= \dfrac{(n-5)(n-1)(n-6)}{(n+6)(n^2+36)}$

(No simplification is possible.)

19. $\dfrac{a^2-9}{a^2} \cdot \dfrac{7a}{a^2+a-12} = \dfrac{(a+3)(a-3)\cdot 7\cdot a}{a\cdot a(a+4)(a-3)}$

$= \dfrac{(a+3)\cancel{(a-3)}\cdot 7\cdot \cancel{a}}{\cancel{a}\cdot a(a+4)\cancel{(a-3)}}$

$= \dfrac{7(a+3)}{a(a+4)}$

21. $\dfrac{4v-8}{5v}\cdot\dfrac{15v^2}{4v^2-16v+16}$

$=\dfrac{(4v-8)15v^2}{5v(4v^2-16v+16)}$

$=\dfrac{\cancel{4}\cancel{(v-2)}\cdot\cancel{5}\cdot 3\cdot v\cdot\cancel{v}}{\cancel{5}\cancel{v}\cdot\cancel{4}\cancel{(v-2)}(v-2)}$

$=\dfrac{3v}{v-2}$

23. $\dfrac{t^2+2t-3}{t^2+4t-5}\cdot\dfrac{t^2-3t-10}{t^2+5t+6}$

$=\dfrac{(t^2+2t-3)(t^2-3t-10)}{(t^2+4t-5)(t^2+5t+6)}$

$=\dfrac{(t+3)(t-1)(t-5)(t+2)}{(t+5)(t-1)(t+3)(t+2)}$

$=\dfrac{\cancel{(t+3)}\cancel{(t-1)}(t-5)\cancel{(t+2)}}{(t+5)\cancel{(t-1)}\cancel{(t+3)}\cancel{(t+2)}}$

$=\dfrac{t-5}{t+5}$

25. $\dfrac{12y+12}{5y+25}\cdot\dfrac{3y^2-75}{8y^2-8}$

$=\dfrac{(12y+12)(3y^2-75)}{(5y+25)(8y^2-8)}$

$=\dfrac{3\cdot\cancel{4(y+1)}\,3\cancel{(y+5)}(y-5)}{5\cancel{(y+5)}\,2\cdot\cancel{4(y+1)}(y-1)}$

$=\dfrac{9(y-5)}{10(y-1)}$

27. $\dfrac{x^2+4x+4}{(x-1)^2}\cdot\dfrac{x^2-2x+1}{(x+2)^2}=\dfrac{(x+2)^2(x-1)^2}{(x-1)^2(x+2)^2}=1$

29. $\dfrac{t^2-4t+4}{t^2-7t+6}\cdot\dfrac{2t^2+7t-15}{t^2-10t+25}$

$=\dfrac{(t^2-4t+4)(2t^2+7t-15)}{(2t^2-7t+6)(t^2-10t+25)}$

$=\dfrac{(t-2)(t-2)(2t-3)(t+5)}{(2t-3)(t-2)(t-5)(t-5)}$

$=\dfrac{\cancel{(t-2)}(t-2)\cancel{(2t-3)}(t+5)}{\cancel{(2t-3)}\cancel{(t-2)}(t-5)(t-5)}$

$=\dfrac{(t-2)(t+5)}{(t-5)^2}$

31. $(10x^2-x-2)\cdot\dfrac{4x^2-8x+3}{10x^2-11x-6}$

$=\dfrac{(10x^2-x-2)(4x^2-8x+3)}{(10x^2-11x-6)}$

$=\dfrac{\cancel{(5x+2)}(2x-1)(2x-1)\cancel{(2x-3)}}{\cancel{(5x+2)}\cancel{(2x-3)}}$

$=(2x-1)^2$

33. $\dfrac{c^3+8}{c^5-4c^3}\cdot\dfrac{c^6-4c^5+4c^4}{c^2-2c+4}$

$=\dfrac{(c+2)(c^2-2c+4)}{c^3(c^2-4)}\cdot\dfrac{c^4(c^2-4c+4)}{c^2-2c+4}$

$=\dfrac{\cancel{(c+2)}\cancel{(c^2-2c+4)}(\cancel{c^3}\cdot c)\cancel{(c-2)}(c-2)}{\cancel{c^3}\cancel{(c+2)}\cancel{(c-2)}\cancel{(c^2-2c+4)}}$

$=c(c-2)$

35. The reciprocal of $\dfrac{2x}{9}$ is $\dfrac{9}{2x}$ because $\dfrac{2x}{9}\cdot\dfrac{9}{2x}=1$.

37. The reciprocal of a^4+3a is $\dfrac{1}{a^4+3a}$ because $\dfrac{a^4+3a}{1}\cdot\dfrac{1}{a^4+3a}=1$.

39. $\dfrac{5}{9}\div\dfrac{3}{4}$

$=\dfrac{5}{9}\cdot\dfrac{4}{3}$ Multiplying by the reciprocal of the divisor

$=\dfrac{5\cdot 4}{9\cdot 3}$

$=\dfrac{20}{27}$

No simplification is possible.

41. $\dfrac{x}{4}\div\dfrac{5}{x}$

$=\dfrac{x}{4}\cdot\dfrac{x}{5}$ Multiplying by the reciprocal of the divisor

$=\dfrac{x\cdot x}{4\cdot 5}$

$=\dfrac{x^2}{20}$

43. $\dfrac{a^5}{b^4}\div\dfrac{a^2}{b}=\dfrac{a^5}{b^4}\cdot\dfrac{b}{a^2}$

$=\dfrac{a^5\cdot b}{b^4\cdot a^2}$

$=\dfrac{a^2\cdot a^3\cdot b}{b\cdot b^3\cdot a^2}$

$=\dfrac{a^2b}{a^2b}\cdot\dfrac{a^3}{b^3}$

$=\dfrac{a^3}{b^3}$

45. $\dfrac{t-3}{6}\div\dfrac{t+1}{8}=\dfrac{t-3}{6}\cdot\dfrac{8}{t+1}$

$=\dfrac{(t-3)(8)}{6\cdot(t+1)}$

$=\dfrac{(t-3)\cdot 4\cdot\cancel{2}}{\cancel{2}\cdot 3(t+1)}$

$=\dfrac{4(t-3)}{3(t+1)}$

47. $\dfrac{4y-8}{y+2} \div \dfrac{y-2}{y^2-4} = \dfrac{4y-8}{y+2} \cdot \dfrac{y^2-4}{y-2}$

$= \dfrac{(4y-8)(y^2-4)}{(y+2)(y-2)}$

$= \dfrac{4\cancel{(y-2)}\cancel{(y+2)}(y-2)}{\cancel{(y+2)}\cancel{(y-2)}(1)}$

$= 4(y-2)$

49. $\dfrac{a}{a-b} \div \dfrac{b}{b-a} = \dfrac{a}{a-b} \cdot \dfrac{b-a}{b}$

$= \dfrac{a(b-a)}{(a-b)(b)}$

$= \dfrac{a(-1)\cancel{(a-b)}}{\cancel{(a-b)}(b)}$

$= \dfrac{-a}{b} = -\dfrac{a}{b}$

51. $\left(n^2+5n+6\right) \div \dfrac{n^2-4}{n+3} = \dfrac{\left(n^2+5n+6\right)}{1} \cdot \dfrac{(n+3)}{n^2-4}$

$= \dfrac{\left(n^2+5n+6\right)(n+3)}{n^2-4}$

$= \dfrac{(n+3)\cancel{(n+2)}(n+3)}{\cancel{(n+2)}(n-2)}$

$= \dfrac{(n+3)^2}{n-2}$

53. $\dfrac{-3+3x}{16} \div \dfrac{x-1}{5} = \dfrac{3x-3}{16} \cdot \dfrac{5}{x-1}$

$= \dfrac{(3x-3)\cdot 5}{16(x-1)}$

$= \dfrac{3(x-1)\cdot 5}{16(x-1)}$

$= \dfrac{3\cancel{(x-1)}\cdot 5}{16\cancel{(x-1)}}$

$= \dfrac{15}{16}$

55. $\dfrac{x-1}{x+2} \div \dfrac{1-x}{4+x^2} = \dfrac{x-1}{x+2} \cdot \dfrac{4+x^2}{1-x}$

$= \dfrac{(x-1)\left(4+x^2\right)}{(x+2)(1-x)}$

$= \dfrac{\cancel{(x-1)}\left(x^2+4\right)}{-1(x+2)\cancel{(x-1)}}$

$= -\dfrac{x^2+4}{x+2}$ or $\dfrac{-x^2-4}{x+2}$

57. $\dfrac{a+2}{a-1} \div \dfrac{3a+6}{a-5} = \dfrac{a+2}{a-1} \cdot \dfrac{a-5}{3a+6}$

$= \dfrac{(a+2)(a-5)}{(a-1)(3a+6)}$

$= \dfrac{(a+2)(a-5)}{(a-1)\cdot 3\cdot (a+2)}$

$= \dfrac{\cancel{(a+2)}(a-5)}{(a-1)\cdot 3\cdot \cancel{(a+2)}}$

$= \dfrac{a-5}{3(a-1)}$

59. $(2x-1) \div \dfrac{2x^2-11x+5}{4x^2-1}$

$= \dfrac{2x-1}{1} \cdot \dfrac{4x^2-1}{2x^2-11x+5}$

$= \dfrac{(2x-1)(4x^2-1)}{1\cdot(2x^2-11x+5)}$

$= \dfrac{(2x-1)(2x+1)(2x-1)}{1\cdot(2x-1)(x-5)}$

$= \dfrac{\cancel{(2x-1)}(2x+1)(2x-1)}{1\cdot\cancel{(2x-1)}(x-5)}$

$= \dfrac{(2x-1)(2x+1)}{x-5}$

61. $\dfrac{w^2-14w+49}{2w^2-3w-14} \div \dfrac{3w^2-20w-7}{w^2-6w-16}$

$= \dfrac{w^2-14w+49}{2w^2-3w-14} \cdot \dfrac{w^2-6w-16}{3w^2-20w-7}$

$= \dfrac{\left(w^2-14w+49\right)\left(w^2-6w-16\right)}{(2w^2-3w-14)(3w^2-20w-7)}$

$= \dfrac{(w-7)\cancel{(w-7)}(w-8)\cancel{(w+2)}}{(2w-7)\cancel{(w+2)}(3w+1)\cancel{(w-7)}}$

$= \dfrac{(w-7)(w-8)}{(2w-7)(3w+1)}$

63. $\dfrac{c^2+10c+21}{c^2-2c-15} \div (5c^2+32c-21)$

$= \dfrac{c^2+10c+21}{c^2-2c-15} \cdot \dfrac{1}{5c^2+32c-21}$

$= \dfrac{(c^2+10c+21)\cdot 1}{(c^2-2c-15)(5c^2+32c-21)}$

$= \dfrac{(c+7)(c+3)}{(c-5)(c+3)(5c-3)(c+7)}$

$= \dfrac{(c+7)(c+3)}{(c+7)(c+3)} \cdot \dfrac{1}{(c-5)(5c-3)}$

$= \dfrac{1}{(c-5)(5c-3)}$

65. $\dfrac{x-y}{x^2+2xy+y^2} \div \dfrac{x^2-y^2}{x^2-5xy+4y^2}$

$= \dfrac{x-y}{x^2+2xy+y^2} \cdot \dfrac{x^2-5xy+4y^2}{x^2-y^2}$

$= \dfrac{\cancel{(x-y)}(x-y)(x-4y)}{(x+y)(x+y)(x+y)\cancel{(x-y)}}$

$= \dfrac{(x-y)(x-4y)}{(x+y)^3}$

67. $\dfrac{x^3-64}{x^3+64} \div \dfrac{x^2-16}{x^2-4x+16}$

$= \dfrac{x^3-64}{x^3+64} \cdot \dfrac{x^2-4x+16}{x^2-16}$

$= \dfrac{(x-4)\left(x^2+4x+16\right)\left(x^2-4x+16\right)}{(x+4)\left(x^2-4x+16\right)(x+4)(x-4)}$

$= \dfrac{\cancel{(x-4)}\left(x^2+4x+16\right)\left(x^2-\cancel{4x}+16\right)}{(x+4)\left(x^2-\cancel{4x}+16\right)(x+4)\cancel{(x-4)}}$

$= \dfrac{\left(x^2+4x+16\right)}{(x+4)^2}$

69. $\dfrac{8a^3+b^3}{2a^2+3ab+b^2} \div \dfrac{8a^2-4ab+2b^2}{4a^2+4ab+b^2}$

$= \dfrac{(2a+b)\left(4a^2-2ab+b^2\right)}{(2a+b)\,(a+b)} \cdot \dfrac{4a^2+4ab+b^2}{8a^2-4ab+2b^2}$

$= \dfrac{\cancel{(2a+b)}\,\cancel{\left(4a^2-2ab+b^2\right)}\,(2a+b)\,(2a+b)}{\cancel{(2a+b)}\,(a+b)\,(2)\,\cancel{\left(4a^2-2ab+b^2\right)}}$

$= \dfrac{(2a+b)^2}{2\,(a+b)}$

71. *Writing Exercise.* Parentheses are required to ensure that numerators and denominators are multiplied correctly. That is, the product of $(x+2)$ and $(3x-1)$ and the product of $(5x-7)$ and $(x+4)$ in the denominator.

73. $\dfrac{3}{4}+\dfrac{5}{6} = \dfrac{3}{4}\cdot\dfrac{3}{3}+\dfrac{5}{6}\cdot\dfrac{2}{2}$

$= \dfrac{9}{12}+\dfrac{10}{12}$

$= \dfrac{19}{12}$

75. $\dfrac{2}{9}-\dfrac{1}{6} = \dfrac{2}{9}\cdot\dfrac{2}{2}-\dfrac{1}{6}\cdot\dfrac{3}{3}$

$= \dfrac{4}{18}-\dfrac{3}{18}$

$= \dfrac{1}{18}$

77. $2x^2-x+1-(x^2-x-2) = 2x^2-x+1-x^2+x+2 = x^2+3$

79. *Writing Exercise.* Yes; consider the product $\dfrac{a}{b}\cdot\dfrac{c}{d}=\dfrac{ac}{bd}$. The reciprocal of the product is $\dfrac{bd}{ac}$. This is equal to the product of the reciprocals of the two original factors: $\dfrac{b}{a}\cdot\dfrac{d}{c}=\dfrac{bd}{ac}$.

81. The reciprocal of $2\frac{1}{3}x$ is $\dfrac{1}{2\frac{1}{3}x} = \dfrac{1}{\frac{7x}{3}} = 1\div\dfrac{7x}{3} = 1\cdot\dfrac{3}{7x} = \dfrac{3}{7x}$

83. $(x-2a)\div\dfrac{a^2x^2-4a^4}{a^2x+2a^3} = \dfrac{x-2a}{1}\cdot\dfrac{a^2x+2a^3}{a^2x^2-4a^4}$

$= \dfrac{(x-2a)(a^2x^2+2a^3)}{(a^2x^2-4a^4)}$

$= \dfrac{\cancel{(x-2a)}\cancel{a^2}\cancel{(x+2a)}}{\cancel{a^2}\cancel{(x-2a)}\cancel{(x+2a)}} = 1$

85. $\dfrac{3x^2-2xy-y^2}{x^2-y^2}\div(3x^2+4xy+y^2)^2$

$= \dfrac{3x^2-2xy-y^2}{x^2-y^2}\cdot\dfrac{1}{(3x^2+4xy+y^2)^2}$

$= \dfrac{\cancel{(3x+y)}\cancel{(x-y)}\cdot 1}{(x+y)\cancel{(x-y)}\cancel{(3x+y)}(3x+y)(x+y)(x+y)}$

$= \dfrac{1}{(x+y)^3(3x+y)}$

87. $\dfrac{a^2-3b}{a^2+2b}\cdot\dfrac{a^2-2b}{a^2+3b}\cdot\dfrac{a^2+2b}{a^2-3b}$

Note that $\dfrac{a^2-3b}{a^2+2b}\cdot\dfrac{a^2+2b}{a^2-3b}$ is the product of reciprocals and thus is equal to 1. Then the product in the original exercise is the remaining factor, $\dfrac{a^2-2b}{a^2+3b}$.

89. $\dfrac{z^2-8z+16}{z^2+8z+16}\div\dfrac{(z-4)^5}{(z+4)^5}\div\dfrac{3z+12}{z^2-16}$

$= \dfrac{(z-4)^2}{(z+4)^2}\cdot\dfrac{(z+4)^5}{(z-4)^5}\cdot\dfrac{(z+4)(z-4)}{3(z+4)}$

$= \dfrac{\cancel{(z-4)^2}\cancel{(z+4)^2}(z+4)^3\cancel{(z+4)}\cancel{(z-4)}}{\cancel{(z+4)^2}\cancel{(z-4)^2}\cancel{(z-4)}(z-4)^2(3)\cancel{(z+4)}}$

$= \dfrac{(z+4)^3}{3(z-4)^2}$

91. $\dfrac{a^4-81b^4}{a^2c-6abc+9b^2c}\cdot\dfrac{a+3b}{a^2+9b^2}\div\dfrac{a^2+6ab+9b^2}{(a-3b)^2}$

$= \dfrac{\cancel{(a^2+9b^2)}\cancel{(a+3b)}(a-3b)}{c\cancel{(a-3b)^2}}\cdot\dfrac{\cancel{a+3b}}{\cancel{a^2+9b^2}}\cdot\dfrac{\cancel{(a-3b)^2}}{\cancel{(a+3b)^2}}$

$= \dfrac{a-3b}{c}$

93. Enter $y_1 = \dfrac{x-1}{x^2+2x+1}\div\dfrac{x^2-1}{x^2-5x+4}$ and $y_2 = \dfrac{x^2-5x+4}{(x+1)^3}$, display the values of y_1 and y_2 in a table, and compare the values. (See the Technology Connection on page 385 in the text.)

Exercise Set 6.3

1. To add two rational expressions when the denominators are the same, add numerators and keep the common denominator. (See page 389 in the text.)

3. The least common multiple of two denominators is usually referred to as the least common denominator and is abbreviated LCD. (See page 390 in the text.)

5. $\dfrac{3}{t}+\dfrac{5}{t}=\dfrac{8}{t}$ Adding numerators

7. $\dfrac{x}{12}+\dfrac{2x+5}{12}=\dfrac{3x+5}{12}$ Adding numerators

9. $\dfrac{4}{a+3}+\dfrac{5}{a+3}=\dfrac{9}{a+3}$

11. $\dfrac{11}{4x-7}-\dfrac{3}{4x-7}=\dfrac{8}{4x-7}$ Subtracting numerators

13. $\dfrac{3y+8}{2y}-\dfrac{y+1}{2y}$

$= \dfrac{3y+8-(y+1)}{2y}$

$= \dfrac{3y+8-y-1}{2y}$ Removing parentheses

$= \dfrac{2y+7}{2y}$

15. $\dfrac{5x+7}{x+3}+\dfrac{x+11}{x+3}$

$= \dfrac{6x+18}{x+3}$ Adding numerators

$= \dfrac{6(x+3)}{x+3}$ Factoring

$= \dfrac{6\cancel{(x+3)}}{\cancel{x+3}}$ Removing a factor equal to 1

$= 6$

17. $\frac{5x+7}{x+3} - \frac{x+11}{x+3} = \frac{5x+7-(x+11)}{x+3}$

$= \frac{5x+7-x-11}{x+3}$

$= \frac{4x-4}{x+3}$

$= \frac{4(x-1)}{x+3}$

19. $\frac{a^2}{a-4} + \frac{a-20}{a-4} = \frac{a^2+a-20}{a-4}$

$= \frac{(a+5)(a-4)}{a-4}$

$= \frac{(a+5)\cancel{(a-4)}}{\cancel{a-4}}$

$= a+5$

21. $\frac{y^2}{y+2} - \frac{5y+14}{y+2} = \frac{y^2-(5y+14)}{y+2}$

$= \frac{y^2-5y-14}{y+2}$

$= \frac{(y-7)(y+2)}{y+2}$

$= \frac{(y-7)\cancel{(y+2)}}{\cancel{y+2}}$

$= y-7$

23. $\frac{t^2-5t}{t-1} + \frac{5t-t^2}{t-1}$

Note that the numerators are opposites, so their sum is 0.

