Solving equations and inequalities is a recurring theme in much of mathematics. In this chapter, we will study some of the principles used to solve equations and inequalities. We will then use equations and inequalities to solve applied problems.

2.1 Solving Equations

Equations and Solutions ▪ The Addition Principle ▪ The Multiplication Principle ▪ Selecting the Correct Approach

Solving equations is essential for problem solving in algebra. In this section, we study two of the most important principles used for this task.

Equations and Solutions

We have already seen that an equation is a number sentence stating that the expressions on either side of the equals sign represent the same number. Some equations, like $3 + 2 = 5$ or $2x + 6 = 2(x + 3)$, are *always* true and some, like $3 + 2 = 6$ or $x + 2 = x + 3$, are *never* true. In this text, we will concentrate on equations like $x + 6 = 13$ or $7x = 141$ that are *sometimes* true, depending on the replacement value for the variable.

Solution of an Equation

Any replacement for the variable that makes an equation true is called a *solution* of the equation. To *solve* an equation means to find all of its solutions.

To determine whether a number is a solution, we substitute that number for the variable throughout the equation. If the values on both sides of the equals sign are the same, then the number that was substituted is a solution.

EXAMPLE 1 Determine whether 7 is a solution of $x + 6 = 13$.

SOLUTION We have

$$\begin{array}{c|ll} x + 6 & 13 & \text{Writing the equation} \\ \hline 7 + 6 & 13 & \text{Substituting 7 for } x \\ 13 & \stackrel{?}{=} 13 \quad \text{TRUE} & 13 = 13 \text{ is a true statement.} \end{array}$$

Since the left-hand side and the right-hand side are the same, 7 is a solution.

CAUTION! Note that in Example 1, the solution is 7, not 13.

EXAMPLE 2 Determine whether $\frac{2}{3}$ is a solution of $156x = 117$.

SOLUTION We have

$156x = 117$		Writing the equation
$156\left(\frac{2}{3}\right) \mid 117$		Substituting $\frac{2}{3}$ for x
$104 \stackrel{?}{=} 117$	FALSE	The statement $104 = 117$ is false.

Since the left-hand side and the right-hand side differ, $\frac{2}{3}$ is not a solution.

The Addition Principle

Consider the equation

$$x = 7.$$

We can easily see that the solution of this equation is 7. Replacing x with 7, we get

$$7 = 7, \quad \text{which is true.}$$

Now consider the equation

$$x + 6 = 13.$$

In Example 1, we found that the solution of $x + 6 = 13$ is also 7. Although the solution of $x = 7$ may seem more obvious, because $x + 6 = 13$ and $x = 7$ have identical solutions, the equations are said to be **equivalent**.

STUDENT NOTES

Be sure to remember the difference between an expression and an equation. For example, $5a - 10$ and $5(a - 2)$ are *equivalent expressions* because they represent the same value for all replacements for a. The *equations* $5a = 10$ and $a = 2$ are *equivalent* because they have the same solution, 2.

Equivalent Equations

Equations with the same solutions are called *equivalent equations*.

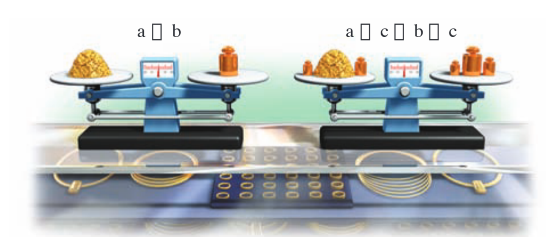
There are principles that enable us to begin with one equation and end up with an equivalent equation, like $x = 7$, for which the solution is obvious. One such principle concerns addition. The equation $a = b$ says that a and b stand for the same number. Suppose this is true, and some number c is added to a. We get the same result if we add c to b, because a and b are the same number.

The Addition Principle

For any real numbers a, b, and c,

$$a = b \quad \text{is equivalent to} \quad a + c = b + c.$$

To visualize the addition principle, consider a balance similar to one a jeweler might use. When the two sides of a balance hold equal weight, the balance is level. If weight is then added or removed, equally, on both sides, the balance will remain level.



When using the addition principle, we often say that we "add the same number to both sides of an equation." We can also "subtract the same number from both sides," since subtraction can be regarded as the addition of an opposite.

EXAMPLE 3

Solve: $x + 5 = -7$.

