2.2 Using the Principles Together

Applying Both Principles ■ Combining Like Terms ■ Clearing Fractions and Decimals ■ Contradictions and Identities

An important strategy for solving new problems is to find a way to make a new problem look like a problem that we already know how to solve. This is precisely the approach taken in this section. You will find that the last steps of the examples in this section are nearly identical to the steps used for solving the equations of Section 2.1. What is new in this section appears in the early steps of each example.

Applying Both Principles

The addition and multiplication principles, along with the laws discussed in Chapter 1, are our tools for solving equations. In this section, we will find that the sequence and manner in which these tools are used is especially important.

EXAMPLE 1 Solve: $5 + 3x = 17$.

SOLUTION Were we to evaluate $5 + 3x$, the rules for the order of operations direct us to *first* multiply by 3 and *then* add 5. Because of this, we can isolate $3x$ and then x by reversing these operations: We first subtract 5 from both sides and then divide both sides by 3. Our goal is an equivalent equation of the form $x = a$.

	$5 + 3x = 17$	
	$5 + 3x - 5 = 17 - 5$	Using the addition principle: subtracting 5 from both sides (adding -5)
	$5 + (-5) + 3x = 12$	Using a commutative law. Try to perform this step mentally.
Isolate the x-term.	$3x = 12$	Simplifying
	$\frac{3x}{3} = \frac{12}{3}$	Using the multiplication principle: dividing both sides by 3 (multiplying by $\frac{1}{3}$)
Isolate x.	$x = 4$	Simplifying

Check:

$$\begin{array}{c|c} 5 + 3x & 17 \\ \hline 5 + 3 \cdot 4 & 17 \\ 5 + 12 & \\ 17 \stackrel{?}{=} 17 & \text{TRUE} \end{array}$$

We use the rules for order of operations: Find the product, $3 \cdot 4$, and then add.

The solution is 4.

TRY EXERCISE 7

EXAMPLE 2

Solve: $\frac{4}{3}x - 7 = 1$.

SOLUTION In $\frac{4}{3}x - 7$, we multiply first and then subtract. To reverse these steps, we first add 7 and then either divide by $\frac{4}{3}$ or multiply by $\frac{3}{4}$.

$$\frac{4}{3}x - 7 = 1$$

$$\frac{4}{3}x - 7 + 7 = 1 + 7 \qquad \text{Adding 7 to both sides}$$

$$\frac{4}{3}x = 8$$

$$\frac{3}{4}\cdot\frac{4}{3}x = \frac{3}{4}\cdot 8 \qquad \text{Multiplying both sides by } \tfrac{3}{4}$$

$$\left.\begin{aligned} 1\cdot x &= \frac{3\cdot \not{4}\cdot 2}{\not{4}} \\ x &= 6 \end{aligned}\right\} \qquad \text{Simplifying}$$

Check:

$\frac{4}{3}x - 7 = 1$	
$\frac{4}{3}\cdot 6 - 7$	1
$8 - 7$	
$1 \overset{?}{=} 1$	TRUE

The solution is 6. TRY EXERCISE 27

STUDY SKILLS

Use the Answer Section Carefully

When using the answers listed at the back of this book, try not to "work backward" from the answer. If you frequently require two or more attempts to answer an exercise correctly, you probably need to work more carefully and/or reread the section preceding the exercise set. Remember that on quizzes and tests you have only one attempt per problem and no answer section to consult.

EXAMPLE 3

Solve: $45 - t = 13$.

SOLUTION We have

$$45 - t = 13$$

$$45 - t - 45 = 13 - 45 \qquad \text{Subtracting 45 from both sides}$$

$$\left.\begin{aligned} 45 + (-t) + (-45) &= 13 - 45 \\ 45 + (-45) + (-t) &= 13 - 45 \end{aligned}\right\} \qquad \text{Try to do these steps mentally.}$$

$$-t = -32 \qquad \text{Try to go directly to this step.}$$

$$(-1)(-t) = (-1)(-32) \qquad \text{Multiplying both sides by } -1 \text{ (Dividing by } -1 \text{ would also work.)}$$

$$t = 32.$$

Check:

$45 - t = 13$	
$45 - 32$	13
$13 \overset{?}{=} 13$	TRUE

The solution is 32. TRY EXERCISE 19

As our skills improve, certain steps can be streamlined.