Then we have $\frac{0}{t-1}$, or 0.

25. $\frac{x-6}{x^2+5x+6} + \frac{9}{x^2+5x+6} = \frac{x+3}{x^2+5x+6}$

$= \frac{x+3}{(x+3)(x+2)}$

$= \frac{\cancel{x+3}}{\cancel{(x+3)}(x+2)}$

$= \frac{1}{x+2}$

27. $\frac{t^2-5t}{t^2+6t+9} + \frac{4t-12}{t^2+6t+9} = \frac{t^2-t-12}{t^2+6t+9}$

$= \frac{(t-4)(t+3)}{(t+3)^2}$

$= \frac{(t-4)\cancel{(t+3)}}{(t+3)\cancel{(t+3)}}$

$= \frac{t-4}{t+3}$

29. $\frac{2y^2+3y}{y^2-7y+12} - \frac{y^2+4y+6}{y^2-7y+12}$

$= \frac{2y^2+3y-\left(y^2+4y+6\right)}{y^2-7y+12}$

$= \frac{2y^2+3y-y^2-4y-6}{y^2-7y+12}$

$= \frac{y^2-y-6}{y^2-7y+12}$

$= \frac{(y-3)(y+2)}{(y-3)(y-4)}$

$= \frac{\cancel{(y-3)}(y+2)}{\cancel{(y-3)}(y-4)}$

$= \frac{y+2}{y-4}$

31. $\frac{3-2x}{x^2-6x+8} + \frac{7-3x}{x^2-6x+8}$

$= \frac{10-5x}{x^2-6x+8}$

$= \frac{5(2-x)}{(x-4)(x-2)}$

$= \frac{5(-1)(x-2)}{(x-4)(x-2)}$

$= \frac{5(-1)\cancel{(x-2)}}{(x-4)\cancel{(x-2)}}$

$= \frac{-5}{x-4}$, or $-\frac{5}{x-4}$, or $\frac{5}{4-x}$

33. $\frac{x-9}{x^2+3x-4} - \frac{2x-5}{x^2+3x-4}$

$= \frac{x-9-(2x-5)}{x^2+3x-4}$

$= \frac{x-9-2x+5}{x^2+3x-4}$

$= \frac{-x-4}{x^2+3x-4}$

$= \frac{-(x+4)}{(x+4)(x-1)}$

$= \frac{-1\cancel{(x+4)}}{\cancel{(x+4)}(x-1)}$

$= \frac{-1}{x-1}$, or $-\frac{1}{x-1}$, or $\frac{1}{1-x}$

35. $15 = 3 \cdot 5$

$36 = 2 \cdot 2 \cdot 3 \cdot 3$

$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$

37. $8 = 2 \cdot 2 \cdot 2$

$9 = 3 \cdot 3$

$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$, or 72

39. $6 = 2 \cdot 3$

$12 = 2 \cdot 2 \cdot 3$

$15 = 3 \cdot 5$

$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 5 = 60$

41. $18t^2 = 2 \cdot 3 \cdot 3 \cdot t \cdot t$

$6t^5 = 2 \cdot 3 \cdot t \cdot t \cdot t \cdot t \cdot t$

$\text{LCM} = 2 \cdot 3 \cdot 3 \cdot t \cdot t \cdot t \cdot t \cdot t = 18t^5$

43. $15a^4b^7 = 3 \cdot 5 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$

$10a^2b^8 = 2 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$

$\text{LCM} = 2 \cdot 3 \cdot 5 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b,$ or $30a^4b^8$

45. $2(y-3) = 2 \cdot (y-3)$

$6(y-3) = 2 \cdot 3 \cdot (y-3)$

$\text{LCM} = 2 \cdot 3 \cdot (y-3),$ or $6(y-3)$

47. $x^2 - 2x - 15 = (x-5)(x+3)$

$x^2 - 9 = (x-3)(x+3)$

$\text{LCM} = (x-5)(x-3)(x+3)$

49. $t^3 + 4t^2 + 4t = t(t^2 + 4t + 4) = t(t+2)(t+2)$

$t^2 - 4t = t(t-4)$

$\text{LCM} = t(t+2)(t+2)(t-4) = t(t+2)^2(t-4)$

51. $6xz^2 = 2 \cdot 3 \cdot x \cdot z \cdot z$

$8x^2y = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$

$15y^3z = 3 \cdot 5 \cdot y \cdot y \cdot y \cdot z$

$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z = 120x^2y^3z^2$

53. $a + 1 = a + 1$

$(a-1)^2 = (a-1)(a-1)$

$a^2 - 1 = (a+1)(a-1)$

$\text{LCM} = (a+1)(a-1)(a-1) = (a+1)(a-1)^2$

55. $2n^2 + n - 1 = (2n-1)(n+1)$

$2n^2 + 3n - 2 = (2n-1)(n+2)$

$\text{LCM} = (2n-1)(n+1)(n+2)$

57. $6x^3 - 24x^2 + 18x = 6x(x^2 - 4x + 3)$

$= 2 \cdot 3 \cdot x(x-1)(x-3)$

$4x^5 - 24x^4 + 20x^3 = 4x^3(x^2 - 6x + 5)$

$= 2 \cdot 2 \cdot x \cdot x \cdot x(x-1)(x-5)$

$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x(x-1)(x-3)(x-5)$

$= 12x^3(x-1)(x-3)(x-5)$

59. $2x^3 - 2 = 2(x^3 - 1) = 2(x-1)(x^2 + x + 1)$

$x^2 - 1 = (x+1)(x-1)$

$\text{LCM} = 2(x+1)(x-1)(x^2 + x + 1)$

61. $6t^4 = 2 \cdot 3 \cdot t \cdot t \cdot t \cdot t$

$18t^2 = 2 \cdot 3 \cdot 3 \cdot t \cdot t$

The LCD is $2 \cdot 3 \cdot 3 \cdot t \cdot t \cdot t \cdot t,$ or $18t^4$.

$$\frac{5}{6t^4} \cdot \frac{3}{3} = \frac{15}{18t^4} \text{ and}$$

$$\frac{s}{18t^2} \cdot \frac{t^2}{t^2} = \frac{st^2}{18t^4}$$

63. $3x^4y^2 = 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$

$9xy^3 = 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

The LCD is $3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y,$ or $9x^4y^3$.

$$\frac{7}{3x^4y^2} \cdot \frac{3y}{3y} = \frac{21y}{9x^4y^3} \text{ and}$$

$$\frac{4}{9xy^3} \cdot \frac{x^3}{x^3} = \frac{4x^3}{9x^4y^3}$$

65. The LCD is $(x+2)(x-2)(x+3)$. (See Exercise 47.)

$$\frac{2x}{x^2-4} = \frac{2x}{(x+2)(x-2)} \cdot \frac{x+3}{x+3} = \frac{2x(x+3)}{(x+2)(x-2)(x+3)}$$

$$\frac{4x}{x^2+5x+6} = \frac{4x}{(x+3)(x+2)} \cdot \frac{x-2}{x-2} = \frac{4x(x-2)}{(x+3)(x+2)(x-2)}$$

67. *Writing Exercise.* If the numbers have a common factor, their product contains that factor more than the greatest number of times it occurs in any one factorization. In this case, their product is not their least common multiple.

69. $-\frac{5}{8} = \frac{-5}{8} = \frac{5}{-8}$

71. $-(x-y) = -x + y,$ or $y - x$

73. $-1(2x-7) = -2x + 7$ or $7 - 2x$

75. *Writing Exercise.* The polynomials contain no common factors other than constants.

77.
$$\frac{6x-1}{x-1} + \frac{3(2x+5)}{x-1} + \frac{3(2x-3)}{x-1}$$
$$= \frac{6x-1+6x+15+6x-9}{x-1}$$
$$= \frac{18x+5}{x-1}$$

79.
$$\frac{x^2}{3x^2-5x-2} - \frac{2x}{3x+1} \cdot \frac{1}{x-2}$$
$$= \frac{x^2}{(3x+1)(x-2)} - \frac{2x}{(3x+1)(x-2)}$$
$$= \frac{x^2-2x}{(3x+1)(x-2)}$$
$$= \frac{x(x-2)}{(3x+1)(x-2)}$$
$$= \frac{x}{3x+1}$$

81. The smallest number of strands that can be used is the LCM of 10 and 3.

$10 = 2 \cdot 5$

$3 = 3$

$\text{LCM} = 2 \cdot 5 \cdot 3 = 30$

The smallest number of strands that can be used is 30.

83. If the number of strands must also be a multiple of 4, we find the smallest multiple of 30 that is also a multiple of 4.

$1 \cdot 30 = 30$, not a multiple of 4

$2 \cdot 30 = 60 = 15 \cdot 4$, a multiple of 4

The smallest number of strands that can be used is 60.

85. $4x^2 - 25 = (2x+5)(2x-5)$

$6x^2 - 7x - 20 = (3x+4)(2x-5)$

$(9x^2 + 24x + 16)^2 = [(3x+4)(3x+4)]^2$
$= (3x+4)(3x+4)(3x+4)(3x+4)$

$\text{LCM} = (2x+5)(2x-5)(3x+4)^4$

87. The first copier prints 20 pages per minute, which is $\dfrac{20}{60}$ or $\dfrac{1}{3}$ copy per second. The second copier prints 18 pages per minutes, which is $\dfrac{18}{60}$ or $\dfrac{3}{10}$ copy per second. The time it takes until the machines begin copying a page at exactly the same time again is the LCM of their copying rates.

$3 = 3$

$10 = 2 \cdot 5$

$\text{LCM} = 2 \cdot 3 \cdot 5 = 30$

It takes 30 seconds

89. The number of minutes after 5:00 A.M. when the shuttles will first leave at the same time again is the LCM of their departure intervals, 25 minutes and 35 minutes.

$25 = 5 \cdot 5$

$35 = 5 \cdot 7$

$\text{LCM} = 5 \cdot 5 \cdot 7$, or 175

Thus, the shuttles will leave at the same time 175 minutes after 5:00 A.M., or at 7:55 A.M.

91. *Writing Exercise.* Evaluate both expressions for some value of the variable for which both are defined. If the results are the same, we can conclude that the answer is probably correct.

Exercise Set 6.4

1. To add or subtract when denominators are different, first find the LCD.

3. Add or subtract the numerators, as indicated. Write the sum or difference over the LCD.

5. $\dfrac{3}{x^2} + \dfrac{5}{x} = \dfrac{3}{x \cdot x} + \dfrac{5}{x}$ $\quad$ $\text{LCD} = x \cdot x$, or x^2

$= \dfrac{3}{x \cdot x} + \dfrac{5}{x} \cdot \dfrac{x}{x}$

$= \dfrac{3+5x}{x^2}$

7. $\left.\begin{array}{l} 6r = 2 \cdot 3 \cdot r \\ 8r = 2 \cdot 2 \cdot 2 \cdot r \end{array}\right\}$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot r$, or $24r$

$\dfrac{1}{6r} - \dfrac{3}{8r} = \dfrac{1}{6r} \cdot \dfrac{4}{4} - \dfrac{3}{8r} \cdot \dfrac{3}{3}$

$= \dfrac{4-9}{24r}$

$= \dfrac{-5}{24r}$, or $-\dfrac{5}{24r}$

9. $\left.\begin{array}{l} uv^2 = u \cdot v \cdot v \\ u^3v = u \cdot u \cdot u \cdot v \end{array}\right\}$ $\text{LCD} = u \cdot u \cdot u \cdot v \cdot v$, or u^3v^2

$\dfrac{3}{uv^2} + \dfrac{4}{u^3v} = \dfrac{3}{uv^2} \cdot \dfrac{u^2}{u^2} + \dfrac{4}{u^3v} \cdot \dfrac{v}{v} = \dfrac{3u^2+4v}{u^3v^2}$

11. $\left.\begin{array}{l} 3xy^2 = 3 \cdot x \cdot y \cdot y \\ x^2y^3 = x \cdot x \cdot y \cdot y \cdot y \end{array}\right\}$ $\text{LCD} = 3 \cdot x \cdot x \cdot y \cdot y \cdot y$, or $3x^2y^3$

$\dfrac{-2}{3xy^2} - \dfrac{6}{x^2y^3} = \dfrac{-2}{3xy^2} \cdot \dfrac{xy}{xy} - \dfrac{6}{x^2y^3} \cdot \dfrac{3}{3} = \dfrac{-2xy-18}{3x^2y^3}$

13. $\left.\begin{array}{l} 8 = 2 \cdot 2 \cdot 2 \\ 6 = 2 \cdot 3 \end{array}\right\}$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3$, or 24

$\dfrac{x+3}{8} + \dfrac{x-2}{6} = \dfrac{x+3}{8} \cdot \dfrac{3}{3} + \dfrac{x-2}{6} \cdot \dfrac{4}{4}$

$= \dfrac{3(x+3)+4(x-2)}{24}$

$= \dfrac{3x+9+4x-8}{24}$

$= \dfrac{7x+1}{24}$

15. $\left.\begin{array}{l} 6 = 2 \cdot 3 \\ 3 = 3 \end{array}\right\}$ $\text{LCD} = 2 \cdot 3$, or 6

$\dfrac{x-2}{6} - \dfrac{x+1}{3} = \dfrac{x-2}{6} - \dfrac{x+1}{3} \cdot \dfrac{2}{2}$

$= \dfrac{x-2}{6} - \dfrac{2x+2}{6}$

$= \dfrac{x-2-(2x+2)}{6}$

$= \dfrac{x-2-2x-2}{6}$

$= \dfrac{-x-4}{6}$, or $\dfrac{-(x+4)}{6}$

17. $\left.\begin{array}{l} 15a = 3 \cdot 5 \cdot a \\ 3a^2 = 3 \cdot a \cdot a \end{array}\right\}$ $\text{LCD} = 5 \cdot 3a \cdot a$, or $15a^2$

$\dfrac{a+3}{15a} + \dfrac{2a-1}{3a^2} = \dfrac{a+3}{15a} \cdot \dfrac{a}{a} + \dfrac{2a-1}{3a^2} \cdot \dfrac{5}{5}$

$= \dfrac{a^2+3a+10a-5}{15a^2}$

$= \dfrac{a^2+13a-5}{15a^2}$

19. $\left.\begin{array}{l}3z = 3\cdot z\\ 4z = 2\cdot 2\cdot z\end{array}\right\}$LCD $= 2\cdot 2\cdot 3\cdot z$, or $12z$

$$\begin{aligned}\frac{4z-9}{3z}-\frac{3z-8}{4z} &= \frac{4z-9}{3z}\cdot\frac{4}{4}-\frac{3z-8}{4z}\cdot\frac{3}{3}\\ &= \frac{16z-36}{12z}-\frac{9z-24}{12z}\\ &= \frac{16z-36-(9z-24)}{12z}\\ &= \frac{16z-36-9z+24}{12z}\\ &= \frac{7z-12}{12z}\end{aligned}$$

21. $\left.\begin{array}{l}cd^2 = c\cdot d\cdot d\\ c^2d = c\cdot c\cdot d\end{array}\right\}$LCD $= c\cdot c\cdot d\cdot d$, or c^2d^2

$$\begin{aligned}\frac{3c+d}{cd^2}+\frac{c-d}{c^2d} &= \frac{3c+d}{cd^2}\cdot\frac{c}{c}+\frac{c-d}{c^2d}\cdot\frac{d}{d}\\ &= \frac{c(3c+d)+d(c-d)}{c^2d^2}\\ &= \frac{3c^2+cd+cd-d^2}{c^2d^2}\\ &= \frac{3c^2+2cd-d^2}{c^2d^2}\end{aligned}$$

23. $\left.\begin{array}{l}3xt^2 = 3\cdot x\cdot t\cdot t\\ x^2t = x\cdot x\cdot t\end{array}\right\}$LCD $= 3\cdot x\cdot x\cdot t\cdot t$, or $3x^2t^2$

$$\begin{aligned}&\frac{4x+2t}{3xt^2}-\frac{5x-3t}{x^2t}\\ &= \frac{4x+2t}{3xt^2}\cdot\frac{x}{x}-\frac{5x-3t}{x^2t}\cdot\frac{3t}{3t}\\ &= \frac{4x^2+2tx}{3x^2t^2}-\frac{15xt-9t^2}{3x^2t^2}\\ &= \frac{4x^2+2tx-(15xt-9t^2)}{3x^2t^2}\\ &= \frac{4x^2+2tx-15xt+9t^2}{3x^2t^2}\\ &= \frac{4x^2-13xt+9t^2}{3x^2t^2}\end{aligned}$$

(Although $4x^2-13xt+9t^2$ can be factored, doing so will not enable us to simplify the result further.)

25. The denominators cannot be factored, so the LCD is their product, $(x-2)(x+2)$.

$$\begin{aligned}\frac{3}{x-2}+\frac{3}{x+2} &= \frac{3}{x-2}\cdot\frac{x+2}{x+2}+\frac{3}{x+2}\cdot\frac{x-2}{x-2}\\ &= \frac{3(x+2)+3(x-2)}{(x-2)(x+2)}\\ &= \frac{3x+6+3x-6}{(x-2)(x+2)}\\ &= \frac{6x}{(x-2)(x+2)}\end{aligned}$$

27. $\dfrac{t}{t+3}-\dfrac{1}{t-1}$ LCD $= (t+3)(t-1)$

$$\begin{aligned}&= \frac{t}{t+3}\cdot\frac{t-1}{t-1}-\frac{1}{t-1}\cdot\frac{t+3}{t+3}\\ &= \frac{t^2-t}{(t+3)(t-1)}-\frac{t+3}{(t+3)(t-1)}\\ &= \frac{t^2-t-(t+3)}{(t+3)(t-1)}\\ &= \frac{t^2-t-t-3}{(t+3)(t-1)}\\ &= \frac{t^2-2t-3}{(t+3)(t-1)}\end{aligned}$$

(Although t^2-2t-3 can be factored, doing so will not enable us to simplify the result further.)