SOLUTION We can add any number we like to both sides. Since -5 is the opposite, or additive inverse, of 5, we add -5 to each side:

$$x + 5 = -7$$

$$x + 5 - 5 = -7 - 5$$ Using the addition principle: adding -5 to both sides or subtracting 5 from both sides

$$x + 0 = -12$$ Simplifying; $x + 5 - 5 = x + 5 + (-5) = x + 0$

$$x = -12.$$ Using the identity property of 0

The equation $x = -12$ is equivalent to the equation $x + 5 = -7$ by the addition principle, so the solution of $x = -12$ is the solution of $x + 5 = -7$.

It is obvious that the solution of $x = -12$ is the number -12. To check the answer in the original equation, we substitute.

Check:

$$\begin{array}{c|c} x + 5 & -7 \\ \hline -12 + 5 & -7 \\ -7 \stackrel{?}{=} -7 & \end{array}$$ TRUE $-7 = -7$ is true.

The solution of the original equation is -12.

TRY EXERCISE 11

In Example 3, note that because we added the *opposite*, or *additive inverse*, of 5, the left side of the equation simplified to x plus the *additive identity*, 0, or simply x. These steps effectively replaced the 5 on the left with a 0. To solve $x + a = b$ for x, we add $-a$ to (or subtract a from) both sides.

EXAMPLE 4

Solve: $-6.5 = y - 8.4$.

SOLUTION The variable is on the right side this time. We can isolate y by adding 8.4 to each side:

$$-6.5 = y - 8.4$$ $y - 8.4$ can be regarded as $y + (-8.4)$.

$$-6.5 + 8.4 = y - 8.4 + 8.4$$ Using the addition principle: Adding 8.4 to both sides "eliminates" -8.4 on the right side.

$$1.9 = y.$$ $y - 8.4 + 8.4 = y + (-8.4) + 8.4 = y + 0 = y$

Check:

$$\begin{array}{c|c} -6.5 & y - 8.4 \\ \hline -6.5 & 1.9 - 8.4 \\ -6.5 \stackrel{?}{=} -6.5 & \end{array}$$ TRUE $-6.5 = -6.5$ is true.

The solution is 1.9.

TRY EXERCISE 15

STUDENT NOTES

We can also think of "undoing" operations in order to isolate a variable. In Example 4, we began with $y - 8.4$ on the right side. To undo the subtraction, we *add* 8.4.

Note that the equations $a = b$ and $b = a$ have the same meaning. Thus, $-6.5 = y - 8.4$ could have been rewritten as $y - 8.4 = -6.5$.

The Multiplication Principle

A second principle for solving equations concerns multiplying. Suppose a and b are equal. If a and b are multiplied by some number c, then ac and bc will also be equal.

The Multiplication Principle

For any real numbers a, b, and c, with $c \neq 0$,

$$a = b \quad \text{is equivalent to} \quad a \cdot c = b \cdot c.$$

EXAMPLE 5 Solve: $\frac{5}{4}x = 10$.

SOLUTION We can multiply both sides by any nonzero number we like. Since $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$, we decide to multiply both sides by $\frac{4}{5}$:

$$\frac{5}{4}x = 10$$

$$\frac{4}{5} \cdot \frac{5}{4}x = \frac{4}{5} \cdot 10$$ Using the multiplication principle: Multiplying both sides by $\frac{4}{5}$ "eliminates" the $\frac{5}{4}$ on the left.

$$1 \cdot x = 8$$ Simplifying

$$x = 8.$$ Using the identity property of 1

Check:

$\frac{5}{4}x = 10$	
$\frac{5}{4} \cdot 8$	10
$\frac{40}{4}$	
10	10

Think of 8 as $\frac{8}{1}$.

$10 \stackrel{?}{=} 10$ TRUE $10 = 10$ is true.

The solution is 8.

TRY EXERCISE 49

In Example 5, to get x alone, we multiplied by the *reciprocal*, or *multiplicative inverse* of $\frac{5}{4}$. We then simplified the left-hand side to x times the *multiplicative identity*, 1, or simply x. These steps effectively replaced the $\frac{5}{4}$ on the left with 1.