EXAMPLE 4

Solve: $16.3 - 7.2y = -8.18$.

SOLUTION We have

$$
\begin{aligned}
16.3 - 7.2y &= -8.18 \\
16.3 - 7.2y - 16.3 &= -8.18 - 16.3 && \text{Subtracting 16.3 from both sides} \\
-7.2y &= -24.48 && \text{Simplifying} \\
\frac{-7.2y}{-7.2} &= \frac{-24.48}{-7.2} && \text{Dividing both sides by } -7.2 \\
y &= 3.4. && \text{Simplifying}
\end{aligned}
$$

Check:

$$
\begin{array}{r|l}
16.3 - 7.2y & -8.18 \\
\hline
16.3 - 7.2(3.4) & -8.18 \\
16.3 - 24.48 & \\
-8.18 \stackrel{?}{=} & -8.18 \quad \text{TRUE}
\end{array}
$$

The solution is 3.4.

TRY EXERCISE 23

Combining Like Terms

If like terms appear on the same side of an equation, we combine them and then solve. Should like terms appear on both sides of an equation, we can use the addition principle to rewrite all like terms on one side.

EXAMPLE 5

Solve.

a) $3x + 4x = -14$

b) $-x + 5 = -8x + 6$

c) $6x + 5 - 7x = 10 - 4x + 7$

d) $2 - 5(x + 5) = 3(x - 2) - 1$

SOLUTION

a)

$$
\begin{aligned}
3x + 4x &= -14 \\
7x &= -14 && \text{Combining like terms} \\
\frac{7x}{7} &= \frac{-14}{7} && \text{Dividing both sides by 7} \\
x &= -2 && \text{Simplifying}
\end{aligned}
$$

The check is left to the student. The solution is -2.

b) To solve $-x + 5 = -8x + 6$, we must first write only variable terms on one side and only constant terms on the other. This can be done by subtracting 5 from both sides, to get all constant terms on the right, and adding $8x$ to both sides, to get all variable terms on the left.

Isolate variable terms on one side and constant terms on the other side.

$$
\begin{aligned}
-x + 5 &= -8x + 6 \\
-x + 8x + 5 &= -8x + 8x + 6 && \text{Adding } 8x \text{ to both sides} \\
7x + 5 &= 6 && \text{Simplifying} \\
7x + 5 - 5 &= 6 - 5 && \text{Subtracting 5 from both sides} \\
7x &= 1 && \text{Combining like terms} \\
\frac{7x}{7} &= \frac{1}{7} && \text{Dividing both sides by 7} \\
x &= \frac{1}{7}
\end{aligned}
$$

The check is left to the student. The solution is $\frac{1}{7}$.

TECHNOLOGY CONNECTION

Most graphing calculators have a TABLE feature that lists the value of a variable expression for different choices of x. For example, to evaluate $6x + 5 - 7x$ for $x = 0, 1, 2, \ldots,$ we first use E to enter $6x + 5 - 7x$ as y_1. We then use F j to specify TblStart $= 0$, ΔTbl $= 1$, and select AUTO twice. By pressing F n , we can generate a table in which the value of $6x + 5 - 7x$ is listed for values of x starting at 0 and increasing by ones.

X	Y1	
0	5	
1	4	
2	3	
3	2	
4	1	
5	0	
6	−1	

X=0

1. Create the above table on your graphing calculator. Scroll up and down to extend the table.
2. Enter $10 - 4x + 7$ as y_2. Your table should now have three columns.
3. For what x-value is y_1 the same as y_2? Compare this with the solution of Example 5(c). Is this a reliable way to solve equations? Why or why not?

c)
$$\begin{aligned}
6x + 5 - 7x &= 10 - 4x + 7 \\
-x + 5 &= 17 - 4x && \text{Combining like terms within each side} \\
-x + 5 + 4x &= 17 - 4x + 4x && \text{Adding } 4x \text{ to both sides} \\
5 + 3x &= 17 && \text{Simplifying. This is identical to Example 1.} \\
3x &= 12 && \text{Subtracting 5 from both sides} \\
\frac{3x}{3} &= \frac{12}{3} && \text{Dividing both sides by 3} \\
x &= 4
\end{aligned}$$