29. $\left.\begin{array}{l}3x = 3\cdot x\\ x+1 = x+1\end{array}\right\}$LCD $= 3x(x+1)$

$$\begin{aligned}\frac{3}{x+1}+\frac{2}{3x} &= \frac{3}{x+1}\cdot\frac{3x}{3x}+\frac{2}{3x}\cdot\frac{x+1}{x+1}\\ &= \frac{9x+2(x+1)}{3x(x+1)}\\ &= \frac{9x+2x+2}{3x(x+1)}\\ &= \frac{11x+2}{3x(x+1)}\end{aligned}$$

31. $\dfrac{3}{2t^2-2t}-\dfrac{5}{2t-2}$

$$\begin{aligned}&= \frac{3}{2t(t-1)}-\frac{5}{2(t-1)}\\ &= \frac{3}{2t(t-1)}-\frac{5}{2(t-1)}\cdot\frac{t}{t}\\ &= \frac{3-5t}{2t(t-1)}\end{aligned}$$

33. $\dfrac{3a}{a^2-9}+\dfrac{a}{a+3}$

$$\begin{aligned}&= \frac{3a}{(a+3)(a-3)}+\frac{a}{a+3}\\ &\qquad\text{LCD} = (a+3)(a-3)\\ &= \frac{3a}{(a+3)(a-3)}+\frac{a}{a+3}\cdot\frac{a-3}{a-3}\\ &= \frac{3a+a(a-3)}{(a+3)(a-3)}\\ &= \frac{3a+a^2-3a}{(a+3)(a-3)}\\ &= \frac{a^2}{(a+3)(a-3)}\end{aligned}$$

35. $\dfrac{6}{z+4} - \dfrac{2}{3z+12} = \dfrac{6}{z+4} - \dfrac{2}{3(z+4)}$

LCD $= 3(z+4)$

$$= \frac{6}{z+4}\cdot\frac{3}{3} - \frac{2}{3(z+4)}$$
$$= \frac{18}{3(z+4)} - \frac{2}{3(z+4)}$$
$$= \frac{16}{3(z+4)}$$

37. $\dfrac{5}{q-1} + \dfrac{2}{(q-1)^2}$

$$= \frac{5}{q-1}\cdot\frac{q-1}{q-1} + \frac{2}{(q-1)^2}$$
$$= \frac{5\,(q-1)+2}{(q-1)^2}$$
$$= \frac{5q-5+2}{(q-1)^2}$$
$$= \frac{5q-3}{(q-1)^2}$$

39. $\dfrac{t-3}{t^3-1} - \dfrac{2}{1-t^3} = \dfrac{t-3}{t^3-1} + \dfrac{2}{t^3-1}$

$$= \frac{t-3+2}{(t-1)(t^2+t+1)} = \frac{t-1}{(t-1)(t^2+t+1)}$$
$$= \frac{1}{t^2+t+1}$$

41. $\dfrac{3a}{4a-20} + \dfrac{9a}{6a-30}$

$$= \frac{3a}{2\cdot 2(a-5)} + \frac{9a}{2\cdot 3(a-5)}$$

LCD $= 2\cdot 2\cdot 3(a-5)$

$$= \frac{3a}{2\cdot 2(a-5)}\cdot\frac{3}{3} + \frac{9a}{2\cdot 3(a-5)}\cdot\frac{2}{2}$$
$$= \frac{9a+18a}{2\cdot 2\cdot 3(a-5)}$$
$$= \frac{27a}{2\cdot 2\cdot 3(a-5)}$$
$$= \frac{\cancel{3}\cdot 9\cdot a}{2\cdot 2\cdot \cancel{3}(a-5)}$$
$$= \frac{9a}{4(a-5)}$$

43. $\dfrac{x}{x-5} + \dfrac{x}{5-x} = \dfrac{x}{x-5} + \dfrac{x}{5-x}\cdot\dfrac{-1}{-1}$

$$= \frac{x}{x-5} + \frac{-x}{x-5}$$
$$= 0$$

45. $\dfrac{6}{a^2+a-2} + \dfrac{4}{a^2-4a+3}$

$$= \frac{6}{(a+2)(a-1)} + \frac{4}{(a-3)(a-1)}$$

LCD $= (a+2)(a-1)(a-3)$

$$= \frac{6}{(a+2)(a-1)}\cdot\frac{a-3}{a-3} + \frac{4}{(a-3)(a-1)}\cdot\frac{a+2}{a+2}$$
$$= \frac{6(a-3)+4(a+2)}{(a+2)(a-1)(a-3)}$$
$$= \frac{6a-18+4a+8}{(a+2)(a-1)(a-3)}$$
$$= \frac{10a-10}{(a+2)(a-1)(a-3)}$$
$$= \frac{10\cancel{(a-1)}}{(a+2)\cancel{(a-1)}(a-3)}$$
$$= \frac{10}{(a+2)(a-3)}$$

47. $\dfrac{x}{x^2+9x+20} - \dfrac{4}{x^2+7x+12}$

$$= \frac{x}{(x+4)(x+5)} - \frac{4}{(x+3)(x+4)}$$

LCD $= (x+3)(x+4)(x+5)$

$$= \frac{x}{(x+4)(x+5)}\cdot\frac{x+3}{x+3} - \frac{4}{(x+3)(x+4)}\cdot\frac{x+5}{x+5}$$
$$= \frac{x(x+3)-4(x+5)}{(x+3)(x+4)(x+5)}$$
$$= \frac{x^2+3x-4x-20}{(x+3)(x+4)(x+5)}$$
$$= \frac{x^2-x-20}{(x+3)(x+4)(x+5)}$$
$$= \frac{\cancel{(x+4)}(x-5)}{(x+3)\cancel{(x+4)}(x+5)}$$
$$= \frac{x-5}{(x+3)(x+5)}$$

49. $\dfrac{3z}{z^2-4z+4} + \dfrac{10}{z^2+z-6}$

$$= \frac{3z}{(z-2)^2} + \frac{10}{(z-2)(z+3)},$$

LCD $= (z-2)^2(z+3)$

$$= \frac{3z}{(z-2)^2}\cdot\frac{z+3}{z+3} + \frac{10}{(z-2)(z+3)}\cdot\frac{z-2}{z-2}$$
$$= \frac{3z(z+3)+10(z-2)}{(z-2)^2(z+3)}$$
$$= \frac{3z^2+9z+10z-20}{(z-2)^2(z+3)}$$
$$= \frac{3z^2+19z-20}{(z-2)^2(z+3)}$$

51. $\dfrac{-7}{x^2+25x+24} - \dfrac{0}{x^2+11x+10}$

Note that $\dfrac{0}{x^2+11x+10}=0$, so the difference is $\dfrac{-7}{x^2+25x+24}$.

53. $$\frac{5x}{4}-\frac{x-2}{-4}=\frac{5x}{4}-\frac{x-2}{-4}\cdot\frac{-1}{-1}$$
$$=\frac{5x}{4}-\frac{2-x}{4}$$
$$=\frac{5x-(2-x)}{4}$$
$$=\frac{5x-2+x}{4}$$
$$=\frac{6x-2}{4}$$
$$=\frac{2\,(3x-1)}{2\cdot 2}$$
$$=\frac{\not{2}\,(3x-1)}{\not{2}\cdot 2}$$
$$=\frac{3x-1}{2}$$

55. $$\frac{y^2}{y-3}+\frac{9}{3-y}=\frac{y^2}{y-3}+\frac{9}{3-y}\cdot\frac{-1}{-1}$$
$$=\frac{y^2}{y-3}+\frac{-9}{-3+y}$$
$$=\frac{y^2-9}{y-3}$$
$$=\frac{(y+3)\cancel{(y-3)}}{\cancel{y-3}}$$
$$=y+3$$

57. $$\frac{c-5}{c^2-64}+\frac{c-5}{64-c^2}=\frac{c-5}{c^2-64}+\frac{c-5}{64-c^2}\cdot\frac{-1}{-1}$$
$$=\frac{c-5}{c^2-64}+\frac{5-c}{c^2-64}$$
$$=\frac{c-5+5-c}{c^2-64}$$
$$=\frac{0}{c^2-64}$$
$$=0$$

59. $$\frac{4-p}{25-p^2}+\frac{p+1}{p-5}$$
$$=\frac{4-p}{(5+p)(5-p)}+\frac{p+1}{p-5}$$
$$=\frac{4-p}{(5+p)(5-p)}\cdot\frac{-1}{-1}+\frac{p+1}{p-5}$$
$$=\frac{p-4}{(p+5)(p-5)}+\frac{p+1}{p-5}\quad \text{LCD}=(p+5)(p-5)$$
$$=\frac{p-4}{(p+5)(p-5)}+\frac{p+1}{p-5}\cdot\frac{p+5}{p+5}$$
$$=\frac{p-4+p^2+6p+5}{(p+5)(p-5)}$$
$$=\frac{p^2+7p+1}{(p+5)(p-5)}$$

61. $$\frac{x}{x-4}-\frac{3}{16-x^2}$$
$$=\frac{x}{x-4}-\frac{3}{(4+x)\,(4-x)}$$
$$=\frac{x}{x-4}\cdot\frac{-1}{-1}-\frac{3}{(4+x)\,(4-x)}$$
$$=\frac{-x}{4-x}-\frac{3}{(4+x)\,(4-x)}\quad \text{LCD}=(4-x)(4+x)$$
$$=\frac{-x}{4-x}\cdot\frac{4+x}{4+x}-\frac{3}{(4+x)\,(4-x)}$$
$$=\frac{-x\,(4+x)-3}{(4-x)\,(4+x)}$$
$$=\frac{-4x-x^2-3}{(4-x)\,(4+x)}$$
$$=\frac{-x^2-4x-3}{(4-x)\,(4+x)},\text{ or }\frac{x^2+4x+3}{(x+4)\,(x-4)}$$

(Although x^2+4x+3 can be factored, doing so will not enable us to simplify the result further.)

63. $$\frac{a}{a^2-1}+\frac{2a}{a-a^2}=\frac{a}{a^2-1}+\frac{2\cdot\not{a}}{\not{a}(1-a)}$$
$$=\frac{a}{(a+1)(a-1)}+\frac{2}{1-a}$$
$$=\frac{a}{(a+1)(a-1)}+\frac{2}{1-a}\cdot\frac{-1}{-1}$$
$$=\frac{a}{(a+1)(a-1)}+\frac{-2}{a-1}$$
$$\text{LCD}=(a+1)(a-1)$$
$$=\frac{a}{(a+1)(a-1)}+\frac{-2}{a-1}\cdot\frac{a+1}{a+1}$$
$$=\frac{a-2a-2}{(a+1)(a-1)}$$
$$=\frac{-a-2}{(a+1)(a-1)},\text{ or}$$
$$=\frac{a+2}{(1+a)(1-a)}$$

65. $$\frac{4x}{x^2-y^2}-\frac{6}{y-x}$$
$$=\frac{4x}{(x+y)(x-y)}-\frac{6}{y-x}$$
$$=\frac{4x}{(x+y)(x-y)}-\frac{6}{y-x}\cdot\frac{-1}{-1}$$
$$=\frac{4x}{(x+y)(x-y)}-\frac{-6}{x-y}\quad \text{LCD}=(x+y)(x-y)$$
$$=\frac{4x}{(x+y)(x-y)}-\frac{-6}{x-y}\cdot\frac{x+y}{x+y}$$
$$=\frac{4x-(-6)(x+y)}{(x+y)(x-y)}$$
$$=\frac{4x+6x+6y}{(x+y)(x-y)}$$
$$=\frac{10x+6y}{(x+y)(x-y)}$$

(Although $10x+6y$ can be factored, doing so will not enable us to simplify the result further.)

67. $\dfrac{x-3}{2-x}-\dfrac{x+3}{x+2}+\dfrac{x+6}{4-x^2}$

$$=\frac{x-3}{2-x}-\frac{x+3}{x+2}+\frac{x+6}{(2+x)(2-x)}$$

$$\text{LCD}=(2+x)(2-x)$$

$$=\frac{x-3}{2-x}\cdot\frac{2+x}{2+x}-\frac{x+3}{x+2}\cdot\frac{2-x}{2-x}+\frac{x+6}{(2+x)(2-x)}$$

$$=\frac{(x-3)(2+x)-(x+3)(2-x)+(x+6)}{(2+x)(2-x)}$$

$$=\frac{x^2-x-6-(-x^2-x+6)+x+6}{(2+x)(2-x)}$$

$$=\frac{x^2-x-6+x^2+x-6+x+6}{(2+x)(2-x)}$$

$$=\frac{2x^2+x-6}{(2+x)(2-x)}$$

$$=\frac{(2x-3)\cancel{(x+2)}}{\cancel{(2+x)}(2-x)}$$

$$=\frac{2x-3}{2-x}$$

69. $\dfrac{2x+5}{x+1}+\dfrac{x+7}{x+5}-\dfrac{5x+17}{(x+1)(x+5)}$

$$\text{LCD}=(x+1)(x+5)$$

$$=\frac{(2x+5)(x+5)+(x+7)(x+1)-(5x+17)}{(x+1)(x+5)}$$

$$=\frac{2x^2+15x+25+x^2+8x+7-5x-17}{(x+1)(x+5)}$$

$$=\frac{3x^2+18x+15}{(x+1)(x+5)}$$

$$=\frac{3\cancel{(x+1)}\cancel{(x+5)}}{\cancel{(x+1)}\cancel{(x+5)}}$$

$$=3$$

71. $\dfrac{1}{x+y}+\dfrac{1}{x-y}-\dfrac{2x}{x^2-y^2}$

$$\text{LCD}=(x+y)(x-y)$$

$$=\frac{1}{x+y}\cdot\frac{x-y}{x-y}+\frac{1}{x-y}\cdot\frac{x+y}{x+y}-\frac{2x}{(x+y)(x-y)}$$

$$=\frac{(x-y)+(x+y)-2x}{(x+y)(x-y)}$$

$$=0$$

73. *Writing Exercise.* Using the least common denominator usually reduces the complexity of computations and requires less simplification of the sum or difference.

75. $\dfrac{-3}{8}\div\dfrac{11}{4}=\dfrac{-3}{8}\cdot\dfrac{4}{11}=-\dfrac{3\cdot\cancel{4}}{2\cdot\cancel{4}\cdot 11}=\dfrac{-3}{22}$

77. $\dfrac{\frac{3}{4}}{\frac{5}{6}}=\dfrac{3}{4}\cdot\dfrac{6}{5}=\dfrac{3\cdot 3\cdot\cancel{2}}{\cancel{2}\cdot 2\cdot 5}=\dfrac{9}{10}$

79. $\dfrac{2x+6}{x-1}\div\dfrac{3x+9}{x-1}=\dfrac{2x+6}{x-1}\cdot\dfrac{x-1}{3x+9}=\dfrac{2\cancel{(x+3)}\cancel{(x-1)}}{\cancel{(x-1)}3\cancel{(x+3)}}=\dfrac{2}{3}$

81. *Writing Exercise.* Their sum is zero. Another explanation is that $-\left(\dfrac{1}{3-x}\right)=\dfrac{1}{-(3-x)}=\dfrac{1}{x-3}$.

83. $P=2\left(\dfrac{3}{x+4}\right)+2\left(\dfrac{2}{x-5}\right)$

$$=\frac{6}{x+4}+\frac{4}{x-5}\qquad \text{LCD}=(x+4)(x-5)$$

$$=\frac{6}{x+4}\cdot\frac{x-5}{x-5}+\frac{4}{x-5}\cdot\frac{x+4}{x+4}$$

$$=\frac{6x-30+4x+16}{(x+4)(x-5)}$$

$$=\frac{10x-14}{(x+4)(x-5)},\ \text{or}\ \frac{10x-14}{x^2-x-20}$$

$$A=\left(\frac{3}{x+4}\right)\left(\frac{2}{x-5}\right)=\frac{6}{(x+4)(x-5)}$$

85. $\dfrac{x^2}{3x^2-5x-2}-\dfrac{2x}{3x+1}\cdot\dfrac{1}{x-2}$

$$=\frac{x^2}{(3x+1)(x-2)}-\frac{2x}{(3x+1)(x-2)}$$

$$=\frac{x^2-2x}{(3x+1)(x-2)}$$

$$=\frac{x(x-2)}{(3x+1)(x-2)}$$

$$=\frac{x}{3x+1}\cdot\frac{x-2}{x-2}$$

$$=\frac{x}{3x+1}$$

87. We recognize that this is the product of the sum and difference of two terms: $(A+B)(A-B)=A^2-B^2$.

$$\left(\frac{x}{x+7}-\frac{3}{x+2}\right)\left(\frac{x}{x+7}+\frac{3}{x+2}\right)$$

$$=\frac{x^2}{(x+7)^2}-\frac{9}{(x+2)^2}\quad \text{LCD}=(x+7)^2(x+2)^2$$

$$=\frac{x^2}{(x+7)^2}\cdot\frac{(x+2)^2}{(x+2)^2}-\frac{9}{(x+2)^2}\cdot\frac{(x+7)^2}{(x+7)^2}$$

$$=\frac{x^2(x+2)^2-9(x+7)^2}{(x+7)^2(x+2)^2}$$

$$=\frac{x^2(x^2+4x+4)-9(x^2+14x+49)}{(x+7)^2(x+2)^2}$$

$$=\frac{x^4+4x^3+4x^2-9x^2-126x-441}{(x+7)^2(x+2)^2}$$

$$=\frac{x^4+4x^3-5x^2-126x-441}{(x+7)^2(x+2)^2}$$

89. $\left(\frac{a}{a-b}+\frac{b}{a+b}\right)\left(\frac{1}{3a+b}+\frac{2a+6b}{9a^2-b^2}\right)$

$= \frac{a}{(a-b)(3a+b)}+\frac{a(2a+6b)}{(a-b)(9a^2-b^2)}+\frac{b}{(a+b)(3a+b)}+\frac{b(2a+6b)}{(a+b)(9a^2-b^2)}$

$= \frac{a}{(a-b)(3a+b)}+\frac{2a^2+6ab}{(a-b)(3a+b)(3a-b)}+\frac{b}{(a+b)(3a+b)}+\frac{2ab+6b^2}{(a+b)(3a+b)(3a-b)}$

LCD $=(a-b)(a+b)(3a+b)(3a-b)$

$= [a(a+b)(3a-b)+(2a^2+6ab)(a+b)+b(a-b)(3a-b)+(2ab+6b^2)(a-b)]/[(a-b)(a+b)(3a+b)(3a-b)]$

$= (3a^3+2a^2b-ab^2+2a^3+8a^2b+6ab^2+b^3-4ab^2+3a^2b+4ab^2-6b^3+2a^2b)/[(a-b)(a+b)(3a+b)(3a-b)]$

$= \frac{5a^3+15a^2b+5ab^2-5b^3}{(a-b)(a+b)(3a+b)(3a-b)}$

$= \frac{5\cancel{(a+b)}(a^2+2ab-b^2)}{(a-b)\cancel{(a+b)}(3a+b)(3a-b)}$

$= \frac{5(a^2+2ab-b^2)}{(a-b)(3a+b)(3a-b)}$

91. Answers may vary. $\frac{a}{a-b}+\frac{3b}{b-a}$

93. *Writing Exercise.* Both y_1 and y_2 are undefined when $x = 5$.

6.4 Connecting the Concepts

1. $= \frac{3}{5x}+\frac{2}{x^2}$ Addition

$= \frac{3}{5x}\cdot\frac{x}{x}+\frac{2}{x^2}\cdot\frac{5}{5}$ LCD $=5x^2$

$= \frac{3x+10}{5x^2}$

3. $= \frac{3}{5x}\div\frac{2}{x^2}$ Division

$= \frac{3}{5x}\cdot\frac{x^2}{2}$

$= \frac{3x}{10}\cdot\frac{x}{x}$

$= \frac{3x}{10}$

5. $= \frac{2x-6}{5x+10}\cdot\frac{x+2}{6x-12}$ Multiplication

$= \frac{\cancel{2}(x-3)\cancel{(x+2)}}{5\cancel{(x+2)}\cancel{2}\cdot 3(x-2)}$

$= \frac{(x-3)}{15(x-2)}$

7. $= \frac{2}{x-5}\div\frac{6}{x-5}$ Division

$= \frac{2}{x-5}\cdot\frac{x-5}{6}$

$= \frac{1}{3}\cdot\frac{2(x-5)}{2(x-5)}$

$= \frac{1}{3}$

9. $= \frac{2}{x+3}+\frac{3}{x+4}$ Addition

$= \frac{2}{x+3}\cdot\frac{x+4}{x+4}+\frac{3}{x+4}\cdot\frac{x+3}{x+3}$ LCD $=(x+3)(x+4)$

$= \frac{2(x+4)+3(x+3)}{(x+3)(x+4)}$

$= \frac{2x+8+3x+9}{(x+3)(x+4)}$

$= \frac{5x+17}{(x+3)(x+4)}$

11. $= \frac{3}{x-4}-\frac{2}{4-x}$ Subtraction

$= \frac{3}{x-4}+\frac{2}{x-4}$

$= \frac{5}{x-4}$

13. $= \frac{a}{6a-9b}-\frac{b}{4a-6b}$ Subtraction

$= \frac{a}{3(2a-3b)}\cdot\frac{2}{2}-\frac{b}{2(2a-3b)}\cdot\frac{3}{3}$ LCD $=6(2a-3b)$

$= \frac{2a-3b}{6(2a-3b)}$

$= \frac{1}{6}$

15. $= \frac{x+1}{x^2-7x+10}+\frac{3}{x^2-x-2}$ Addition

$= \frac{x+1}{(x-5)(x-2)}\cdot\frac{x+1}{x+1}+\frac{3}{(x-2)(x+1)}\cdot\frac{x-5}{x-5}$ LCD$=(x+1)(x-2)(x-5)$

$= \frac{x^2+2x+1+3x-15}{(x+1)(x-2)(x-5)}$

$= \frac{x^2+5x-14}{(x+1)(x-2)(x-5)}$

$= \frac{(x+7)\cancel{(x-2)}}{(x+1)\cancel{(x-2)}(x-5)}$

$= \frac{x+7}{(x+1)(x-5)}$

17. $= \frac{t+2}{10}+\frac{2t+1}{15}$ Addition

$= \frac{3(t+2)}{3(10)}+\frac{2(2t+1)}{2(15)}$ LCD $=30$

$= \frac{3t+6+4t+2}{30}=\frac{7t+8}{30}$

19. $= \frac{a^2-2a+1}{a^2-4}\div\left(a^2-3a+2\right)$ Division

$= \frac{(a-1)\cancel{(a-1)}}{(a+2)(a-2)}\cdot\frac{1}{(a-2)\cancel{(a-1)}}$

$= \frac{a-1}{(a+2)(a-2)^2}$

Exercise Set 6.5

1. The LCD is the LCM of x^2, x, 2, and $4x$. It is $4x^2$.

$$\frac{\frac{5}{x^2}+\frac{1}{x}}{\frac{7}{2}-\frac{3}{4x}}\cdot\frac{4x^2}{4x^2}=\frac{\frac{5}{x^2}\cdot 4x^2+\frac{1}{x}\cdot 4x^2}{\frac{7}{2}\cdot 4x^2-\frac{3}{4x}\cdot 4x^2}$$

Choice (d) is correct.