Because division is the same as multiplying by a reciprocal, the multiplication principle also tells us that we can "divide both sides by the same nonzero number." That is,

$$\text{if } a = b, \text{ then } \frac{1}{c} \cdot a = \frac{1}{c} \cdot b \quad \text{and} \quad \frac{a}{c} = \frac{b}{c} \quad (\text{provided } c \neq 0).$$

In a product like $3x$, the multiplier 3 is called the **coefficient**. *When the coefficient of the variable is an integer or a decimal, it is usually easiest to solve an equation by dividing on both sides. When the coefficient is in fraction notation, it is usually easiest to multiply by the reciprocal.*

EXAMPLE 6 Solve: **(a)** $-4x = 9$; **(b)** $-x = 5$; **(c)** $\frac{2y}{9} = \frac{8}{3}$.

STUDENT NOTES

In Example 6(a), we can think of undoing the multiplication $-4 \cdot x$ by *dividing* by -4.

SOLUTION

a) In $-4x = 9$, the coefficient of x is an integer, so we *divide* on both sides:

$$\frac{-4x}{-4} = \frac{9}{-4}$$ Using the multiplication principle: Dividing both sides by -4 is the same as multiplying by $-\frac{1}{4}$.

$$1 \cdot x = -\frac{9}{4}$$ Simplifying

$$x = -\frac{9}{4}.$$ Using the identity property of 1

Check:

$$\begin{array}{r|l} -4x & = 9 \\ \hline -4\left(-\frac{9}{4}\right) & 9 \\ 9 & \stackrel{?}{=} 9 \quad \text{TRUE} \end{array}$$

$9 = 9$ is true.

The solution is $-\frac{9}{4}$.

b) To solve an equation like $-x = 5$, remember that when an expression is multiplied or divided by -1, its sign is changed. Here we divide both sides by -1 to change the sign of $-x$:

$$-x = 5 \qquad \text{Note that } -x = -1 \cdot x.$$

$$\frac{-x}{-1} = \frac{5}{-1} \qquad \text{Dividing both sides by } -1. \text{ (Multiplying by } -1 \text{ would also work. Note that the reciprocal of } -1 \text{ is } -1.)$$

$$x = -5. \qquad \text{Note that } \frac{-x}{-1} \text{ is the same as } \frac{x}{1}.$$

Check:

$$\begin{array}{r|l} -x & = 5 \\ \hline -(-5) & 5 \\ 5 & \stackrel{?}{=} 5 \quad \text{TRUE} \end{array}$$

$5 = 5$ is true.

The solution is -5.

c) To solve an equation like $\frac{2y}{9} = \frac{8}{3}$, we rewrite the left-hand side as $\frac{2}{9} \cdot y$ and then use the multiplication principle, multiplying by the reciprocal of $\frac{2}{9}$:

$$\frac{2y}{9} = \frac{8}{3}$$

$$\frac{2}{9} \cdot y = \frac{8}{3} \qquad \text{Rewriting } \frac{2y}{9} \text{ as } \frac{2}{9} \cdot y$$

$$\frac{9}{2} \cdot \frac{2}{9} \cdot y = \frac{9}{2} \cdot \frac{8}{3} \qquad \text{Multiplying both sides by } \frac{9}{2}$$

$$1y = \frac{3 \cdot \cancel{3} \cdot \cancel{2} \cdot 4}{\cancel{2} \cdot \cancel{3}} \qquad \text{Removing a factor equal to 1: } \frac{3 \cdot 2}{2 \cdot 3} = 1$$

$$y = 12.$$

Check:

$$\begin{array}{c|c} \frac{2y}{9} & = \frac{8}{3} \\ \hline \frac{2 \cdot 12}{9} & \frac{8}{3} \\ \frac{24}{9} & \\ \frac{8}{3} & \stackrel{?}{=} \frac{8}{3} \quad \text{TRUE} \end{array}$$

$\frac{8}{3} = \frac{8}{3}$ is true.

The solution is 12.

TRY EXERCISE 35

Selecting the Correct Approach

It is important that you be able to determine which principle should be used to solve a particular equation.

EXAMPLE 7 Solve: **(a)** $-\frac{2}{3} + x = \frac{5}{2}$; **(b)** $12.6 = 3t$.