Check:

$6x + 5 - 7x = 10 - 4x + 7$		
$6 \cdot 4 + 5 - 7 \cdot 4$	$10 - 4 \cdot 4 + 7$	
$24 + 5 - 28$	$10 - 16 + 7$	
$1 \overset{?}{=} 1$		TRUE

The student can confirm that 4 checks and is the solution.

d)
$$\begin{aligned}
2 - 5(x + 5) &= 3(x - 2) - 1 \\
2 - 5x - 25 &= 3x - 6 - 1 && \text{Using the distributive law. This is now similar to part (c) above.} \\
-5x - 23 &= 3x - 7 && \text{Combining like terms on each side} \\
-5x - 23 + 7 &= 3x \\
-23 + 7 &= 3x + 5x && \text{Adding 7 and } 5x \text{ to both sides. This isolates the } x\text{-terms on one side and the constant terms on the other.} \\
-16 &= 8x && \text{Simplifying} \\
\frac{-16}{8} &= \frac{8x}{8} && \text{Dividing both sides by 8} \\
-2 &= x && \text{This is equivalent to } x = -2.
\end{aligned}$$

The student can confirm that −2 checks and is the solution.

TRY EXERCISE 39

Clearing Fractions and Decimals

Equations are generally easier to solve when they do not contain fractions or decimals. The multiplication principle can be used to "clear" fractions or decimals, as shown here.

Clearing Fractions	Clearing Decimals
$\frac{1}{2}x + 5 = \frac{3}{4}$	$2.3x + 7 = 5.4$
$4\left(\frac{1}{2}x + 5\right) = 4 \cdot \frac{3}{4}$	$10(2.3x + 7) = 10 \cdot 5.4$
$2x + 20 = 3$	$23x + 70 = 54$

In each case, the resulting equation is equivalent to the original equation, but easier to solve.

The easiest way to clear an equation of fractions is to multiply *both sides* of the equation by the smallest, or *least*, common denominator of the fractions in the equation.

EXAMPLE 6

Solve: **(a)** $\frac{2}{3}x - \frac{1}{6} = 2x$; **(b)** $\frac{2}{5}(3x + 2) = 8$.

SOLUTION

a) We multiply both sides by 6, the least common denominator of $\frac{2}{3}$ and $\frac{1}{6}$.

$$6\left(\frac{2}{3}x - \frac{1}{6}\right) = 6 \cdot 2x \qquad \text{Multiplying both sides by 6}$$

$$6 \cdot \frac{2}{3}x - 6 \cdot \frac{1}{6} = 6 \cdot 2x$$

CAUTION! Be sure the distributive law is used to multiply *all* the terms by 6.

$$4x - 1 = 12x \qquad \text{Simplifying. Note that the fractions are cleared: } 6 \cdot \tfrac{2}{3} = 4,\ 6 \cdot \tfrac{1}{6} = 1, \text{ and } 6 \cdot 2 = 12.$$

$$-1 = 8x \qquad \text{Subtracting } 4x \text{ from both sides}$$

$$\frac{-1}{8} = \frac{8x}{8} \qquad \text{Dividing both sides by 8}$$

$$-\frac{1}{8} = x$$

The student can confirm that $-\frac{1}{8}$ checks and is the solution.

b) To solve $\frac{2}{5}(3x + 2) = 8$, we can multiply both sides by $\frac{5}{2}$ (or divide by $\frac{2}{5}$) to "undo" the multiplication by $\frac{2}{5}$ on the left side.

$$\frac{5}{2} \cdot \frac{2}{5}(3x + 2) = \frac{5}{2} \cdot 8 \qquad \text{Multiplying both sides by } \tfrac{5}{2}$$

$$3x + 2 = 20 \qquad \text{Simplifying; } \tfrac{5}{2} \cdot \tfrac{2}{5} = 1 \text{ and } \tfrac{5}{2} \cdot \tfrac{8}{1} = 20$$

$$3x = 18 \qquad \text{Subtracting 2 from both sides}$$

$$x = 6 \qquad \text{Dividing both sides by 3}$$

The student can confirm that 6 checks and is the solution.