3. We subtract to get a single rational expression in the numerator and add to get a single rational expression in the denominator.

$$\frac{\frac{4}{5x}-\frac{1}{10}}{\frac{8}{x^2}+\frac{7}{2}}=\frac{\frac{4}{5x}\cdot\frac{2}{2}-\frac{1}{10}\cdot\frac{x}{x}}{\frac{8}{x^2}\cdot\frac{2}{2}+\frac{7}{2}\cdot\frac{x^2}{x^2}}$$

$$=\frac{\frac{8}{10x}-\frac{x}{10x}}{\frac{16}{2x^2}+\frac{7x^2}{2x^2}}$$

$$=\frac{\frac{8-x}{10x}}{\frac{16+7x^2}{2x^2}}$$

Choice (b) is correct.

5. $\dfrac{1+\frac{1}{4}}{2+\frac{3}{4}}$ LCD is 4

$=\dfrac{1+\frac{1}{4}}{2+\frac{3}{4}}\cdot\dfrac{4}{4}$ Multiplying by $\frac{4}{4}$

$=\dfrac{\left(1+\frac{1}{4}\right)4}{\left(2+\frac{3}{4}\right)4}$ Multiplying numerator and denominator by 4

$=\dfrac{1\cdot 4+\frac{1}{4}\cdot 4}{2\cdot 4+\frac{3}{4}\cdot 4}$

$=\dfrac{4+1}{8+3}$

$=\dfrac{5}{11}$

7. $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}-\frac{1}{6}}$

$=\dfrac{\frac{1}{2}\cdot\frac{3}{3}+\frac{1}{3}\cdot\frac{2}{2}}{\frac{1}{4}\cdot\frac{3}{3}-\frac{1}{6}\cdot\frac{2}{2}}$ Getting a common denominator in numerator and in denominator

$=\dfrac{\frac{3}{6}+\frac{2}{6}}{\frac{3}{12}-\frac{2}{12}}$

$=\dfrac{\frac{5}{6}}{\frac{1}{12}}$ Adding in the numerator; subtracting in the denominator

$=\dfrac{5}{6}\cdot\dfrac{12}{1}$ Multiplying by the reciprocal of the divisor

$=\dfrac{5\cdot 6\cdot 2}{6}$

$=\dfrac{5\cdot \not{6}\cdot 2}{\not{6}}$

$=10$

9. $\dfrac{\frac{x}{4}+x}{\frac{4}{x}+x}$ LCD is $4x$

$=\dfrac{\frac{x}{4}+x}{\frac{4}{x}+x}\cdot\dfrac{4x}{4x}$

$=\dfrac{\left(\frac{x}{4}+x\right)(4x)}{\left(\frac{4}{x}+x\right)(4x)}$

$=\dfrac{x^2+4x^2}{16+4x^2}$

$\dfrac{5x^2}{16+4x^2}$

11. $\dfrac{\frac{10}{t}}{\frac{2}{t^2}-\frac{5}{t}}$

$=\dfrac{\frac{10}{t}}{\frac{2}{t^2}-\frac{5}{t}}\cdot\dfrac{t^2}{t^2}$

$=\dfrac{\frac{10}{t}\cdot t^2}{\left(\frac{2}{t^2}-\frac{5}{t}\right)t^2}$

$=\dfrac{10t}{\frac{2}{t^2}\cdot t^2-\frac{5}{t}\cdot t^2}$

$=\dfrac{10t}{2-5t}$, or $\dfrac{-10t}{5t-2}$

13. $$\frac{\dfrac{2a-5}{3a}}{\dfrac{a-7}{6a}}$$

$$= \frac{2a-5}{3a}\cdot\frac{6a}{a-7}$$ Multiplying by the reciprocal of the divisor

$$= \frac{(2a-5)\cdot 2\cdot 3a}{3a\cdot(a-7)}$$

$$= \frac{(2a-5)\cdot 2\cdot \cancel{3a}}{\cancel{3a}\cdot(a-7)}$$

$$= \frac{2(2a-5)}{a-7}$$

$$= \frac{4a-10}{a-7}$$

15. $$\frac{\dfrac{x}{6}-\dfrac{3}{x}}{\dfrac{1}{3}+\dfrac{1}{x}}$$ LCD is $6x$

$$= \frac{\dfrac{x}{6}-\dfrac{3}{x}}{\dfrac{1}{3}+\dfrac{1}{x}}\cdot\frac{6x}{6x}$$

$$= \frac{\dfrac{x}{6}\cdot 6x-\dfrac{3}{x}\cdot 6x}{\dfrac{1}{3}\cdot 6x+\dfrac{1}{x}\cdot 6x}$$

$$= \frac{x^2-18}{2x+6}$$

17. $$\frac{\dfrac{1}{s}-\dfrac{1}{5}}{\dfrac{s-5}{s}}$$ LCD is $5s$

$$= \frac{\dfrac{1}{s}-\dfrac{1}{5}}{\dfrac{s-5}{s}}\cdot\frac{5s}{5s}$$

$$= \frac{\dfrac{1}{s}\cdot 5s-\dfrac{1}{5}\cdot 5s}{\left(\dfrac{s-5}{s}\right)(5s)}$$

$$= \frac{5-s}{(s-5)(5)}$$

$$= \frac{-\cancel{(s-5)}}{\cancel{(s-5)}(5)}$$

$$= -\frac{1}{5}$$

19. $$\frac{\dfrac{1}{t^2}+1}{\dfrac{1}{t}-1}$$ LCD is t^2

$$= \frac{\dfrac{1}{t^2}+1}{\dfrac{1}{t}-1}\cdot\frac{t^2}{t^2}$$

$$= \frac{\dfrac{1}{t^2}\cdot t^2+1\cdot t^2}{\dfrac{1}{t}\cdot t^2-1\cdot t^2}$$

$$= \frac{1+t^2}{t-t^2}$$

(Although the denominator can be factored, doing so will not enable us to simplify further.)

21. $$\frac{\dfrac{x^2}{x^2-y^2}}{\dfrac{x}{x+y}}$$

$$= \frac{x^2}{x^2-y^2}\cdot\frac{x+y}{x}$$ Multiplying by the reciprocal of the divisor

$$= \frac{x^2(x+y)}{(x^2-y^2)(x)}$$

$$= \frac{x\cdot x\cdot(x+y)}{(x+y)(x-y)(x)}$$

$$= \frac{\cancel{x}\cdot x\cdot\cancel{(x+y)}}{\cancel{(x+y)}(x-y)(\cancel{x})}$$

$$= \frac{x}{x-y}$$

23. $$\frac{\dfrac{7}{c^2}+\dfrac{4}{c}}{\dfrac{6}{c}-\dfrac{3}{c^3}}$$ LCD is c^3

$$= \frac{\dfrac{7}{c^2}+\dfrac{4}{c}}{\dfrac{6}{c}-\dfrac{3}{c^3}}\cdot\frac{c^3}{c^3}$$

$$= \frac{\dfrac{7}{c^2}\cdot c^3+\dfrac{4}{c}\cdot c^3}{\dfrac{6}{c}\cdot c^3-\dfrac{3}{c^3}\cdot c^3}$$

$$= \frac{7c+4c^2}{6c^2-3}$$

(Although the numerator and the denominator can be factored, doing so will not enable us to simplify further.)

25. $$\frac{\dfrac{2}{7a^4}-\dfrac{1}{14a}}{\dfrac{3}{5a^2}+\dfrac{2}{15a}} = \frac{\dfrac{2}{7a^4}\cdot\dfrac{2}{2}-\dfrac{1}{14a}\cdot\dfrac{a^3}{a^3}}{\dfrac{3}{5a^2}\cdot\dfrac{3}{3}+\dfrac{2}{15a}\cdot\dfrac{a}{a}}$$

$$= \frac{\dfrac{4-a^3}{14a^4}}{\dfrac{9+2a}{15a^2}}$$

$$= \frac{4-a^3}{14a^4}\cdot\frac{15a^2}{9+2a}$$

$$= \frac{15\cdot\cancel{a^2}(4-a^3)}{14a^2\cdot\cancel{a^2}(9+2a)}$$

$$= \frac{15(4-a^3)}{14a^2(9+2a)},\text{ or }\frac{60-15a^3}{126a^2+28a^3}$$

27. $$\frac{\dfrac{x}{5y^3}+\dfrac{3}{10y}}{\dfrac{3}{10y}+\dfrac{x}{5y^3}}$$

Observe that, by the commutative law of addition, the numerator and denominator are equivalent, so the result is 1.

29. $$\frac{\frac{3}{ab^4}+\frac{4}{a^3b}}{\frac{5}{a^3b}-\frac{3}{ab}} = \frac{\frac{3}{ab^4}\cdot\frac{a^2}{a^2}+\frac{4}{a^3b}\cdot\frac{b^3}{b^3}}{\frac{5}{a^3b}-\frac{3}{ab}\cdot\frac{a^2}{a^2}}$$
$$= \frac{\frac{3a^2+4b^3}{a^3b^4}}{\frac{5-3a^2}{a^3b}}$$
$$= \frac{3a^2+4b^3}{a^3b^4}\cdot\frac{a^3b}{5-3a^2}$$
$$= \frac{a^3b(3a^2+4b^3)}{a^3b\cdot b^3(5-3a^2)}$$
$$= \frac{3a^2+4b^3}{b^3(5-3a^2)},\ \text{or}\ \frac{3a^2+4b^3}{5b^3-3a^2b^3}$$

31. $$\frac{t-\frac{9}{t}}{t+\frac{4}{t}} = \frac{t\cdot\frac{t}{t}-\frac{9}{t}}{t\cdot\frac{t}{t}+\frac{4}{t}}$$
$$= \frac{\frac{t^2-9}{t}}{\frac{t^2+4}{t}}$$
$$= \frac{t^2-9}{t}\cdot\frac{t}{t^2+4}$$
$$= \frac{t(t^2-9)}{t(t^2+4)}$$
$$= \frac{t^2-9}{t^2+4}$$

33. $$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}}\cdot\frac{a^3b^3}{a^3b^3}$$
$$= \frac{\frac{1}{a}\cdot a^3b^3+\frac{1}{b}\cdot a^3b^3}{\frac{1}{a^3}\cdot a^3b^3+\frac{1}{b^3}\cdot a^3b^3}$$
$$= \frac{a^2b^3+a^3b^2}{b^3+a^3}$$
$$= \frac{a^2b^2\,(b+a)}{(b+a)\,(b^2-ab+a^2)}$$
$$= \frac{a^2b^2}{b^2-ab+a^2}$$

35. $$\frac{3+\frac{4}{ab^3}}{\frac{3+a}{a^2b}} = \frac{3+\frac{4}{ab^3}}{\frac{3+a}{a^2b}}\cdot\frac{a^2b^3}{a^2b^3}$$
$$= \frac{3\cdot a^2b^3+\frac{4}{ab^3}\cdot a^2b^3}{\frac{3+a}{a^2b}\cdot a^2b^3}$$
$$= \frac{3a^2b^3+4a}{b^2(3+a)},\ \text{or}\ \frac{3a^2b^3+4a}{3b^2+ab^2}$$

37. $$\frac{t+5+\frac{3}{t}}{t+2+\frac{1}{t}} \qquad \text{LCD is } t$$
$$= \frac{t+5+\frac{3}{t}}{t+2+\frac{1}{t}}\cdot\frac{t}{t}$$
$$= \frac{t\cdot t+5\cdot t+\frac{3}{t}\cdot t}{t\cdot t+2\cdot t+\frac{1}{t}\cdot t}$$
$$= \frac{t^2+5t+3}{t^2+2t+1}$$
$$= \frac{t^2+5t+3}{(t+1)^2}$$

39. $$\frac{x-2-\frac{1}{x}}{x-5-\frac{4}{x}} = \frac{x-2-\frac{1}{x}}{x-5-\frac{4}{x}}\cdot\frac{x}{x}$$
$$= \frac{x\cdot x-2\cdot x-\frac{1}{x}\cdot x}{x\cdot x-5\cdot x-\frac{4}{x}\cdot x}$$
$$= \frac{x^2-2x-1}{x^2-5x-4}$$

41. *Writing Exercise.* Yes; Method 2, multiplying by the LCD, does not require division of rational expressions.

43. $$\begin{aligned} 3x-5+2(4x-1) &= 12x-3 \\ 3x-5+8x-2 &= 12x-3 \\ 11x-7 &= 12x-3 \\ -7 &= x-3 \\ -4 &= x \end{aligned}$$
The solution is -4.

45. $$\begin{aligned} \frac{3}{4}x-\frac{5}{8} &= \frac{3}{8}x+\frac{7}{4} \qquad \text{LCD is 8} \\ 8\left(\frac{3}{4}x-\frac{5}{8}\right) &= 8\left(\frac{3}{8}x+\frac{7}{4}\right) \\ 8\cdot\frac{3}{4}x-8\cdot\frac{5}{8} &= 8\cdot\frac{3}{8}x+8\cdot\frac{7}{4} \\ 6x-5 &= 3x+14 \\ 3x-5 &= 14 \\ 3x &= 19 \\ x &= \frac{19}{3} \end{aligned}$$
The solution is $\frac{19}{3}$.

47. $$\begin{aligned} x^2-7x+12 &= 0 \\ (x-3)\,(x-4) &= 0 \end{aligned}$$
$$x-3=0,\ \text{or}\ x-4=0$$
$$x=3\ \text{or}\ x=4$$

49. *Writing Exercise.* Although either method could be used, Method 2 requires fewer steps.

51. $\dfrac{\frac{x-5}{x-6}}{\frac{x-7}{x-8}}$

This expression is undefined for any value of x that makes a denominator 0. We see that $x-6=0$ when $x=6$, $x-7=0$ when $x=7$, and $x-8=0$ when $x=8$, so the expression is undefined for the x-values 6, 7, and 8.

53. $\dfrac{\frac{2x+3}{5x+4}}{\frac{3}{7}-\frac{x^2}{21}}$

This expression is undefined for any value of x that makes a denominator 0. First we find the value of x for which $5x+4=0$.

$$5x+4=0$$
$$5x=-4$$
$$x=-\frac{4}{5}$$

Then we find the value of x for which $\frac{3}{7}-\frac{x^2}{21}=0$:

$$\frac{3}{7}-\frac{x^2}{21}=0$$
$$21\left(\frac{3}{7}-\frac{x^2}{21}\right)=21\cdot 0$$
$$21\cdot\frac{3}{7}-21\cdot\frac{x^2}{21}=0$$
$$9-x^2=0$$
$$9=x^2$$
$$\pm 3=x$$

The expression is undefined for the x-values $-\frac{4}{5}$, -3 and 3.

55. For the complex rational expression

$\dfrac{\frac{A}{B}}{\frac{C}{D}}$ the LCD is BD.

$$=\frac{\frac{A}{B}\cdot BD}{\frac{C}{D}\cdot BD}$$
$$=\frac{\frac{ABD}{B}}{\frac{CBD}{D}}=\frac{\frac{A\cancel{B}D}{\cancel{B}}}{\frac{BC\cancel{D}}{\cancel{D}}}$$
$$=\frac{AD}{BC}$$
$$=\frac{A}{B}\cdot\frac{D}{C}$$

57. $$\frac{\frac{x}{x+5}+\frac{3}{x+2}}{\frac{2}{x+2}-\frac{x}{x+5}}=\frac{\frac{x}{x+5}+\frac{3}{x+2}}{\frac{2}{x+2}-\frac{x}{x+5}}\cdot\frac{(x+5)(x+2)}{(x+5)(x+2)}$$
$$=\frac{x(x+2)+3(x+5)}{2(x+5)-x(x+2)}$$
$$=\frac{x^2+2x+3x+15}{2x+10-x^2-2x}$$
$$=\frac{x^2+5x+15}{-x^2+10}$$

59. $\left[\dfrac{\frac{x-1}{x-1}-1}{\frac{x+1}{x-1}+1}\right]^5$

Consider the numerator of the complex rational expression:

$$\frac{x-1}{x-1}-1=1-1=0$$

Since the denominator, $\frac{x+1}{x-1}+1$ is not equal to 0, the simplified form of the original expression is 0.

61. $$\frac{\frac{z}{1-\frac{z}{2+2z}}-2z}{\frac{2z}{5z-2}-3}=\frac{\frac{z}{\frac{2+2z-z}{2+2z}}-2z}{\frac{2z-15z+6}{5z-2}}$$
$$=\frac{\frac{z}{\frac{2+z}{2+2z}}-2z}{\frac{-13z+6}{5z-2}}$$
$$=\frac{z\cdot\frac{2+2z}{2+z}-2z}{\frac{-13z+6}{5z-2}}$$
$$=\frac{\frac{z(2+2z)-2z(2+z)}{2+z}}{\frac{-13z+6}{5z-2}}$$
$$=\frac{\frac{2z+2z^2-4z-2z^2}{2+z}}{\frac{-13z+6}{5z-2}}$$
$$=\frac{\frac{-2z}{2+z}}{\frac{-13z+6}{5z-2}}$$
$$=\frac{-2z}{2+z}\cdot\frac{5z-2}{-13z+6}$$
$$=\frac{-2z(5z-2)}{(2+z)(-13z+6)},\text{ or}$$
$$\frac{2z(5z-2)}{(2+z)(13z-6)}$$

63. *Writing Exercise.* When a variable appears only in the numerator(s) of the rational expression(s) that are in the numerator of the complex rational expression, there will be no restrictions on the variables.

Exercise Set 6.6

1. The statement is false. See Example 2(c).

3. The statement is true. See page 416 in the text.

5. Because no variable appears in a denominator, no restrictions exist.

$$\frac{3}{5}-\frac{2}{3}=\frac{x}{6},\ \text{LCD}=30$$
$$30\left(\frac{3}{5}-\frac{2}{3}\right)=30\cdot\frac{x}{6}$$
$$30\cdot\frac{3}{5}-30\cdot\frac{2}{3}=30\cdot\frac{x}{6}$$
$$18-20=5x$$
$$-2=5x$$
$$\frac{-2}{5}=x$$

Check:
$$\begin{array}{c|c} \frac{3}{5}-\frac{2}{3} & \frac{x}{6} \\ \hline \frac{3}{5}-\frac{2}{3} & \frac{\frac{-2}{5}}{6} \\ \frac{18}{30}-\frac{20}{30} & \frac{-2}{5}\cdot\frac{1}{6} \end{array}$$
$$-\frac{2}{30}\overset{?}{=}\frac{-2}{30}\quad\text{TRUE}$$

This checks, so the solution is $-\frac{2}{5}$.

7. Note that x cannot be 0.

$$\frac{1}{3}+\frac{5}{6}=\frac{1}{x},\ \text{LCD}=6x$$
$$6x\left(\frac{1}{3}+\frac{5}{6}\right)=6x\cdot\frac{1}{x}$$
$$6x\cdot\frac{1}{3}+6x\cdot\frac{5}{6}=6x\cdot\frac{1}{x}$$
$$2x+5x=6$$
$$7x=6$$
$$x=\frac{6}{7}$$

Check:
$$\begin{array}{c|c} \frac{1}{3}+\frac{5}{6} & \frac{1}{x} \\ \hline \frac{1}{3}+\frac{5}{6} & \frac{1}{\frac{6}{7}} \\ \frac{2}{6}+\frac{5}{6} & 1\cdot\frac{7}{6} \end{array}$$
$$\frac{7}{6}\overset{?}{=}\frac{7}{6}\quad\text{TRUE}$$

This checks, so the solution is $\frac{6}{7}$.