SOLUTION

a) To undo addition of $-\frac{2}{3}$, we subtract $-\frac{2}{3}$ from both sides. Subtracting $-\frac{2}{3}$ is the same as adding $\frac{2}{3}$.

$$\begin{aligned} -\tfrac{2}{3} + x &= \tfrac{5}{2} \\ -\tfrac{2}{3} + x + \tfrac{2}{3} &= \tfrac{5}{2} + \tfrac{2}{3} && \text{Using the addition principle} \\ x &= \tfrac{5}{2} + \tfrac{2}{3} \\ x &= \tfrac{5}{2}\cdot\tfrac{3}{3} + \tfrac{2}{3}\cdot\tfrac{2}{2} && \text{Finding a common denominator} \\ x &= \tfrac{15}{6} + \tfrac{4}{6} \\ x &= \tfrac{19}{6} \end{aligned}$$

Check:

$$\begin{array}{c|c} \multicolumn{2}{c}{-\frac{2}{3} + x = \frac{5}{2}} \\ \hline -\frac{2}{3} + \frac{19}{6} & \frac{5}{2} \\ -\frac{4}{6} + \frac{19}{6} & \\ \frac{15}{6} & \\ \frac{5\cdot\cancel{3}}{2\cdot\cancel{3}} & \\ \frac{5}{2} \overset{?}{=} \frac{5}{2} & \text{TRUE} \end{array}$$

$-\frac{2}{3}\cdot\frac{2}{2} = -\frac{4}{6}$

Removing a factor equal to 1: $\frac{3}{3} = 1$

$\frac{5}{2} = \frac{5}{2}$ is true.

The solution is $\frac{19}{6}$.

b) To undo multiplication by 3, we either divide both sides by 3 or multiply both sides by $\frac{1}{3}$:

$$\begin{aligned} 12.6 &= 3t \\ \frac{12.6}{3} &= \frac{3t}{3} && \text{Using the multiplication principle} \\ 4.2 &= t. && \text{Simplifying} \end{aligned}$$

Check:

$$\begin{array}{c|c} \multicolumn{2}{c}{12.6 = 3t} \\ \hline 12.6 & 3(4.2) \\ 12.6 & \overset{?}{=} 12.6 \quad \text{TRUE} \end{array}$$

$12.6 = 12.6$ is true.

The solution is 4.2.

TRY EXERCISES 59 and 67

STUDY SKILLS

Seeking Help?

A variety of resources are available to help make studying easier and more enjoyable.

- **Textbook supplements.** See the preface for a description of the supplements for this textbook: the *Student's Solutions Manual*, a complete set of videos on DVD, MathXL tutorial exercises on CD, and complete online courses in MathXL and MyMathLab.
- **Your college or university.** Your own college or university probably has resources to enhance your math learning: a learning lab or tutoring center, study skills workshops or group tutoring sessions tailored for the course you are taking, or a bulletin board or network where you can locate the names of experienced private tutors.
- **Your instructor.** Find out your instructor's office hours and make it a point to visit when you need additional help. Many instructors also welcome student e-mail.

2.1 EXERCISE SET

For Extra Help MyMathLab MathXL PRACTICE WATCH DOWNLOAD

Concept Reinforcement *For each of Exercises 1–6, match the statement with the most appropriate choice from the column on the right.*

1. ____ The equations $x + 3 = 7$ and $6x = 24$
2. ____ The expressions $3(x - 2)$ and $3x - 6$
3. ____ A replacement that makes an equation true
4. ____ The role of 9 in $9ab$
5. ____ The principle used to solve $\frac{2}{3}\cdot x = -4$
6. ____ The principle used to solve $\frac{2}{3} + x = -4$

a) Coefficient
b) Equivalent expressions
c) Equivalent equations
d) The multiplication principle
e) The addition principle
f) Solution

For each of Exercises 7–10, match the equation with the step, from the column on the right, that would be used to solve the equation.

7. $6x = 30$ **a)** Add 6 to both sides.

8. $x + 6 = 30$ **b)** Subtract 6 from both sides.

9. $\frac{1}{6}x = 30$ **c)** Multiply both sides by 6.

10. $x - 6 = 30$ **d)** Divide both sides by 6.

To the student and the instructor: *The* TRY EXERCISES *for examples are indicated by a shaded block on the exercise number. Complete step-by-step solutions for these exercises appear online at www.pearsonhighered.com/bittingerellenbogen.*

Solve using the addition principle. Don't forget to check!