TRY EXERCISE 69

To clear an equation of decimals, we count the greatest number of decimal places in any one number. If the greatest number of decimal places is 1, we multiply both sides by 10; if it is 2, we multiply by 100; and so on. This procedure is the same as multiplying by the least common denominator after converting the decimals to fractions.

EXAMPLE 7

Solve: $16.3 - 7.2y = -8.18$.

STUDENT NOTES

Compare the steps of Examples 4 and 7. Note that although the two approaches differ, they yield the same solution. Whenever you can use two approaches to solve a problem, try to do so, both as a check and as a valuable learning experience.

SOLUTION The greatest number of decimal places in any one number is *two*. Multiplying by 100 will clear all decimals.

$$100(16.3 - 7.2y) = 100(-8.18) \qquad \text{Multiplying both sides by 100}$$

$$100(16.3) - 100(7.2y) = 100(-8.18) \qquad \text{Using the distributive law}$$

$$1630 - 720y = -818 \qquad \text{Simplifying}$$

$$-720y = -818 - 1630 \qquad \text{Subtracting 1630 from both sides}$$

$$-720y = -2448 \qquad \text{Combining like terms}$$

$$y = \frac{-2448}{-720} \qquad \text{Dividing both sides by } -720$$

$$y = 3.4$$

In Example 4, the same solution was found without clearing decimals. Finding the same answer in two ways is a good check. The solution is 3.4.

TRY EXERCISE 75

An Equation-Solving Procedure

1. Use the multiplication principle to clear any fractions or decimals. (This is optional, but can ease computations. See Examples 6 and 7.)
2. If necessary, use the distributive law to remove parentheses. Then combine like terms on each side. (See Example 5.)
3. Use the addition principle, as needed, to isolate all variable terms on one side. Then combine like terms. (See Examples 1–7.)
4. Multiply or divide to solve for the variable, using the multiplication principle. (See Examples 1–7.)
5. Check all possible solutions in the original equation. (See Examples 1–4.)

Contradictions and Identities

All of the equations we have examined so far had a solution. Equations that are true for some values (solutions), but not for others, are called **conditional equations**. Equations that have no solution, such as $x + 1 = x + 2$, are called **contradictions**. If, when solving an equation, we obtain an equation that is false for any value of x, the equation has no solution.

EXAMPLE 8 Solve: $3x - 5 = 3(x - 2) + 4$.

SOLUTION

$$3x - 5 = 3(x - 2) + 4$$
$$3x - 5 = 3x - 6 + 4 \qquad \text{Using the distributive law}$$
$$3x - 5 = 3x - 2 \qquad \text{Combining like terms}$$
$$-3x + 3x - 5 = -3x + 3x - 2 \qquad \text{Using the addition principle}$$
$$-5 = -2$$

Since the original equation is equivalent to $-5 = -2$, which is false regardless of the choice of x, the original equation has no solution. There is no solution of $3x - 5 = 3(x - 2) + 4$. The equation is a contradiction. It is *never* true.

TRY EXERCISE 45

Some equations, like $x + 1 = x + 1$, are true for all replacements. Such an equation is called an **identity**.

EXAMPLE 9 Solve: $2x + 7 = 7(x + 1) - 5x$.

SOLUTION

$$2x + 7 = 7(x + 1) - 5x$$
$$2x + 7 = 7x + 7 - 5x \qquad \text{Using the distributive law}$$
$$2x + 7 = 2x + 7 \qquad \text{Combining like terms}$$

The equation $2x + 7 = 2x + 7$ is true regardless of the replacement for x, so all real numbers are solutions. Note that $2x + 7 = 2x + 7$ is equivalent to $2x = 2x$, $7 = 7$, or $0 = 0$. All real numbers are solutions and the equation is an identity.