9. Note that t cannot be 0.

$$\frac{1}{8}+\frac{1}{12}=\frac{1}{t},\ \text{LCD}\ =48t$$
$$48t\left(\frac{1}{8}+\frac{1}{12}\right)=48t\cdot\frac{1}{t}$$
$$48t\cdot\frac{1}{8}+48t\cdot\frac{1}{12}=48t\cdot\frac{1}{t}$$
$$6t+4t=48$$
$$10t=48$$
$$t=\frac{24}{5}$$

Check:
$$\begin{array}{c|c} \frac{1}{8}+\frac{1}{12} & \frac{1}{t} \\ \hline \frac{1}{8}+\frac{1}{12} & \frac{\frac{1}{24}}{5} \\ \frac{3}{24}+\frac{2}{24} & 1\cdot\frac{5}{24} \end{array}$$
$$\frac{5}{24}\overset{?}{=}\frac{5}{24}\quad\text{TRUE}$$

This checks, so the solution is $\frac{24}{5}$.

11. Note that y cannot be 0.

$$y+\frac{4}{y}=-5\ \text{LCD}\ =y$$
$$y\left(y+\frac{4}{y}\right)=-5\cdot y$$
$$y\cdot y+y\cdot\frac{4}{y}=-5\cdot y$$
$$y^2+4=-5y$$
$$y^2+5y+4=0$$
$$(y+4)(y+1)=0$$
$$y+4=0\ \text{or}\ y+1=0$$
$$y=-4\ \text{or}\ y=-1$$

Check:
$$\begin{array}{c|c} y+\frac{4}{y} & -5 \\ \hline -4+\frac{4}{-4} & -5 \\ -4-1 & \end{array}$$
$$-5\overset{?}{=}-5\quad\text{TRUE}$$

$$\begin{array}{c|c} y+\frac{4}{y} & -5 \\ \hline -1+\frac{4}{-1} & -5 \\ -1-4 & \end{array}$$
$$-5\overset{?}{=}-5\quad\text{TRUE}$$

Both of these check, so the solutions are -4 and -1.

13. Note that x cannot be 0.

$$\frac{x}{6}-\frac{6}{x}=0,\ \text{LCD}=6x$$
$$6x\left(\frac{x}{6}-\frac{6}{x}\right)=6x\cdot 0$$
$$6x\cdot\frac{x}{6}-6x\cdot\frac{6}{x}=6x\cdot 0$$
$$x^2-36=0$$
$$(x+6)(x-6)=0$$
$$x+6=0 \quad or \quad x-6=0$$
$$x=-6 \quad or \quad x=6$$

Check:

$$\frac{x}{6}-\frac{6}{x}=0: \quad \frac{-6}{6}-\frac{6}{-6} = -1+1 \ \big|\ 0, \qquad 0 \stackrel{?}{=} 0 \ \text{TRUE}$$

$$\frac{x}{6}-\frac{6}{x}=0: \quad \frac{6}{6}-\frac{6}{6} = 1-1 \ \big|\ 0, \qquad 0 \stackrel{?}{=} 0 \ \text{TRUE}$$

Both of these check, so the two solutions are -6 and 6.

15. Note that x cannot be 0.

$$\frac{2}{x}=\frac{5}{x}-\frac{1}{4},\ \text{LCD}\ =4x$$
$$4x\cdot\frac{2}{x}=4x\left(\frac{5}{x}-\frac{1}{4}\right)$$
$$4x\cdot\frac{2}{x}=4x\cdot\frac{5}{x}-4x\cdot\frac{1}{4}$$
$$8=20-x$$
$$-12=-x$$
$$12=x$$

Check:

$$\frac{2}{x}=\frac{5}{x}-\frac{1}{4}: \quad \frac{12}{12}\ \Big|\ \frac{5}{12}-\frac{1}{4} = \frac{5}{12}-\frac{3}{12} = \frac{2}{12}$$
$$\frac{2}{12}\stackrel{?}{=}\frac{2}{12} \qquad \text{TRUE}$$

This checks, so the solution is 12.

17. Note that t cannot be 0.

$$\frac{5}{3t}+\frac{3}{t}=1,\ \text{LCD}=3t$$
$$3t\left(\frac{5}{3t}+\frac{3}{t}\right)=3t\cdot 1$$
$$3t\cdot\frac{5}{3t}+3t\cdot\frac{3}{t}=3t\cdot 1$$
$$5+9=3t$$
$$14=3t$$
$$\frac{14}{3}=t$$

Check:

$$\frac{5}{3t}+\frac{3}{t}=1: \quad \frac{5}{3\cdot\frac{14}{3}}+\frac{3}{\frac{14}{3}} = \frac{5}{14}+\frac{9}{14} = \frac{14}{14}\ \Big|\ 1$$
$$1\stackrel{?}{=}1 \quad \text{TRUE}$$

This checks, so the solution is $\frac{14}{3}$.

19. To avoid the division by 0, we must have $n-6\neq 0$, or $n\neq 6$.

$$\frac{n+2}{n-6}=\frac{1}{2},\ \text{LCD}\ =2\,(n-6)$$
$$2\,(n-6)\cdot\frac{n+2}{n-6}=2\,(n-6)\cdot\frac{1}{2}$$
$$2\,(n+2)=n-6$$
$$2n+4=n-6$$
$$n=-10$$

Check:

$$\frac{n+2}{n-6}=\frac{1}{2}: \quad \frac{-10+2}{-10-6} = \frac{-8}{-16}\ \Big|\ \frac{1}{2}$$
$$\frac{1}{2}\stackrel{?}{=}\frac{1}{2} \quad \text{TRUE}$$

This checks, so the solution is -10.

21. Note that x cannot be 0.

$$\begin{aligned}
x+\frac{12}{x} &= -7,\ \text{LCD is } x\\
x\left(x+\frac{12}{x}\right) &= x\cdot(-7)\\
x\cdot x+x\cdot\frac{12}{x} &= -7x\\
x^2+12 &= -7x\\
x^2+7x+12 &= 0\\
(x+3)(x+4) &= 0\\
x+3=0 \quad &or \quad x+4=0\\
x=-3 \quad &or \quad x=-4
\end{aligned}$$

Both numbers check, so the solutions are -3 and -4.

23. To avoid division by 0, we must have $x-4\neq 0$ and $x+1\neq 0$, or $x\neq 4$ and $x\neq -1$.

$$\begin{aligned}
\frac{3}{x-4} &= \frac{5}{x+1},\ \text{LCD } = (x-4)(x+1)\\
(x-4)(x+1)\cdot\frac{3}{x-4} &= (x-4)(x+1)\cdot\frac{5}{x+1}\\
3(x+1) &= 5(x-4)\\
3x+3 &= 5x-20\\
23 &= 2x\\
\frac{23}{2} &= x
\end{aligned}$$

This checks, so the solution is $\frac{23}{2}$.

25. Because no variable appears in a denominator, no restrictions exist.

$$\begin{aligned}
\frac{a}{6}-\frac{a}{10} &= \frac{1}{6},\ \text{LCD} = 30\\
30\left(\frac{a}{6}-\frac{a}{10}\right) &= 30\cdot\frac{1}{6}\\
30\cdot\frac{a}{6}-30\cdot\frac{a}{10} &= 30\cdot\frac{1}{6}\\
5a-3a &= 5\\
2a &= 5\\
a &= \frac{5}{2}
\end{aligned}$$

This checks, so the solution is $\frac{5}{2}$.

27. Because no variable appears in a denominator, no restrictions exist.

$$\begin{aligned}
\frac{x+1}{3}-1 &= \frac{x-1}{2},\ \text{LCD} = 6\\
6\left(\frac{x+1}{3}-1\right) &= 6\cdot\frac{x-1}{2}\\
6\cdot\frac{x+1}{3}-6\cdot 1 &= 6\cdot\frac{x-1}{2}\\
2(x+1)-6 &= 3(x-1)\\
2x+2-6 &= 3x-3\\
2x-4 &= 3x-3\\
-1 &= x
\end{aligned}$$

This checks, so the solution is -1.

29. To avoid division by 0, we must have $y-3\neq 0$, or $y\neq 3$.

$$\begin{aligned}
\frac{y+3}{y-3} &= \frac{6}{y-3},\ \text{LCD } = y-3\\
(y-3)\cdot\frac{y+3}{y-3} &= (y-3)\cdot\frac{6}{y-3}\\
y+3 &= 6\\
y &= 3
\end{aligned}$$

Because of the restriction $y\neq 3$, the number 3 must be rejected as a solution. The equation has no solution.

31. To avoid division by 0, we must have $x+4\neq 0$ and $x\neq 0$, or $x\neq -4$ and $x\neq 0$.

$$\begin{aligned}
\frac{3}{x+4} &= \frac{5}{x},\ \text{LCD} = x(x+4)\\
x(x+4)\cdot\frac{3}{x+4} &= x(x+4)\cdot\frac{5}{x}\\
3x &= 5(x+4)\\
3x &= 5x+20\\
-2x &= 20\\
x &= -10
\end{aligned}$$

This checks, so the solution is -10.

33. To avoid division by 0, we must have $n+2\neq 0$ and $n+1\neq 0$, or $n\neq -2$ and $n\neq -1$.

$$\begin{aligned}
\frac{n+1}{n+2} &= \frac{n-3}{n+1},\ \text{LCD } = (n+2)(n+1)\\
(n+2)(n+1)\cdot\frac{n+1}{n+2} &= (n+2)(n+1)\cdot\frac{n-3}{n+1}\\
(n+1)(n+1) &= (n+2)(n-3)\\
n^2+2n+1 &= n^2-n-6\\
3n &= -7\\
n &= \frac{-7}{3}
\end{aligned}$$

This checks, so the solution is $\frac{-7}{3}$.

35. To avoid division by 0, we must have $t-2\neq 0$, or $t\neq 2$.

$$\begin{aligned}
\frac{5}{t-2}+\frac{3t}{t-2} &= \frac{4}{t^2-4t+4},\ \text{LCD is } (t-2)^2\\
(t-2)^2\left(\frac{5}{t-2}+\frac{3t}{t-2}\right) &= (t-2)^2\cdot\frac{4}{(t-2)^2}\\
5(t-2)+3t(t-2) &= 4\\
5t-10+3t^2-6t &= 4\\
3t^2-t-10 &= 4\\
3t^2-t-14 &= 0\\
(3t-7)(t+2) &= 0\\
3t-7=0 \quad &or \quad t+2=0\\
3t=7 \quad &or \quad t=-2\\
t=\frac{7}{3} \quad &or \quad t=-2
\end{aligned}$$

Both numbers check. The solutions are $\frac{7}{3}$ and -2.

37. To avoid division by 0, we must have $x+5\neq 0$ and $x-5\neq 0$, or $x\neq -5$ and $x\neq 5$.

$$\frac{x}{x+5}-\frac{5}{x-5}=\frac{14}{x^2-25}.$$
$$\text{LCD}=(x+5)(x-5)$$

$$(x+5)(x-5)\cdot\left(\frac{x}{x+5}-\frac{5}{x-5}\right)=(x+5)(x-5)\cdot\frac{14}{(x+5)(x-5)}$$
$$x(x-5)-5(x+5)=14$$
$$x^2-5x-5x-25=14$$
$$x^2-10x-39=0$$
$$(x-13)(x+3)=0$$
$$x-13=0 \text{ or } x+3=0$$
$$x=13 \text{ or } x=-3$$

Both numbers check. The solutions are $-3, 13$.

39. To avoid division by 0, we must have $t-3\neq 0$ and $t+3\neq 0$, or $t\neq 3$ and $t\neq -3$.

$$\frac{5}{t-3}-\frac{30}{t^2-9}=1,$$
$$\text{LCD}=(t-3)(t+3)$$

$$(t-3)(t+3)\cdot\left(\frac{5}{t-3}-\frac{30}{t^2-9}\right)=(t-3)(t+3)\cdot 1$$
$$5(t+3)-30=(t-3)(t+3)$$
$$5t+15-30=t^2-9$$
$$0=t^2-5t+6$$
$$0=(t-3)(t-2)$$
$$t-3=0 \text{ or } t-2=0$$
$$t=3 \text{ or } t=2$$

Because of the restriction $t\neq 3$, we must reject the number 3 as a solution. The number 2 checks, so it is the solution.

41. To avoid division by 0, we must have $6-a\neq 0$ (or equivalently $a-6\neq 0$) or $a\neq 6$.

$$\frac{7}{6-a}=\frac{a+1}{a-6}$$
$$\frac{-1}{-1}\cdot\frac{7}{6-a}=\frac{a+1}{a-6}$$
$$\frac{-7}{a-6}=\frac{a+1}{a-6}, \text{ LCD }=a-6$$
$$(a-6)\cdot\frac{-7}{a-6}=(a-6)\cdot\frac{a+1}{a-6}$$
$$-7=a+1$$
$$-8=a$$

This checks.

43. $\dfrac{-2}{x+2}=\dfrac{x}{x+2}$

To avoid division by 0, we must have $x+2\neq 0$, or $x\neq -2$. Now observe that the denominators are the same, so the numerators must be the same. Thus, we have $-2=x$, but because of the restriction $x\neq -2$ this cannot be a solution. The equation has no solution.

45. Note that x cannot be 0.

$$\frac{12}{x}=\frac{x}{3}, \text{ LCD }=3x$$
$$3x\cdot\frac{12}{x}=3x\cdot\frac{x}{3}$$
$$36=x^2$$
$$0=x^2-36$$
$$0=(x+6)(x-6)$$
$$x+6=0 \text{ or } x-6=0$$
$$x=-6 \text{ or } x=6$$

This checks.

47. *Writing Exercise.* When solving rational equations, we multiply each side by the LCM of the denominators in order to clear fractions.

49. ***Familiarize***. Let $x=$ the first odd integer. Then $x+2=$ the next odd integer.

Translate.

$$\underbrace{\text{The sum of two consecutive odd integers}}_{x+(x+2)}\ \underset{=}{\text{is}}\ \underset{276}{276.}$$

Carry out. We solve the equation.

$$x+(x+2)=276$$
$$2x+2=276$$
$$2x=274$$
$$x=137$$

When $x=137$, then $x+2=137+2=139$.

Check. The numbers 137 and 139 are consecutive odd integers and $137+139=276$. These numbers check.

State. The integers are 137 and 139.

51. ***Familiarize***. Let $b=$ the base of the triangle, in cm. Then $b+3=$ the height. Recall that the area of a triangle is given by $\frac{1}{2}\times$ base $\times$ height.

Translate.

$$\underbrace{\text{The area of the triangle}}_{\frac{1}{2}\cdot b\cdot(b+3)}\ \underset{=}{\text{is}}\ \underset{54}{54\text{ cm}^2.}$$

Carry out. We solve the equation.

$$\frac{1}{2}b(b+3)=54$$
$$2\cdot\frac{1}{2}b(b+3)=2\cdot 54$$
$$b(b+3)=108$$
$$b^2+3b=108$$
$$b^2+3b-108=0$$
$$(b-9)(b+12)=0$$
$$b-9=0 \quad \textit{or} \quad b+12=0$$
$$b=9 \quad \textit{or} \quad b=-12$$

Check. The length of the base cannot be negative so we need to check only 9. If the base is 9 cm, then the height is $9+3$, or 12 cm, and the area is $\frac{1}{2}\cdot 9\cdot 12$, or 54 cm^2. The answer checks.

State. The base measures 9 cm, and the height measures 12 cm.

53. To find the rate, in centimeters per day, we divide the amount of growth by the number of days. From June 9 to June 24 is $24-9=15$ days.

$$\begin{aligned}\text{Rate, in cm per day} &= \frac{0.9 \text{ cm}}{15 \text{ days}}\\ &= 0.06 \text{ cm/day}\\ &= 0.06 \text{ cm per day}\end{aligned}$$

55. *Writing Exercise*. Begin with an equation. Then divide on both sides of the equation by an expression whose value is zero for at least one solution of the equation. See Exercises 43 and 44 for examples.

57. To avoid division by 0, we must have $x-3\neq 0$, or $x\neq 3$.

$$\begin{aligned}1+\frac{x-1}{x-3} &= \frac{2}{x-3}-x, \text{ LCD}=x-3\\ (x-3)\left(1+\frac{x-1}{x-3}\right) &= (x-3)\left(\frac{2}{x-3}-x\right)\\ (x-3)\cdot 1+(x-3)\cdot\frac{x-1}{x-3} &= (x-3)\cdot\frac{2}{x-3}-(x-3)x\\ x-3+x-1 &= 2-x^2+3x\\ 2x-4 &= 2-x^2+3x\\ x^2-x-6 &= 0\\ (x-3)(x+2) &= 0\end{aligned}$$

$x-3=0$ *or* $x+2=0$

$x=3$ *or* $x=-2$

Because of the restriction $x\neq 3$, we must reject the number 3 as a solution. The number -2 checks, so it is the solution.

59. To avoid division by 0, we must have $x+2\neq 0$ and $x-2\neq 0$, or $x\neq -2$ and $x\neq 2$.

$$\begin{aligned}\frac{12-6x}{x^2-4} &= \frac{3x}{x+2}-\frac{3-2x}{2-x},\\ \frac{12-6x}{(x+2)(x-2)} &= \frac{3x}{x+2}-\frac{3-2x}{2-x}\cdot\frac{-1}{-1}\\ \frac{12-6x}{(x+2)(x-2)} &= \frac{3x}{x+2}-\frac{2x-3}{x-2},\\ &\text{LCD} = (x+2)(x-2)\\ (x+2)(x-2)\cdot\frac{12-6x}{(x+2)(x-2)} &=\\ &(x+2)(x-2)\left(\frac{3x}{x+2}-\frac{2x-3}{x-2}\right)\\ 12-6x &=\\ &3x(x-2)-(x+2)(2x-3)\\ 12-6x &=\\ &3x^2-6x-2x^2-x+6\\ 0 &= x^2-x-6\\ 0 &= (x-3)(x+2)\end{aligned}$$

$x-3=0$ *or* $x+2=0$

$x=3$ *or* $x=-2$

Because of the restriction $x\neq -2$, we must reject the number -2 as a solution. The number 3 checks, so it is the solution.

61. To avoid division by 0, we must have $a+3\neq 0$, or $a\neq -3$.

$$\begin{aligned}7-\frac{a-2}{a+3} &= \frac{a^2-4}{a+3}+5, \text{ LCD} = a+3\\ (a+3)\left(7-\frac{a-2}{a+3}\right) &= (a+3)\left(\frac{a^2-4}{a+3}+5\right)\\ 7(a+3)-(a-2) &= a^2-4+5(a+3)\\ 7a+21-a+2 &= a^2-4+5a+15\\ 6a+23 &= a^2+5a+11\\ 0 &= a^2-a-12\\ 0 &= (a-4)(a+3)\end{aligned}$$

$a-4=0$ *or* $a+3=0$

$a=4$ *or* $a=-3$

Because of the restriction $a\neq -3$, we must reject the number -3 as a solution. The number 4 checks, so it is the solution.

63. To avoid division by 0, we must have $x-1\neq 0$, or $x\neq 1$.

$$\begin{aligned}\frac{1}{x-1}+x-5 &= \frac{5x-4}{x-1}-6, \text{ LCD} = x-1\\ (x-1)\left(\frac{1}{x-1}+x-5\right) &= (x-1)\left(\frac{5x-4}{x-1}-6\right)\\ 1+x(x-1)-5(x-1) &= 5x-4-6(x-1)\\ 1+x^2-x-5x+5 &= 5x-4-6x+6\\ x^2-6x+6 &= -x+2\\ x^2-5x+4 &= 0\\ (x-1)(x-4) &= 0\end{aligned}$$

$x-1=0$ *or* $x-4=0$

$x=1$ *or* $x=4$

Because of the restriction $x\neq 1$, we must reject the number 1 as a solution. The number 4 checks, so it is the solution.

65. Note that x cannot be 0.

$$\begin{aligned}\frac{\frac{1}{x}+1}{x} &= \frac{\frac{1}{x}}{2}\\ \left(\frac{1}{x}+1\right)\cdot\frac{1}{x} &= \frac{1}{x}\cdot\frac{1}{2}\\ \frac{1}{x^2}+\frac{1}{x} &= \frac{1}{2x}, \text{ LCD } = 2x^2\\ 2+2x &= x\\ 2 &= -x\\ -2 &= x\end{aligned}$$

This checks.

67.