11. $x + 10 = 21$
12. $t + 9 = 47$
13. $y + 7 = -18$
14. $x + 12 = -7$
15. $-6 = y + 25$
16. $-5 = x + 8$
17. $x - 18 = 23$
18. $x - 19 = 16$
19. $12 = -7 + y$
20. $15 = -8 + z$
21. $-5 + t = -11$
22. $-6 + y = -21$
23. $r + \frac{1}{3} = \frac{8}{3}$
24. $t + \frac{3}{8} = \frac{5}{8}$
25. $x - \frac{3}{5} = -\frac{7}{10}$
26. $x - \frac{2}{3} = -\frac{5}{6}$
27. $x - \frac{5}{6} = \frac{7}{8}$
28. $y - \frac{3}{4} = \frac{5}{6}$
29. $-\frac{1}{5} + z = -\frac{1}{4}$
30. $-\frac{2}{3} + y = -\frac{3}{4}$
31. $m - 2.8 = 6.3$
32. $y - 5.3 = 8.7$
33. $-9.7 = -4.7 + y$
34. $-7.8 = 2.8 + x$

Solve using the multiplication principle. Don't forget to check!

35. $8a = 56$
36. $6x = 72$
37. $84 = 7x$
38. $45 = 9t$
39. $-x = 38$
40. $100 = -x$
Aha! **41.** $-t = -8$
42. $-68 = -r$
43. $-7x = 49$
44. $-4x = 36$
45. $-1.3a = -10.4$
46. $-3.4t = -20.4$
47. $\frac{y}{8} = 11$
48. $\frac{a}{4} = 13$
49. $\frac{4}{5}x = 16$
50. $\frac{3}{4}x = 27$
51. $\frac{-x}{6} = 9$
52. $\frac{-t}{4} = 8$
53. $\frac{1}{9} = \frac{z}{-5}$
54. $\frac{2}{7} = \frac{x}{-3}$
Aha! **55.** $-\frac{3}{5}r = -\frac{3}{5}$
56. $-\frac{2}{5}y = -\frac{4}{15}$
57. $\frac{-3r}{2} = -\frac{27}{4}$
58. $\frac{5x}{7} = -\frac{10}{14}$

Solve. The icon 🖩 indicates an exercise designed to give practice using a calculator.

59. $4.5 + t = -3.1$
60. $\frac{3}{4}x = 18$
61. $-8.2x = 20.5$
62. $t - 7.4 = -12.9$
63. $x - 4 = -19$
64. $y - 6 = -14$
65. $t - 3 = -8$
66. $t - 9 = -8$
67. $-12x = 14$
68. $-15x = 20$
69. $48 = -\frac{3}{8}y$
70. $14 = t + 27$
71. $a - \frac{1}{6} = -\frac{2}{3}$
72. $-\frac{x}{6} = \frac{2}{9}$
73. $-24 = \frac{8x}{5}$
74. $\frac{1}{5} + y = -\frac{3}{10}$
75. $-\frac{4}{3}t = -12$
76. $\frac{17}{35} = -x$
🖩 **77.** $-483.297 = -794.053 + t$
🖩 **78.** $-0.2344x = 2028.732$

79. When solving an equation, how do you determine what number to add, subtract, multiply, or divide by on both sides of that equation?

80. What is the difference between equivalent expressions and equivalent equations?

Skill Review

To prepare for Section 2.2, review the rules for order of operations (Section 1.8).

Simplify. [1.8]

81. $3 \cdot 4 - 18$

82. $14 - 2(7 - 1)$

83. $16 \div (2 - 3 \cdot 2) + 5$

84. $12 - 5 \cdot 2^3 + 4 \cdot 3$

Synthesis

85. To solve $-3.5 = 14t$, Anita adds 3.5 to both sides. Will this form an equivalent equation? Will it help solve the equation? Explain.

86. Explain why it is not necessary to state a subtraction principle: For any real numbers a, b, and c, $a = b$ is equivalent to $a - c = b - c$.

Solve for x. Assume a, c, m $\neq 0$.

87. $mx = 11.6m$

88. $x - 4 + a = a$

89. $cx + 5c = 7c$

90. $c \cdot \frac{21}{a} = \frac{7cx}{2a}$

91. $7 + |x| = 30$

92. $ax - 3a = 5a$

93. If $t - 3590 = 1820$, find $t + 3590$.

94. If $n + 268 = 124$, find $n - 268$.

95. Lydia makes a calculation and gets an answer of 22.5. On the last step, she multiplies by 0.3 when she should have divided by 0.3. What should the correct answer be?

96. Are the equations $x = 5$ and $x^2 = 25$ equivalent? Why or why not?