TRY EXERCISE 33

2.2 EXERCISE SET

For Extra Help MyMathLab MathXL PRACTICE WATCH DOWNLOAD

Concept Reinforcement *In each of Exercises 1–6, match the equation with an equivalent equation from the column on the right that could be the next step in finding a solution.*

1. ____ $3x - 1 = 7$ a) $6x - 6 = 2$
2. ____ $4x + 5x = 12$ b) $4x + 2x = 3$
3. ____ $6(x - 1) = 2$ c) $3x = 7 + 1$
4. ____ $7x = 9$ d) $8x + 2x = 6 + 5$
5. ____ $4x = 3 - 2x$ e) $9x = 12$
6. ____ $8x - 5 = 6 - 2x$ f) $x = \frac{9}{7}$

Solve and check. Label any contradictions or identities.

7. $2x + 9 = 25$
8. $3x - 11 = 13$
9. $6z + 5 = 47$
10. $5z + 2 = 57$
11. $7t - 8 = 27$
12. $6x - 5 = 2$
13. $3x - 9 = 1$
14. $5x - 9 = 41$
15. $8z + 2 = -54$
16. $4x + 3 = -21$
17. $-37 = 9t + 8$
18. $-39 = 1 + 5t$
19. $12 - t = 16$
20. $9 - t = 21$
21. $-6z - 18 = -132$
22. $-7x - 24 = -129$
23. $5.3 + 1.2n = 1.94$
24. $6.4 - 2.5n = 2.2$
25. $32 - 7x = 11$
26. $27 - 6x = 99$
27. $\frac{3}{5}t - 1 = 8$
28. $\frac{2}{3}t - 1 = 5$
29. $6 + \frac{7}{2}x = -15$
30. $6 + \frac{5}{4}x = -4$
31. $-\frac{4a}{5} - 8 = 2$
32. $-\frac{8a}{7} - 2 = 4$
33. $4x = x + 3x$
34. $-3z + 8z = 45$
35. $4x - 6 = 6x$
36. $5x - x = x + 3x$
37. $2 - 5y = 26 - y$
38. $6x - 5 = 7 + 2x$
39. $7(2a - 1) = 21$
40. $5(3 - 3t) = 30$
41. Aha! $11 = 11(x + 1)$
42. $9 = 3(5x - 2)$
43. $2(3 + 4m) - 6 = 48$
44. $3(5 + 3m) - 8 = 7$
45. $3(x + 4) = 3(x - 1)$
46. $5(x - 7) = 3(x - 2) + 2x$
47. $2r + 8 = 6r + 10$
48. $3b - 2 = 7b + 4$
49. $6x + 3 = 2x + 3$
50. $5y + 3 = 2y + 15$
51. $5 - 2x = 3x - 7x + 25$
52. $10 - 3x = x - 2x + 40$
53. $7 + 3x - 6 = 3x + 5 - x$
54. $5 + 4x - 7 = 4x - 2 - x$
55. $4y - 4 + y + 24 = 6y + 20 - 4y$
56. $5y - 10 + y = 7y + 18 - 5y$
57. $4 + 7x = 7(x + 1)$
58. $3(t + 2) + t = 2(3 + 2t)$
59. $19 - 3(2x - 1) = 7$

60. $5(d + 4) = 7(d - 2)$

61. $7(5x - 2) = 6(6x - 1)$

62. $5(t + 1) + 8 = 3(t - 2) + 6$

63. $2(3t + 1) - 5 = t - (t + 2)$

64. $4x - (x + 6) = 5(3x - 1) + 8$

65. $2(7 - x) - 20 = 7x - 3(2 + 3x)$

66. $5(x - 7) = 3(x - 2) + 2x$

67. $19 - (2x + 3) = 2(x + 3) + x$

68. $13 - (2c + 2) = 2(c + 2) + 3c$

Clear fractions or decimals, solve, and check.