6.6 Connecting the Concepts

1. Expression; $\frac{4x^2-8x}{4x^2+4x}=\frac{4x\,(x-2)}{4x\,(x+1)}=\frac{x-2}{x+1}$

3. Equation; $\frac{3}{y} - \frac{1}{4} = \frac{1}{y}$ Note: $y \neq 0$

$$4y\left(\frac{3}{y} - \frac{1}{4}\right) = 4y\left(\frac{1}{y}\right) \qquad \text{LCD } = 4y$$
$$12 - y = 4$$
$$-y = -8$$
$$y = 8$$

The solution is 8.

5. Equation; $\frac{5}{x+3} = \frac{3}{x+2}$ Note $x \neq -2, -3$

$$(x+2)(x+3) \cdot \frac{5}{x+3} = (x+2)(x+3) \cdot \frac{3}{x+2}$$
$$\text{LCD } = (x+2)(x+3)$$
$$5(x+2) = 3(x+3)$$
$$5x + 10 = 3x + 9$$
$$5x = 3x - 1$$
$$2x = -1$$
$$x = \frac{-1}{2}$$

7. Expression; $\frac{2a}{a+1} - \frac{4a}{1-a^2}$

$$= \frac{2a}{a+1} - \frac{4a}{a^2-1}$$
$$= \frac{2a(a-1) + 4a}{(a+1)(a-1)} \qquad \text{LCD } = (a+1)(a-1)$$
$$= \frac{2a^2 - 2a + 4}{(a+1)(a-1)}$$
$$= \frac{2a^2 - 2a}{(a+1)(a-1)}$$
$$= \frac{2a(a-1)}{(a+1)(a-1)}$$
$$= \frac{2a}{a+1}$$

9. Expression; $\frac{18x^2}{25} \div \frac{12x}{5} = \frac{18x^2}{25} \cdot \frac{5}{12x}$

$$= \frac{3x}{10} \cdot \frac{5 \cdot 6x}{5 \cdot 6x}$$
$$= \frac{3x}{10}$$

Exercise Set 6.7

1. 1 cake in 2 hours = $\frac{1 \text{ cake}}{2 \text{ hr}} = \frac{1}{2}$ cake per hour

3. Sandy: $\frac{1 \text{ cake}}{2 \text{ hr}} = \frac{1}{2}$ cake per hour

Eric: $\frac{1 \text{ cake}}{3 \text{ hr}} = \frac{1}{3}$ cake per hour

Together: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ cake per hour

5. 1 lawn in 3 hours = $\frac{1 \text{ lawn}}{3 \text{ hr}} = \frac{1}{3}$ lawn per hour

7. ***Familiarize***. Let t represent the number of hours it takes Bryan and Caroline to refinish the floors working together.

Translate. Bryan takes 8 hr and Caroline takes 6 hr to complete the job, so we have $\frac{t}{8} + \frac{t}{6} = 1$

Carry out. We solve the equation. Multiply on both sides by the LCD, 24.

Check. If Bryan does the job alone in 8 hr, then in $3\frac{3}{7}$ hr he does $\frac{24/7}{8}$, or $\frac{3}{7}$ of the job. If Caroline does the job alone in 6 hr, then in $3\frac{3}{7}$ hr she does $\frac{24/7}{6}$, or $\frac{4}{7}$ of the job. Together, they do $\frac{3}{7} + \frac{4}{7}$, or 1 entire job. The result checks.

State. It would take Bryan and Caroline $3\frac{3}{7}$ hr to finish the job working together.

9. ***Familiarize***. The job takes Aficio 7 minutes working alone and MX 6 minutes working alone. Then in 1 minute Aficio does $\frac{1}{7}$ of the job and MX does $\frac{1}{6}$ of the job. Working together, they can do $\frac{1}{7} + \frac{1}{6}$, or $\frac{13}{42}$ of the job in 1 minute. In 2 minutes, Aficio does $2\left(\frac{1}{7}\right)$ of the job and MX does $2\left(\frac{1}{6}\right)$ of the job. Working together they can do $2\left(\frac{1}{7}\right) + 2\left(\frac{1}{6}\right)$, or $\frac{13}{21}$ of the job in 2 minutes. In 4 minutes Aficio does $4\left(\frac{1}{7}\right)$ of the job and MX does $4\left(\frac{1}{6}\right)$ of the job. Working together they can do $4\left(\frac{1}{7}\right) + 4\left(\frac{1}{6}\right)$, or $\frac{26}{21}$ of the job which is more of the job then needs to be done. The answer is somewhere between 2 minutes and 4 minutes.

Translate. If they work together t minutes, then Aficio does $t\left(\frac{1}{7}\right)$ of the job and MX does $t\left(\frac{1}{6}\right)$ of the job. We want some number t such that

$$\left(\frac{1}{7} + \frac{1}{6}\right)t = 1, \text{ or } \frac{13}{42}t = 1.$$

Carry out. We solve the equation.

$$\frac{13}{42}t = 1$$
$$\frac{42}{13} \cdot \frac{13}{42}t = \frac{42}{13} \cdot 1$$
$$t = \frac{42}{13}$$

Check. We repeat computations. The answer checks. We also expected the result to be between 2 minutes and 4 minutes.

State. Working together, it takes Aficio and MX 42/13 mins, or 3 3/13 mins.

11. ***Familiarize***. In 1 minute the Wayne pump does $\frac{1}{42}$ of the job and the Craftsman pump does $\frac{1}{35}$ of the job. Working together, they do $\frac{1}{42} + \frac{1}{35}$ of the job in 1 minute. Suppose it takes t minutes to do the job working together.

Translate. We find t such that $t\left(\frac{1}{42}\right) + t\left(\frac{1}{35}\right) = 1$, or $\frac{t}{42} + \frac{t}{35} = 1$.

Carry out. We solve the equation. We multiply both sides by the LCD, 210.

$$210\left(\frac{t}{42} + \frac{t}{35}\right) = 210 \cdot 1$$
$$5t + 6t = 210$$
$$11t = 210$$
$$t = \frac{210}{11}$$

Check. In $\frac{210}{11}$ min the Wayne pump does $\frac{210}{11} \cdot \frac{1}{42}$, or $\frac{5}{11}$ of the job and the Craftsman pump does $\frac{210}{11} \cdot \frac{1}{35}$, or $\frac{6}{11}$ of the job. Together they do $\frac{5}{11} + \frac{6}{11}$, or 1 entire job. The answer checks.

State. The two pumps can pump out the basement in $\frac{210}{11}$ min, or $19\frac{1}{11}$ min, working together.

13. ***Familiarize***. Let t represent the time, in minutes, that it takes the K5400 to print the brochures working alone. Then $2t$ represents the time it takes the H470 to do the job, working alone. In 1 minute the K5400 does $\frac{1}{t}$ of the job and the H470 does $\frac{1}{2t}$ of the job.

Translate. Working together, they can do the entire job in 45 min, so we want to find t such that $45\left(\frac{1}{t}\right)+45\left(\frac{1}{2t}\right) = 1$, or $\frac{45}{t} + \frac{45}{2t} = 1$.

Carry out. We solve the equation. We multiply both sides by the LCD, $2t$.

$$2t\left(\frac{45}{t} + \frac{45}{2t}\right) = 2t \cdot 1$$
$$90 + 45 = 2t$$
$$135 = 2t$$
$$\frac{135}{2} = t, \text{ or } 67\tfrac{1}{2}$$

Check. If the K5400 can do the job in $\frac{135}{2}$ min, then in 45 min it does $45 \cdot \frac{1}{135/2}$, or $\frac{2}{3}$ of the job. If it takes the H470 $2 \cdot \frac{135}{2}$, or 135 min, to do the job, then in 45 min it does $45 \cdot \frac{1}{135}$, or $\frac{1}{3}$ of the job. Working together, the two machines do $\frac{2}{3} + \frac{1}{3}$, or 1 entire job, in 45 min.

State. Working alone, it takes the K5400 $67\frac{1}{2}$ min and the H470 135 min to print the brochure.

15. ***Familiarize***. Let t represent the number of minutes it takes the Airgle machine to purify the air working alone. Then $t-15$ represents the time it takes the Austin machine to purify the air, working alone. In 1 minute the Airgle does $\frac{1}{t}$ of the job and the Austin does $\frac{1}{t-15}$ of the job.

Translate. Working together, the two machines can purify the air in 10 min to find t such that $10\left(\frac{1}{t}\right) + 10\left(\frac{1}{t-15}\right) = 1$, or $\frac{10}{t} + \frac{10}{t-15} = 1$.

Carry out. We solve the equation. First we multiply both sides by the LCD, $t(t-15)$.

$$t(t-15)\left(\frac{10}{t} + \frac{10}{t-15}\right) = t(t-15) \cdot 1$$
$$10(t-15) + 10t = t(t-15)$$
$$10t - 150 + 10t = t^2 - 15t$$
$$0 = t^2 - 35t + 150$$
$$0 = (t-5)(t-30)$$
$$t = 5 \text{ or } t = 30$$

Check. If $t = 5$, then $t - 15 = 5 - 15 = -10$. Since negative time has no meaning in this application, 5 cannot be a solution. If $t = 30$, then $t - 15 = 30 - 15 = 15$. In 10 min the Airgle machine does $10 \cdot \frac{1}{30}$, or $\frac{1}{3}$ of the job. In 10 min the Austin does $10 \cdot \frac{1}{15}$, or $\frac{2}{3}$ of the job. Together they do $\frac{1}{3} + \frac{2}{3}$, or 1 entire job. The answer checks.

State. Working alone, the Airgle machine can purify the air in 30 min and the Austin machine can purify the air in 15 min.

17. ***Familiarize***.Let t represent the number of minutes it takes Chris to do the job working alone. Then $t + 120$ represents the time it takes Kim to do the job working alone. We will convert hours to minutes:

Translate. In 175 min Chris and Kim will do one entire job, so we have $175\left(\frac{1}{t}\right)+175\left(\frac{1}{t+120}\right) = 1$, or $\frac{175}{t}+\frac{175}{t+120} = 1$

Carry out.We solve the equation. Multiply on both sides by the LCD, $t(t+120)$.

$$t(t+120)\left(\frac{175}{t} + \frac{175}{t+120}\right) = t(t+120) \cdot 1$$
$$175(t+120) + 175t = t(t+120)$$
$$175t + 21,000 + 175t = t^2 + 120t$$
$$0 = t^2 - 230t - 21,000$$
$$0 = (t-300)(t+70)$$
$$t = 300 \text{ or } t = -70$$

Check.Since negative time has no meaning in this problem -70 is not a solution of the original problem. If Chris does the job alone in 300 min, then in 175 min he does $\frac{175}{300} = \frac{7}{12}$ of the job. If Kim does the job alone in 300 + 120, or 420 min, then in 175 min she does $\frac{175}{420} = \frac{5}{12}$ of the job. Together, they do $\frac{7}{12} + \frac{5}{12}$, or 1 entire job, in 175 min. The result checks.

State. It would take Chris 300 min, or 5 hours to do the job alone.

19. ***Familiarize***. We complete the table shown in the text.

$d = r \cdot t$

	Distance	Speed	Time
B & M	330	$r-14$	$\frac{330}{r-14}$
AMTRAK	400	r	$\frac{400}{r}$

Translate. Since the time must be the same for both trains, we have the equation

$$\frac{330}{r-14} = \frac{400}{r}.$$

Carry out. We first multiply by the LCD, $r(r-14)$.

$$r(r-14) \cdot \frac{330}{r-14} = r(r-14) \cdot \frac{400}{r}$$
$$330r = 400(r-14)$$
$$330r = 400r - 5600$$
$$-70r = -5600$$
$$r = 80$$

If the speed of the AMTRAK train is 80 km/h, then the speed of the B & M train is 80 − 14, or 66 km/h.

Check. The speed of the B&M train is 14 km/h slower than the speed of the AMTRAK train. At 66 km/h the B&M train travels 330 km in 330/66, or 5 hr. At 80 km/h the AMTRAK train travels 400 km in 400/80, or 5 hr. The times are the same, so the answer checks.

State. The speed of the AMTRAK train is 80 km/h, and the speed of the B&M freight train is 66 km/h.

21. ***Familiarize***. We first make a drawing. Let r = the kayak's speed in still water in mph. Then $r-3$ = the speed upstream and $r+3$ = the speed downstream.

We organize the information in a table. The time is the same both upstream and downstream so we use t for each time.

	Distance	Speed	Time
Upstream	4	$r-3$	t
Downstream	10	$r+3$	t

Translate. Using the formula Time = Distance/Rate in each row of the table and the fact that the times are the same, we can write an equation. $\frac{4}{r-3} = \frac{10}{r+3}$

Carry out. We solve the equation.
$\frac{4}{r-3} = \frac{10}{r+3}$, LCD is $(r-3)(r+3)$

$$(r-3)(r+3)\cdot\frac{4}{r-3} = (r-3)(r+3)\cdot\frac{10}{r+3}$$
$$4(r+3) = 10(r-3)$$
$$4r+12 = 10r-30$$
$$42 = 6r$$
$$7 = r$$

Check. If $r=7$ mph, then $r-3$ is 4 mph and $r+3$ is 10 mph. The time upstream is $\frac{4}{4}$, or 1 hour. The time downstream is $\frac{10}{10}$, or 1 hour. Since the times are the same, the answer checks.

State. The speed of the kayak in still water is 7 mph.

23. ***Familiarize***. We first make a drawing. Let r = Roslyn's speed on a nonmoving sidewalk in ft/sec. Then her speed moving forward on the moving sidewalk is r + 1.8, and her speed in the opposite direction is $r-1.8$.

We organize the information in a table. The time is the same both forward and in the opposite direction, so we use t for each time.

	Distance	Speed	Time
Forward	105	$r+1.8$	t
Opposite direction	51	$r-1.8$	t

Translate. Using the formula Time = Distance/Rate in each row of the table and the fact that the times are the same, we can write an equation. $\frac{105}{r+1.8} = \frac{51}{r-1.8}$

Carry out. We solve the equation.

$$\frac{105}{r+1.8} = \frac{51}{r-1.8}$$
LCD is $(r+1.8)(r-1.8)$
$$(r+1.8)(r-1.8)\frac{105}{r+1.8} = (r+1.8)(r-1.8)\frac{51}{r-1.8}$$
$$105(r-1.8) = 51(r+1.8)$$
$$105r-189 = 51r+91.8$$
$$54r = 280.8$$
$$r = 5.2$$

Check. If Roslyn's speed on a nonmoving sidewalk is 5.2 ft/sec, then her speed moving forward on the moving sidewalk is 5.2 + 1.8, or 7 ft/sec, and her speed moving in the opposite direction on the sidewalk is 5.2 - 1.8, or 3.4 ft/sec. Moving 105 ft at 7 ft/sec takes $\frac{105}{7} = 15$ sec. Moving 51 ft at 3.4 ft/sec takes $\frac{51}{3.4} = 15$ sec. Since the times are the same, the answer checks.

State. Roslyn's would be walking 5.2 ft/sec on a nonmoving sidewalk.

25. ***Familiarize***. Let t = the time it takes Caledonia to drive to town and organize the given information in a table.

	Distance	Speed	Time
Caledonia	15	r	t
Manley	20	r	$t+1$

Translate. We can replace the r's in the table above using the formula $r = d/t$.

	Distance	Speed	Time
Caledonia	15	$\frac{15}{t}$	t
Manley	20	$\frac{20}{t+1}$	$t+1$

Since the speeds are the same for both riders, we have the equation

$$\frac{15}{t} = \frac{20}{t+1}.$$

Carry out. We multiply by the LCD, $t(t+1)$.

$$t(t+1)\cdot\frac{15}{t} = t(t+1)\cdot\frac{20}{t+1}$$
$$15(t+1) = 20t$$
$$15t+15 = 20t$$
$$15 = 5t$$
$$3 = t$$

If $t=3$, then $t+1 = 3+1$, or 4.

Check. If Caledonia's time is 3 hr and Manley's time is 4 hr, then Manley's time is 1 hr more than Caledonia's. Caledonia's speed is 15/3, or 5 mph. Manley's speed is 20/4, or 5 mph. Since the speeds are the same, the answer checks.

State. It takes Caledonia 3 hr to drive to town.

27. ***Familiarize***. We let r = the speed of the river. Then $15+r$ = LeBron's speed downstream in km/h and $15-r$ = his speed upstream in km/h. The times are the same. Let t represent the time. We organize the information in a table.

	Distance	Speed	Time
Upstream	140	$15+r$	t
Downstream	35	$15-r$	t

Translate. Using the formula Time = Distance/Rate in each row of the table and the fact that the times are the same, we can write an equation. $\frac{140}{15+r} = \frac{35}{15-r}$

Carry out. We solve the equation.

$$\begin{aligned}
\tfrac{140}{15+r} &= \tfrac{35}{15-r},\ \text{LCD is } (15+r)(15-r)\\
(15+r)(15-r)\cdot\tfrac{140}{15+r} &= (15+r)(15-r)\cdot\tfrac{35}{15-r}\\
140(15-r) &= 35(15+r)\\
2100-140r &= 525+35r\\
1575 &= 175r\\
9 &= r
\end{aligned}$$

Check. If $r = 9$, then the speed downstream is 15 + 9, or 24 km/h and the speed upstream is 15 - 9, or 6 km/h. The time for the trip downstream is $\frac{140}{24}$, or $5\frac{5}{6}$ hours. The time for the trip upstream is $\frac{35}{6}$, or $5\frac{5}{6}$ hours. The times are the same. The values check.

State. The speed of the river is 9 km/h.

29. ***Familiarize***. Let c = the speed of the current, in km/h. Then $7 + c$ = the speed downriver and $7 - c$ = the speed upriver. We organize the information in a table.

	Distance	Speed	Time
Downriver	45	$7+c$	t_1
Upriver	45	$7-c$	t_2

Translate. Using the formula Time = Distance/Rate we see that $t_1 = \frac{45}{7+c}$ and $t_2 = \frac{45}{7-c}$. The total time upriver and back is 14 hr, so $t_1 + t_2 = 14$, or $\frac{45}{7+c} + \frac{45}{7-c} = 14$.

Carry out. We solve the equation. Multiply both sides by the LCD, $(7 + c)(7 - c)$.

$$\begin{aligned}
(7+c)(7-c)\left(\tfrac{45}{7+c}+\tfrac{45}{7-c}\right) &= (7+c)(7-c)14\\
45(7-c)+45(7+c) &= 14(49-c^2)\\
315-45c+315+45c &= 686-14c^2\\
14c^2-56 &= 0\\
14(c+2)(c-2) &= 0\\
c+2=0 \quad \text{or} &\quad c-2=0\\
c=-2 \quad \text{or} &\quad c=2
\end{aligned}$$

Check. Since speed cannot be negative in this problem, -2 cannot be a solution of the original problem. If the speed of the current is 2 km/h, the barge travels upriver at 7 - 2, or 5 km/h. At this rate it takes $\frac{45}{5}$, or 9 hr, to travel 45 km. The barge travels downriver at 7 + 2, or 9 km/h. At this rate it takes $\frac{45}{9}$, or 5 hr, to travel 45 km. The total travel time is 9 + 5, or 14 hr. The answer checks.

State. The speed of the current is 2 km/h.

31. ***Familiarize***. Let r = the speed at which the train actually traveled in mph, and let t = the actual travel time in hours. We organize the information in a table.

	Distance	Speed	Time
Actual speed	120	r	t
Faster speed	120	$r+10$	$t-2$

Translate. From the first row of the table we have $120 = rt$, and from the second row we have $120 = (r + 10)(t - 2)$. Solving the first equation for t, we have $t = \frac{120}{r}$. Substituting for t in the second equation, we have $120 = (r + 10)\left(\frac{120}{r} - 2\right)$.

Carry out. We solve the equation.

$$\begin{aligned}
120 &= (r+10)\left(\tfrac{120}{r}-2\right)\\
120 &= 120-2r+\tfrac{1200}{r}-20\\
20 &= -2r+\tfrac{1200}{r}\\
r\cdot 20 &= r\left(-2r+\tfrac{1200}{r}\right)\\
20r &= -2r^2+1200\\
2r^2+20r-1200 &= 0\\
2(r^2+10r-600) &= 0\\
2(r+30)(r-20) &= 0\\
r=-30 \ &\text{or}\ r=20
\end{aligned}$$

Check. Since speed cannot be negative in this problem, cannot be a solution of the original problem. If the speed is 20 mph, it takes $\frac{120}{20}$, or 6 hr, to travel 120 mi. If the speed is 10 mph faster, or 30 mph, it takes $\frac{120}{30}$, or 4 hr, to travel 120 mi. Since 4 hr is 2 hr less time than 6 hr, the answer checks.