69. $\frac{2}{3} + \frac{1}{4}t = 2$

70. $-\frac{5}{6} + x = -\frac{1}{2} - \frac{2}{3}$

71. $\frac{2}{3} + 4t = 6t - \frac{2}{15}$

72. $\frac{1}{2} + 4m = 3m - \frac{5}{2}$

73. $\frac{1}{3}x + \frac{2}{5} = \frac{4}{5} + \frac{3}{5}x - \frac{2}{3}$

74. $1 - \frac{2}{3}y = \frac{9}{5} - \frac{1}{5}y + \frac{3}{5}$

75. $2.1x + 45.2 = 3.2 - 8.4x$

76. $0.91 - 0.2z = 1.23 - 0.6z$

77. $0.76 + 0.21t = 0.96t - 0.49$

78. $1.7t + 8 - 1.62t = 0.4t - 0.32 + 8$

79. $\frac{2}{5}x - \frac{3}{2}x = \frac{3}{4}x + 3$

80. $\frac{5}{16}y + \frac{3}{8}y = 2 + \frac{1}{4}y$

81. $\frac{1}{3}(2x - 1) = 7$

82. $\frac{1}{5}(4x - 1) = 7$

83. $\frac{3}{4}(3t - 4) = 15$

84. $\frac{3}{2}(2x + 5) = -\frac{15}{2}$

85. $\frac{1}{6}\left(\frac{3}{4}x - 2\right) = -\frac{1}{5}$

86. $\frac{2}{3}\left(\frac{7}{8} - 4x\right) - \frac{5}{8} = \frac{3}{8}$

87. $0.7(3x + 6) = 1.1 - (x - 3)$

88. $0.9(2x - 8) = 4 - (x + 5)$

89. $a + (a - 3) = (a + 2) - (a + 1)$

90. $0.8 - 4(b - 1) = 0.2 + 3(4 - b)$

91. Tyla solves $45 - t = 13$ (Example 3) by adding $t - 13$ to both sides. Is this approach preferable to the one used in Example 3? Why or why not?

92. Why must the rules for the order of operations be understood before solving the equations in this section?

Skill Review

To prepare for Section 2.3, review evaluating algebraic expressions (Section 1.8).

Evaluate. [1.8]

93. $3 - 5a$, for $a = 2$

94. $12 \div 4 \cdot t$, for $t = 5$

95. $7x - 2x$, for $x = -3$

96. $t(8 - 3t)$, for $t = -2$

Synthesis

97. What procedure would you use to solve an equation like $0.23x + \frac{17}{3} = -0.8 + \frac{3}{4}x$? Could your procedure be streamlined? If so, how?

98. Dave is determined to solve $3x + 4 = -11$ by first using the multiplication principle to "eliminate" the 3. How should he proceed and why?

Solve. Label any contradictions or identities.

99. $8.43x - 2.5(3.2 - 0.7x) = -3.455x + 9.04$

100. $0.008 + 9.62x - 42.8 = 0.944x + 0.0083 - x$

101. $-2[3(x - 2) + 4] = 4(5 - x) - 2x$

102. $0 = t - (-6) - (-7t)$

103. $2x(x + 5) - 3(x^2 + 2x - 1) = 9 - 5x - x^2$

104. $x(x - 4) = 3x(x + 1) - 2(x^2 + x - 5)$

105. $9 - 3x = 2(5 - 2x) - (1 - 5x)$

Aha! **106.** $[7 - 2(8 \div (-2))]x = 0$

107. $\dfrac{x}{14} - \dfrac{5x + 2}{49} = \dfrac{3x - 4}{7}$

108. $\dfrac{5x + 3}{4} + \dfrac{25}{12} = \dfrac{5 + 2x}{3}$

109. $2\{9 - 3[-2x - 4]\} = 12x + 42$

110. $-9t + 2 = 2 - 9t - 5(8 \div 4(1 + 3^4))$

COLLABORATIVE CORNER

Step-by-Step Solutions

Focus: Solving linear equations

Time: 20 minutes

Group size: 3

In general, there is more than one correct sequence of steps for solving an equation. This makes it important that you write your steps clearly and logically so that others can follow your approach.

ACTIVITY

1. Each group member should select a different one of the following equations and, on a fresh sheet of paper, perform the first step of the solution.

   4 - 31x - 32 = 7x + 612 - x2

   5 - 73x - 21x - 624 = 3x + 412x - 72 + 9

   4x - 732 + 31x - 52 + x4 = 4 - 91- 3x - 192

2. Pass the papers around so that the second and third steps of each solution are performed by the other two group members. Before writing, make sure that the previous step is correct. If a mistake is discovered, return the problem to the person who made the mistake for repairs. Continue passing the problems around until all equations have been solved.
3. Each group should reach a consensus on what the three solutions are and then compare their answers to those of other groups.