State. The speed was 20 mph.

33. We write a proportion and then solve it.

$$\begin{aligned}
\frac{b}{6} &= \frac{7}{4}\\
b &= \frac{7}{4}\cdot 6\\
b &= \frac{42}{4},\ \text{or } 10.5
\end{aligned}$$

(Note that the proportions $\frac{6}{b} = \frac{4}{7}$, $\frac{b}{7} = \frac{6}{4}$, or $\frac{7}{b} = \frac{4}{6}$ could also be used.)

35. We write a proportion and then solve it.

$$\begin{aligned}
\frac{4}{f} &= \frac{6}{4}\\
4f\cdot\frac{4}{f} &= 4f\cdot\frac{6}{4}\\
16 &= 6f\\
\frac{8}{3} &= f \qquad \text{Simplifying}
\end{aligned}$$

(One of the following proportions could also be used: $\frac{f}{4} = \frac{4}{6}$, $\frac{4}{f} = \frac{9}{6}$, $\frac{f}{4} = \frac{6}{9}$, $\frac{4}{9} = \frac{f}{6}$, $\frac{9}{4} = \frac{6}{f}$)

37. From the blueprint we see that 9 in. represents 36 ft and that p in. represent 15 ft. We use a proportion to find p.

$$\begin{aligned}
\frac{9}{36} &= \frac{p}{15}\\
180\cdot\frac{9}{36} &= 180\cdot\frac{p}{15}\\
45 &= 12p\\
\frac{15}{4} &= p,\ \text{or}\\
3\frac{3}{4} &= p
\end{aligned}$$

The length of p is $3\frac{3}{4}$ in.

39. From the blueprint we see that 9 in. represents 36 ft and that 5 in. represents r ft. We use a proportion to find r.

$$\frac{9}{36} = \frac{5}{r}$$

$$36r \cdot \frac{9}{36} = 36r \cdot \frac{5}{r}$$

$$9r = 180$$

$$r = 20$$

The length of r is 20 ft.

41. Consider the two similar right triangles in the drawing. One has legs 4 ft and 6 ft. The other has legs 10ft and l ft. We use a proportion to find l.

$$\frac{4}{6} = \frac{10}{l}$$

$$6l \cdot \frac{4}{6} = 6l \cdot \frac{10}{l}$$

$$4l = 60$$

$$l = 15\text{ft}$$

43. Consider the two similar right triangles in the drawing. One has legs 5 and 7. The other has legs 9 and r. We use a proportion to find r.

$$\frac{5}{7} = \frac{9}{r}$$

$$7r \cdot \frac{5}{7} = 7r \cdot \frac{9}{r}$$

$$5r = 63$$

$$r = \frac{63}{5}, \text{ or } 12.6$$

45. ***Familiarize***. Brett had 384 text messages in 8 days. Let n = the number of text messages in 30 days.

Translate.

$$\begin{array}{r} \text{Messages} \rightarrow \\ \text{Days} \rightarrow \end{array} \frac{384}{8} = \frac{n}{30} \begin{array}{l} \leftarrow \text{Messages} \\ \leftarrow \text{Days} \end{array}$$

Carry out. We solve the proportion.

$$120 \cdot \frac{384}{8} = 120 \cdot \frac{n}{30}$$

$$5760 = 4n$$

$$1440 = n$$

Check. $\frac{384}{8} = 48$, $\frac{1440}{30} = 48$

The ratios are the same so the answer checks.

State. He will send or receive 1440 messages in 30 days.

47. ***Familiarize***. Persons caught on 295 mi stretch is 12,334. Let n = the number caught on 5525 mi border.

Translate.

$$\begin{array}{r} \text{Persons} \rightarrow \\ \text{distance} \rightarrow \end{array} \frac{12334}{295} = \frac{n}{5525} \begin{array}{l} \leftarrow \text{Persons} \\ \leftarrow \text{distance} \end{array}$$

Carry out. We solve the proportion.

$$325,975 \cdot \frac{12334}{295} = 325,975 \cdot \frac{n}{5525}$$

$$13,629,070 = 59n$$

$$231,001 \approx n$$

Check. $\frac{12334}{295} \approx 41.81$, $\frac{231001}{5525} \approx 41.81$

The ratios are the same, so the answer checks.

State. Over the entire border about 231,001 people may be caught.

49. ***Familiarize***. Let g = the number if gal of gas for a 810 mi trip. We can use a proportion to solve for g.

Translate.

$$\begin{array}{r} \text{gal} \rightarrow \\ \text{miles} \rightarrow \end{array} \frac{4}{180} = \frac{g}{810} \begin{array}{l} \leftarrow \text{gal} \\ \leftarrow \text{miles} \end{array}$$

Carry out. We solve the proportion.

We multiply by 1620 to get g alone.

$$1620 \cdot \frac{4}{180} = 1620 \cdot \frac{g}{810}$$

$$36 = 2g$$

$$18 = g$$

Check.

$\frac{4}{180} \approx 0.02$, $\frac{18}{810} \approx 0.02$

The ratios are the same, so the answer checks.

State. For a trip of 810 mi, 18 gal of gas are needed.

51. ***Familiarize***. Let w = the wing width of a stork, in cm. We can use a proportion.

Translate. We translate to a proportion.

$$\begin{array}{r} \text{wing span} \rightarrow \\ \text{wing width} \rightarrow \end{array} \frac{180}{24} = \frac{200}{w} \begin{array}{l} \leftarrow \text{wing span} \\ \leftarrow \text{wing width} \end{array}$$

Carry out. We solve the proportion.

$$24w \cdot \frac{180}{24} = 24w \cdot \frac{200}{w}$$

$$180w = 4800$$

$$w = \frac{80}{3} = 26\frac{2}{3}\text{cm}$$

Check.

$\frac{180}{24} = 7.5$, $\frac{200}{26\frac{2}{3}} = 7.5$

The ratios are the same, so the answer checks.

State. The wing width of a stork is $26\frac{2}{3}$cm.

53. ***Familiarize***. Let D = the number of defective bulbs in a batch of 1430 bulbs. We can use a proportion to find D.

Translate.

$$\begin{array}{r} \text{defective bulbs} \rightarrow \\ \text{batch size} \rightarrow \end{array} \frac{8}{220} = \frac{D}{1430} \begin{array}{l} \leftarrow \text{defective bulbs} \\ \leftarrow \text{batch size} \end{array}$$

Carry out. We solve the proportion.

$$2860 \cdot \frac{8}{220} = 2860 \cdot \frac{D}{1430}$$
$$104 = 2D$$
$$52 = D$$

Check. $\frac{8}{220} = 0.0\overline{36}$, $\frac{52}{1430} = 0.0\overline{36}$

The ratios are the same, so the answer checks.

State. In a batch of 1430 bulbs, 52 defective bulbs can be expected.

55. ***Familiarize***. Let z = the number of ounces of water needed by a Bolognese. We can use a proportion to solve for z.

Translate. We translate to a proportion.

$$\begin{matrix}\text{dog weight} \longrightarrow \\ \text{water} \longrightarrow\end{matrix} \frac{8}{12} = \frac{5}{z} \begin{matrix}\longleftarrow \text{dog weight} \\ \longleftarrow \text{water}\end{matrix}$$

Carry out. We solve the proportion.

$$12z \cdot \frac{8}{12} = 12z \cdot \frac{5}{z}$$
$$8z = 60$$
$$z = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}\text{oz}$$

Check.
$\frac{8}{12} = 0.\overline{6}$, $\frac{5}{7\frac{1}{2}} = 0.\overline{6}$

The ratios are the same, so the answer checks.

State. For a 5-lb Bolognese, approximately $7\frac{1}{2}$oz of water is required per day.

57. ***Familiarize***. Let p = the number of Whale in the pod. We use a proportion to solve for p.

Translate.

$$\begin{matrix}\text{sighted} \rightarrow \\ \text{pod} \rightarrow\end{matrix} \frac{12}{27} = \frac{40}{p} \begin{matrix}\leftarrow \text{sighted} \\ \leftarrow \text{pod}\end{matrix}$$

Carry out. We solve the proportion.

$$27p \cdot \frac{12}{27} = 27p \cdot \frac{40}{p}$$
$$12p = 1080$$
$$p = 90$$

Check. $\frac{12}{27} = \frac{4}{9}$, $\frac{40}{90} = \frac{4}{p}$

The ratios are the same, so the answer checks.

State. There are 90 whales in the pod, when 40 whales are sighted.

59. ***Familiarize***. The ratio of the weight of an object on the moon to the weight of an object on Earth is 0.16 to 1.

a) We wish to find how much a 12-ton rocket would weigh on the moon.

b) We wish to find how much a 180-lb astronaut would weigh on the moon.

Translate. We translate to proportions.

a)
$$\begin{matrix}\text{Weight on the moon} \rightarrow \\ \text{Weight on Earth} \rightarrow\end{matrix} \frac{0.16}{1} = \frac{T}{12} \begin{matrix}\leftarrow \text{Weight on the moon} \\ \leftarrow \text{Weight on Earth}\end{matrix}$$

b)
$$\begin{matrix}\text{Weight on the moon} \rightarrow \\ \text{Weight on Earth} \rightarrow\end{matrix} \frac{0.16}{1} = \frac{P}{180} \begin{matrix}\leftarrow \text{Weight on the moon} \\ \leftarrow \text{Weight on Earth}\end{matrix}$$

Carry out. We solve each proportion.

a) $\frac{0.16}{1} = \frac{T}{12}$ b) $\frac{0.16}{1} = \frac{P}{180}$

$12(0.16) = T$ $180(0.16) = P$

$1.92 = T$ $28.8 = P$

Check. $\frac{0.16}{1} = 0.16$, $\frac{1.92}{12} = 0.16$, $\frac{28.8}{180} = 0.16$

The ratios are the same, so the answer checks.

State.

a) A 12-ton rocket would weigh 1.92 tons on the moon.

b) A 180-lb astronaut would weigh 28.8 lb on the moon.

61. *Writing Exercise*. No. If the workers work at different rates, two workers will complete a task in more than half the time of the faster person working alone but in less than half the slower person's time. This is illustrated in Example 1.

63. Graph: $y = 2x - 6$.

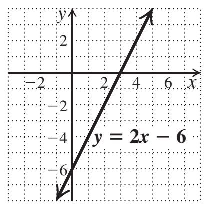
We select some x-values and compute y-values.

If $x = 1$, then $y = 2 \cdot 1 - 6 = -4$.

If $x = 3$, then $y = 2 \cdot 3 - 6 = 0$.

If $x = 5$, then $y = 2 \cdot 5 - 6 = 4$.

x	y	(x, y)
1	-4	$(1, -4)$
3	0	$(3, 0)$
5	4	$(5, 4)$



65. Graph: $3x + 2y = 12$.

We can replace either variable with a number and then calculate the other coordinate. We will find the intercepts and one other point.

If $y = 0$, we have:

$$3x + 2 \cdot 0 = 12$$
$$3x = 12$$
$$x = 4$$

The x-intercept is $(4, 0)$.

If $x = 0$, we have:

$$3 \cdot 0 + 2y = 12$$
$$2y = 12$$
$$y = 6$$

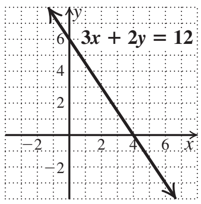
The y-intercept is $(0, 6)$.

If $y = -3$, we have:

$$3x + 2(-3) = 12$$
$$3x - 6 = 12$$
$$3x = 18$$
$$x = 6$$

The point $(6, -3)$ is on the graph.

We plot these points and draw a line through them.



67. Graph: $y = -\dfrac{3}{4}x + 2$

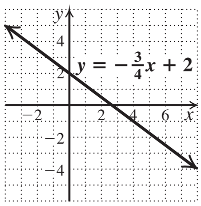
We select some x-values and compute y-values. We use multiples of 4 to avoid fractions.

If $x = -4$, then $y = -\dfrac{3}{4}(-4) + 2 = 5$.

If $x = 0$, then $y = -\dfrac{3}{4} \cdot 0 + 2 = 2$.

If $x = 4$, then $y = -\dfrac{3}{4} \cdot 4 + 2 = -1$.

x	y	(x, y)
-4	5	$(-4, 5)$
0	2	$(0, 2)$
4	-1	$(4, -1)$



69. *Writing Exercise.* Yes; if the steamrollers working together take more than half as long as the slower steam-roller would working alone, then they do more than one entire job. That is, in half the time it takes the slower steamroller to do the job alone, the faster steamroller can do more than half of the job alone, and together they do more than $\frac{1}{2} + \frac{1}{2}$, or 1 entire job in that time.

71. ***Familiarize***. If the drainage gate is closed, $\frac{1}{9}$ of the bog is filled in 1 hr. If the bog is not being filled, $\frac{1}{11}$ of the bog is drained in 1 hr. If the bog is being filled with the drainage gate left open, $\frac{1}{9} - \frac{1}{11}$ of the bog is filled in 1 hr. Let t = the time it takes to fill the bog with the drainage gate left open.

Translate. We want to find t such that $t\left(\frac{1}{9} - \frac{1}{11}\right) = 1$, or $\frac{t}{9} - \frac{t}{11} = 1$.

Carry out. We solve the equation. First we multiply by the LCD, 99.

Check. In $\frac{99}{2}$ hr, we have

$$\frac{99}{2}\left(\frac{1}{9} - \frac{1}{11}\right) = \frac{11}{2} - \frac{9}{2} = \frac{2}{2} = 1$$

full bog.

State. It will take $\frac{99}{2}$, or $49\frac{1}{2}$ hr, to fill the bog.

73. ***Familiarize***. Let p = the number of people per hour moved by the 60 cm-wide escalator. Then $2p$ = the number of people per hour moved by the 100 cm-wide escalator. We convert 1575 people per 14 minutes to people per hour: $\frac{1575 \text{ people}}{14 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 6750$ people/hr

Translate. We use the information that together the escalators move 6750 people per hour to write an equation. $p + 2p = 6750$

Carry out. We solve the equation.

$$p + 2p = 6750$$
$$3p = 6750$$
$$p = 2250$$

Check. If the 60 cm-wide escalator moves 2250 people per hour, then the 100 cm-wide escalator moves , or 4500 people per hour. Together, they move , or 6750 people per hour. The answer checks.

State. The 60 cm-wide escalator moves 2250 people per hour.

75. ***Familiarize***. Let d = the distance, in miles, the paddle-boat can cruise upriver before it is time to turn around. The boat's speed upriver is 12 - 5, or 7 mph, and its speed downriver is 12 + 5, or 17 mph. We organize the information in a table.

	Distance	Speed	Time
Upriver	d	7	t_1
Downriver	d	17	t_2

Translate. Using the formula Time = Distance/Rate we see that $t_1 = \frac{d}{7}$ and $t_2 = \frac{d}{17}$. The time upriver and back is 3 hr, so $t_1 + t_2 = 3$, or $\frac{d}{7} + \frac{d}{17} = 3$.

Carry out. We solve the equation.

$$7 \cdot 17\left(\frac{d}{7} + \frac{d}{17}\right) = 7 \cdot 17 \cdot 3$$
$$17d + 7d = 357$$
$$24d = 357$$
$$d = \frac{119}{8}$$

Check. Traveling $\frac{119}{8}$ mi upriver at a speed of 7 mph takes $\frac{119/8}{7} = \frac{17}{8}$ hr. Traveling $\frac{119}{8}$ mi downriver at a speed of 17 mph takes $\frac{119/8}{17} = \frac{7}{8}$ hr. The total time is $\frac{17}{8} + \frac{7}{8} = \frac{24}{8} = 3$ hr. The answer checks.

State. The pilot can go $\frac{119}{8}$, or $14\frac{7}{8}$ mi upriver before it is time to turn around.

77. We will begin by finding how long it will take Alma and Kevin to grade a batch of exams, working together. Then we will find what percentage of the job was done by Alma.

Solve: $\left(\frac{1}{3}+\frac{1}{4}\right)t = 1$, or $\frac{7}{12}\cdot t = 1$

$t = \frac{12}{7}$ hr

Now, since Alma can do the job alone in 3 hr, she does $\frac{1}{3}$ of the job in 1 hr and in $\frac{12}{7}$ hr she does $\frac{12}{7}\cdot\frac{1}{3} \approx 0.57 \approx 57\%$ of the job.

79. ***Familiarize***. Let t represent the time it takes the printers to print 500 pages working together.

Translate. The faster machine can print 500 pages in 40 min, and it takes the slower printer 50 min to do the same job. Then we have $\frac{t}{40} + \frac{t}{50} = 1$.

Carry out. We solve the equation.

$$\frac{t}{40} + \frac{t}{50} = 1, \text{ LCD is } 200$$
$$200\left(\frac{t}{40} + \frac{t}{50}\right) = 200\cdot 1$$
$$5t + 4t = 200\cdot 1$$
$$9t = 200$$
$$t = \frac{200}{9}$$

In $\frac{200}{9}$ min, the faster printer does $\frac{200/9}{40}$, or $\frac{200}{9}\cdot\frac{1}{40}$, or $\frac{5}{9}$ of the job. Then starting at page 1, it would print $\frac{5}{9}\cdot 500$, or $277\frac{7}{9}$ pages. Thus, in $\frac{200}{9}$ min, the two machines will meet on page 278.

Check. We can check to see that the slower machine is also printing page 278 after $\frac{200}{9}$ min. In $\frac{200}{9}$ min, the slower machine does $\frac{200/9}{50}$, or $\frac{200}{9}\cdot\frac{1}{50}$, or $\frac{4}{9}$ of the job. Then it would print $\frac{4}{9}\cdot 500$, or $222\frac{2}{9}$ pages. Working backward from page 500, this machine would be on page $500 - 222\frac{2}{9}$, or $277\frac{2}{9}$. Thus, both machines are printing page 278 after $\frac{200}{9}$ min. The answer checks.

State. The two machines will meet on page 278.

81. Find a second proportion:

$$\frac{A}{B} = \frac{C}{D} \quad \text{Given}$$
$$\frac{D}{A}\cdot\frac{A}{B} = \frac{D}{A}\cdot\frac{C}{D} \quad \text{Multiplying by } \frac{D}{A}$$
$$\frac{D}{B} = \frac{C}{A}$$

Find a third proportion:

$$\frac{A}{B} = \frac{C}{D} \quad \text{Given}$$
$$\frac{B}{C}\cdot\frac{A}{B} = \frac{B}{C}\cdot\frac{C}{D} \quad \text{Multiplying by } \frac{B}{C}$$
$$\frac{A}{C} = \frac{B}{D}$$

Find a fourth proportion:

$$\frac{A}{B} = \frac{C}{D} \quad \text{Given}$$
$$\frac{DB}{AC}\cdot\frac{A}{B} = \frac{DB}{AC}\cdot\frac{C}{D} \quad \text{Multiplying by } \frac{DB}{AC}$$
$$\frac{D}{C} = \frac{B}{A}$$

83. ***Familiarize***. Let $r =$ the speed in mph Garry would have to travel for the last half of the trip in order to average a speed of 45 mph for the entire trip. We organize the information in a table.

	Distance	Speed	Time
First half	50	40	t_1
Last half	50	r	t_2

The total distance is 50 + 50, or 100 mi. The total time is $t_1 + t_2$, or $\frac{50}{40} + \frac{50}{r}$, or $\frac{5}{4} + \frac{50}{r}$. The average speed is 45 mph.

Translate.

Average speed $= \frac{\text{Totaldistance}}{\text{Totaltime}}$

$45 = \frac{100}{\frac{5}{4}+\frac{50}{r}}$

Carry out. We solve the equation.

$$45 = \frac{100}{\frac{5}{4}+\frac{50}{r}}$$
$$45 = \frac{100}{\frac{5r+200}{4r}}$$
$$45 = 100\cdot\frac{4r}{5r+200}$$
$$45 = \frac{400r}{5r+200}$$
$$(5r+200)(45) = (5r+200)\cdot\frac{400r}{5r+200}$$
$$225r + 9000 = 400r$$
$$9000 = 175r$$
$$\frac{360}{7} = r$$

Check. Traveling 50 mi at 40 mph takes $\frac{50}{40}$, or $\frac{5}{4}$ hr. Traveling 50 mi at $\frac{360}{7}$ mph takes $\frac{50}{360/7}$, or $\frac{35}{36}$ hr. Then the total time is $\frac{5}{4} + \frac{35}{36} = \frac{80}{36} = \frac{20}{9}$ hr. The average speed when traveling 100 mi for $\frac{20}{9}$ hr is $\frac{100}{20/9} = 45$ mph. The answer checks.

State. Garry would have to travel at a speed of $\frac{360}{7}$, or $51\frac{3}{7}$ mph for the last half of the trip so that the average speed for the entire trip would be 45 mph.

85. *Writing Exercise.*

$$\frac{A+B}{B} = \frac{C+D}{D}$$
$$\frac{A}{B} + \frac{B}{B} = \frac{C}{D} + \frac{D}{D}$$
$$\frac{A}{B} + 1 = \frac{C}{D} + 1$$
$$\frac{A}{B} = \frac{C}{D}$$

The equations are equivalent.

Chapter 6 Review

1. False; some rational expressions like $\frac{y^2+4}{y+2}$ cannot be simplified.

3. False; when $t=3$, then $\frac{t-3}{t^2-4}=\frac{3-3}{3^2-4}=\frac{0}{5}=0$.

5. True; see page 384 in the text.

7. False; see page 390 in the text.

9. $\frac{17}{-x^2}$

Set the denominator equal to 0 and solve for x.
$-x^2=0$
$x=0$
The expression is undefined for $x=0$.

11. $\frac{x-5}{x^2-36}$

Set the denominator equal to 0 and solve for x.

$$\begin{aligned} x^2-36&=0\\ (x+6)(x-6)&=0\\ x+6=0 &\text{ or } x-6=0\\ x=-6 &\text{ or } x=6 \end{aligned}$$

The expression is undefined for $x=-6$ and $x=6$.

13. $\frac{-6}{(t+2)^2}$

Set the denominator equal to 0 and solve for t.

$$\begin{aligned} (t+2)^2&=0\\ t+2&=0\\ t&=-2 \end{aligned}$$

The expression is undefined for $t=-2$.

15. $\frac{14x^2-x-3}{2x^2-7x+3}=\frac{(2x-1)(7x+3)}{(2x-1)(x-3)}=\frac{7x+3}{x-3}$

17. $\frac{5x^2-20y^2}{2y-x}=\frac{-5\left(4y^2-x^2\right)}{(2y-x)}$
$=\frac{-5(2y+x)(2y-x)}{(2y-x)}=-5(2y+x)$

19.

$$\begin{aligned} \frac{6y-12}{2y^2+3y-2}\cdot\frac{y^2-4}{8y-8}&=\frac{6(y-2)(y-2)(y+2)}{(2y-1)(y+2)(8)(y-1)}\\ &=\frac{3(y-2)^2}{4(2y-1)(y-1)}\cdot\frac{2(y+2)}{2(y+2)}\\ &=\frac{3(y-2)^2}{4(2y-1)(y-1)} \end{aligned}$$

21.

$$\begin{aligned} \frac{4x^4}{x^2-1}\div\frac{2x^3}{x^2-2x+1}&=\frac{4x^4}{(x+1)(x-1)}\cdot\frac{(x-1)(x-1)}{2x^3}\\ &=\frac{2x(x-1)}{x+1}\cdot\frac{2x^3(x-1)}{2x^3(x-1)}\\ &=\frac{2x(x-1)}{x+1} \end{aligned}$$

23.

$$\begin{aligned} \left(t^2+3t-4\right)\div\frac{t^2-1}{t+4}&=(t+4)(t-1)\cdot\frac{(t+4)}{(t+1)(t-1)}\\ &=\frac{(t+4)^2}{t+1}\cdot\frac{t-1}{t-1}\\ &=\frac{(t+4)^2}{t+1} \end{aligned}$$

25. $x^2-x=x(x-1)$
$x^5-x^3=x^3\left(x^2-1\right)=x\cdot x\cdot x\cdot(x+1)(x-1)$
$x^4=x\cdot x\cdot x\cdot x$
$\text{LCM}=x\cdot x\cdot x\cdot x\cdot(x+1)(x-1)=x^4(x+1)(x-1)$

27. $\frac{x+6}{x+3}+\frac{9-4x}{x+3}=\frac{x+6+9-4x}{x+3}=\frac{-3x+15}{x+3}$

29.

$$\begin{aligned} \frac{3x-1}{2x}-\frac{x-3}{x}&=\frac{3x-1}{2x}-\frac{x-3}{x}\cdot\frac{2}{2}\\ \text{LCD }&=2x\\ &=\frac{3x-1-2(x-3)}{2x}\\ &=\frac{3x-1-2x+6}{2x}\\ &=\frac{x+5}{2x} \end{aligned}$$

31.

$$\begin{aligned} \frac{y^2}{y-2}+\frac{6y-8}{2-y}&=\frac{y^2}{y-2}+\frac{8-6y}{y-2}\\ &=\frac{y^2-6y+8}{y-2}\\ &=\frac{(y-4)(y-2)}{y-2}\\ &=y-4 \end{aligned}$$

33.

$$\begin{aligned} \frac{d^2}{d-2}+\frac{4}{2-d}&=\frac{d^2}{d-2}-\frac{4}{d-2}\\ &=\frac{d^2-4}{d-2}\\ &=\frac{(d+2)(d-2)}{d-2}\\ &=d+2 \end{aligned}$$

35.

$$\begin{aligned} &\frac{3x}{x+2}-\frac{x}{x-2}+\frac{8}{x^2-4}\\ &=\frac{3x}{x+2}\cdot\frac{x-2}{x-2}-\frac{x}{x-2}\cdot\frac{x+2}{x+2}+\frac{8}{(x+2)(x-2)}\\ &\text{LCD }=(x+2)(x-2)\\ &=\frac{3x^2-6x-x^2-2x+8}{(x+2)(x-2)}\\ &=\frac{2x^2-8x+8}{(x+2)(x-2)}=\frac{2\left(x^2-4x+4\right)}{(x+2)(x-2)}\\ &=\frac{2(x-2)(x-2)}{(x+2)(x-2)}\\ &=\frac{2(x-2)}{x+2} \end{aligned}$$

37.

$$\begin{aligned} \frac{\frac{1}{z}+1}{\frac{1}{z^2}-1}&=\frac{\frac{1}{z}+1}{\frac{1}{z^2}-1}\cdot\frac{z^2}{z^2}\\ \text{LCD }&=z^2\\ &=\frac{z+z^2}{1-z^2}\\ &=\frac{z(1+z)}{(1-z)(1+z)}\\ &=\frac{z}{1-z} \end{aligned}$$

39. $$\frac{\frac{c}{d}-\frac{d}{c}}{\frac{1}{c}+\frac{1}{d}} = \frac{\frac{c}{d}-\frac{d}{c}}{\frac{1}{c}+\frac{1}{d}}\cdot\frac{cd}{cd}$$
$$\text{LCD} = cd$$
$$= \frac{c^2-d^2}{d+c}$$
$$= \frac{(c-d)(c+d)}{c+d}$$
$$= c-d$$

41. $\dfrac{3}{x+4} = \dfrac{1}{x-1}$ Note $x \neq 1, -4$
$$(x-1)(x+4)\frac{3}{x+4} = (x-1)(x+4)\frac{1}{x-1}$$
$$\text{LCD} = (x-1)(x+4)$$
$$3(x-1) = x+4$$
$$3x-3 = x+4$$
$$3x = x+7$$
$$2x = 7$$
$$x = \frac{7}{2}$$

43. ***Familiarize***. The job takes Jackson 12 hours working alone and Charis 9 hours working alone. Then in 1 hour Jackson does $\frac{1}{12}$ of the job and Charis does $\frac{1}{9}$ of the job. Working together, they can do $\frac{1}{9}+\frac{1}{12}$, or $\frac{7}{36}$ of the job in 1 hour.

Translate. If they work together t hours, then Jackson does $t\left(\frac{1}{9}\right)$ of the job and Charis does $t\left(\frac{1}{12}\right)$ of the job. We want some number t such that
$$\left(\frac{1}{9}+\frac{1}{12}\right)t = 1, \text{ or } \frac{7}{36}t = 1.$$

Carry out. We solve the equation.
$$\frac{7}{36}t = 1$$
$$\frac{36}{7}\cdot\frac{7}{36}t = \frac{36}{7}\cdot 1$$
$$t = \frac{36}{7} \text{ or } 5\frac{1}{7}$$

Check. The check can be done by repeating the computation.

State. Working together, it takes them $5\frac{1}{7}$ hrs to complete the job.

45. ***Familiarize***. Let r = the rate by rail, in km/hr. The car's speed is $r+15$. Also set t = the time, in hours that the train and car travel. We organize the information in a table.

	Distance	Speed	Time
train	60	r	t
car	70	$r+15$	t

Translate. We can replace the t's in the table above using the formula $r = d/t$.

	Distance	Speed	Time
train	60	r	$\frac{60}{r}$
car	70	$r+15$	$\frac{70}{r+15}$

Since the times are the same for both vehicles, we have the equation
$$\frac{60}{r} = \frac{70}{r+15}.$$

Carry out. We multiply by the LCD, $r(r+15)$.
$$r(r+15)\cdot\frac{60}{r} = r(r+15)\cdot\frac{70}{r+15}$$
$$60(r+15) = 70r$$
$$60r+900 = 70r$$
$$900 = 10r$$
$$90 = r$$

If $r = 90$, then $r+15 = 105$.

Check. If the train's speed is 90 km/hr and the car's speed is 105 km/hr, then the car's speed is 15 km/hr faster than the train. The car's time is 105/70 or 1.5hr. The train's time is 90/60, or 1.5hr. Since the speeds are the same, the answer checks.

State. The car travels at 105 km/hr and the train travels at 90 km/hr.

47. ***Familiarize***. The ratio of seal tagged to the total number of seal in the harbor, T, is $\frac{33}{T}$. Of the 40 seals caught later, there were 24 are tagged. The ratio of tagged seals to seals caught is $\frac{24}{40}$.

Translate. We translate to a proportion.

Seals originally tagged	$\rightarrow 33$	$= 24 \leftarrow$	Tagged seals caught later
Seals in harbor	$\rightarrow T$	$40 \leftarrow$	Seals caught later

$$\frac{33}{T} = \frac{24}{40}$$

Carry out. We solve the proportion.
$$40T\cdot\frac{33}{T} = 40T\cdot\frac{24}{40}$$
$$1320 = 24T$$
$$55 = T$$

Check.
$$\frac{33}{55} = 0.6, \ \frac{24}{40} = 0.6$$
The ratios are the same, so the answer checks.

State. We estimate that there are 55 seals in the harbor.

49. *Writing Exercise.* The LCM of denominators is used to clear fractions when simplifying complex rational expressions using the method of multiplying by the LCD, and when solving rational equations.

51. $\dfrac{2a^2+5a-3}{a^2}\cdot\dfrac{5a^3+30a^2}{2a^2+7a-4}\div\dfrac{a^2+6a}{a^2+7a+12}$

$=\dfrac{(2a-1)(a+3)}{a^2}\cdot\dfrac{5a^2(a+6)}{(2a-1)(a+4)}\cdot\dfrac{(a+3)(a+4)}{a(a+6)}$

$=\dfrac{5(a+3)^2}{a}$

53. $\dfrac{5(x-y)}{(x-y)(x+2y)}-\dfrac{5(x-3y)}{(x+2y)(x-3y)}$

$=\dfrac{5}{x+2y}-\dfrac{5}{x+2y}=0$

Chapter 6 Test

1. $\dfrac{2-x}{5x}$

We find the number which makes the denominator 0.

$5x=0$

$x=0$

The expression is undefined for $x=0$.

3. $\dfrac{x-7}{x^2-1}$

We find the number which makes the denominator 0.

$x^2-1=0$

$(x+1)(x-1)=0$

$x+1=0$ or $x-1=0$

$x=-1$ or $x=1$

The expression is undefined for $x=-1$ and $x=1$.

5. $\dfrac{6x^2+17x+7}{2x^2+7x+3}=\dfrac{(3x+7)(2x+1)}{(x+3)(2x+1)}=\dfrac{(3x+7)(2x+1)}{(x+3)(2x+1)}$

$=\dfrac{3x+7}{x+3}$

7. $\dfrac{25y^2-1}{9y^2-6y}\div\dfrac{5y^2+9y-2}{3y^2+y-2}$

$=\dfrac{25y^2-1}{9y^2-6y}\cdot\dfrac{3y^2+y-2}{5y^2+9y-2}$

$=\dfrac{(5y+1)(5y-1)}{3y(3y-2)}\cdot\dfrac{(3y-2)(y+1)}{(5y-1)(y+2)}$

$=\dfrac{(5y+1)(5y-1)(3y-2)(y+1)}{3y(3y-2)(5y-1)(y+2)}$

$=\dfrac{(5y+1)(y+1)}{3y(y+2)}$

9. $(x^2+6x+9)\cdot\dfrac{(x-3)^2}{x^2-9}$

$=\dfrac{(x+3)(x+3)}{1}\cdot\dfrac{(x-3)(x-3)}{(x+3)(x-3)}$

$=\dfrac{(x+3)(x+3)(x-3)(x-3)}{(x+3)(x-3)}$

$=(x+3)(x-3)$

11. $\dfrac{2+x}{x^3}+\dfrac{7-4x}{x^3}=\dfrac{2+x+7-4x}{x^3}=\dfrac{-3x+9}{x^3}$

13. $\dfrac{2x-4}{x-3}+\dfrac{x-1}{3-x}=\dfrac{2x-4}{-1(3-x)}+\dfrac{x-1}{-1(3-x)}$

$=\dfrac{-(2x-4)}{3-x}+\dfrac{x-1}{3-x}$

$=\dfrac{-2x+4+x-1}{3-x}$

$=\dfrac{3-x}{3-x}$

$=1$

15. $\dfrac{7}{t-2}+\dfrac{4}{t}$ LCD is $t(t-2)$

$=\dfrac{7}{t-2}\cdot\dfrac{t}{t}+\dfrac{4}{t}\cdot\dfrac{t-2}{t-2}$

$=\dfrac{7t}{t(t-2)}+\dfrac{4(t-2)}{t(t-2)}$

$=\dfrac{7t+4t-8}{t(t-2)}$

$=\dfrac{11t-8}{t(t-2)}$

17. $\dfrac{1}{x-1}+\dfrac{4}{x^2-1}-\dfrac{2}{x^2-2x+1}=$

$\dfrac{1}{x-1}+\dfrac{4}{(x+1)(x-1)}-\dfrac{2}{(x-1)(x-1)}$

LCD is $(x-1)(x-1)(x+1)$

$=\dfrac{1}{x-1}\cdot\dfrac{(x-1)(x+1)}{(x-1)(x+1)}+$

$\dfrac{4}{(x-1)(x+1)}\cdot\dfrac{x-1}{x-1}-\dfrac{2(x+1)}{(x+1)(x-1)^2}$

$=\dfrac{(x-1)(x+1)}{(x+1)(x-1)^2}+\dfrac{4(x-1)}{(x+1)(x-1)^2}-\dfrac{2(x+1)}{(x+1)(x-1)^2}$

$=\dfrac{(x-1)(x+1)+4(x-1)-2(x+1)}{(x+1)(x-1)^2}$

$=\dfrac{x^2-1+4x-4-2x-2}{(x+1)(x-1)^2}$

$=\dfrac{x^2+2x-7}{(x+1)(x-1)^2}$

19. $\dfrac{\dfrac{x}{8}-\dfrac{8}{x}}{\dfrac{1}{8}+\dfrac{1}{x}}$ LCD is $8x$

$=\dfrac{8x}{8x}\cdot\dfrac{\dfrac{x}{8}-\dfrac{8}{x}}{\dfrac{1}{8}+\dfrac{1}{x}}=\dfrac{\dfrac{8x^2}{8}-\dfrac{64x}{x}}{\dfrac{8x}{8}+\dfrac{8x}{x}}$

$=\dfrac{x^2-64}{x+8}=\dfrac{(x+8)(x-8)}{x+8}=\dfrac{(x+8)(x-8)}{x+8}$

$=x-8$

21. To avoid division by 0, we must have $x\neq 0$ and $x-2\neq 0$, or $x\neq 0$ and $x\neq 2$.

$\dfrac{15}{x}-\dfrac{15}{x-2}=-2$ LCD $=x(x-2)$

$$x(x-2)\left(\frac{15}{x}-\frac{15}{x-2}\right)=x(x-2)(-2)$$
$$15(x-2)-15x=-2x(x-2)$$
$$15x-30-15x=-2x^2+4x$$
$$2x^2-4x-30=0$$
$$2(x^2-2x-15)=0$$
$$2(x-5)(x+3)=0$$

$x-5=0$ or $x+3=0$
$x=5$ or $x=-3$
The solutions are -3 and 5.

23. ***Familiarize.*** Burning 320 calories corresponds to walking 4 mi, and we wish to find the number of miles m that correspond to burning 100 calories. We can use a proportion.

Translate.
calories burned → $\frac{320}{4}=\frac{100}{m}$ ← calories burned
miles walked → ← miles walked

Carry out. We solve the proportion.

$$4m\cdot\frac{320}{4}=4m\cdot\frac{100}{m}$$
$$320m=400$$
$$m=\frac{5}{4}=1\frac{1}{4}$$

Check. $\frac{320}{4}=80,\ \frac{100}{5/4}=80$

The ratios are the same sot he answer checks.
State. Walking $1\frac{1}{4}$ mi corresponds to burning 100 calories.

25. ***Familiarize.*** Let $t=$ the number of hours it would take Rema to mulch the flower beds, working alone. Then $t+6$ = the number of hours it would take Perez, working alone. Note that $2\frac{6}{7}=\frac{20}{7}$ In $\frac{20}{7}$ hr Rema does $\frac{20}{7}\cdot\frac{1}{t}$ of the job and Perez does $\frac{20}{7}\cdot\frac{1}{t+6}$ of the job, and together they do 1 complete job.

Translate. We use the information above to write an equation. $\frac{20}{7}\cdot\frac{1}{t}+\frac{20}{7}\cdot\frac{1}{t+6}=1$

Carry out. We solve the equation.

$$\frac{20}{7}\cdot\frac{1}{t}+\frac{20}{7}\cdot\frac{1}{t+6}=1,\ \text{LCD}=7t(t+6)$$
$$7t(t+6)\cdot\left(\frac{20}{7}\cdot\frac{1}{t}+\frac{20}{7}\cdot\frac{1}{t+6}\right)=7t(t+6)\cdot 1$$
$$20(t+6)+20t=7t(t+6)$$
$$20t+120+20t=7t^2+42t$$
$$0=7t^2+2x-120$$
$$0=(7t+30)(t-4)$$

$7t+30=0$ or $t-4=0$
$t=-\frac{30}{7}$ or $t=4$

Check. Time cannot be negative in this application, so we check only 4. If Rema can mulch the flower beds working alone in 4 hr, then it would take Perez 4 + 6, or 10 hr, working alone. In $\frac{20}{7}$ hr they would do $\frac{20}{7}\cdot\frac{1}{4}+\frac{20}{7}\cdot\frac{1}{10}=\frac{5}{7}+\frac{2}{7}=1$ complete job. The answer checks.

State. It would take Rema 4 hr to mulch the flower beds working alone, and it would take Perez 10 hr working alone.

27. ***Familiarize.*** Let $x=$ the number. Then $-\frac{1}{x}$ is the opposite of the numbers reciprocal.

Translate. The square of the number, x^2 is equivalent to $-\frac{1}{x}$, so we write a proportion. $x^2=-\frac{1}{x}$

Carry out. We solve the equation.

$$x\cdot x^2=x\cdot\left(-\frac{1}{x}\right)$$
$$x^3=-1$$
$$x=\sqrt[3]{-1}=-1$$

Check. $(-1)^2=1$, $-\frac{1}{-1}=1$, so the ratios are equivalent.

State. The number is -1.