

Many applications of mathematics involve relationships among two or more quantities. An equation that represents such a relationship will use two or more letters and is known as a **formula**. Most of the letters in this book are variables, but some are constants. For example, c in $E = mc^2$ represents the speed of light.

Evaluating Formulas

EXAMPLE**1**

Event promotion. Event promoters use the formula

$$p = \frac{1.2x}{s}$$

to determine a ticket price p for an event with x dollars of expenses and s anticipated ticket sales. Grand Events expects expenses for an upcoming concert to be \$80,000 and anticipates selling 4000 tickets. What should the ticket price be?

Source: *The Indianapolis Star*, 2/27/03



SOLUTION We substitute 80,000 for x and 4000 for s in the formula and calculate p :

$$p = \frac{1.2x}{s} = \frac{1.2(80,000)}{4000} = 24.$$

The ticket price should be \$24.

TRY EXERCISE 1

Solving for a Variable

In the Northeast, the formula $B = 30a$ is used to determine the minimum furnace output B , in British thermal units (Btu's), for a well-insulated home with a square feet of flooring. Suppose that a contractor has an extra furnace and wants to determine the size of the largest (well-insulated) house in which it can be used. The contractor can substitute the amount of the furnace's output in Btu's—say, 63,000—for B , and then solve for a :

$$\begin{aligned} 63,000 &= 30a && \text{Replacing } B \text{ with } 63,000 \\ 2100 &= a. && \text{Dividing both sides by } 30 \end{aligned}$$

The home should have no more than 2100 ft² of flooring.

Were these calculations to be performed for a variety of furnaces, the contractor would find it easier to first solve $B = 30a$ for a , and *then* substitute values for B . Solving for a variable can be done in much the same way that we solved equations in Sections 2.1 and 2.2.

EXAMPLE 2

Solve for a : $B = 30a$.

SOLUTION We have

$$\begin{aligned} B &= 30a && \text{We want this letter alone.} \\ \frac{B}{30} &= a. && \text{Dividing both sides by } 30 \end{aligned}$$

The equation $a = B/30$ gives a quick, easy way to determine the floor area of the largest (well-insulated) house that a furnace supplying B Btu's could heat.

TRY EXERCISE 9

TECHNOLOGY CONNECTION

Suppose that after calculating $63,000 \div 30$, we wish to find $72,000 \div 30$. Pressing **2ND** **ENTRY** gives the following.

63000/30	2100
63000/30	█

Moving the cursor left, we can change 63,000 to 72,000 and press **ENTER**.

63000/30	2100
72000/30	2400

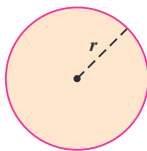
- Verify the work above and then use **2ND** **ENTRY** to find $72,000 \div 90$.

To see how solving a formula is just like solving an equation, compare the following. In (A), we solve as usual; in (B), we show steps but do not simplify; and in (C), we *cannot* simplify because a , b , and c are unknown.

<p>A. $5x + 2 = 12$</p> $5x = 12 - 2$ $5x = 10$ $x = \frac{10}{5} = 2$	<p>B. $5x + 2 = 12$</p> $5x = 12 - 2$ $x = \frac{12 - 2}{5}$	<p>C. $ax + b = c$</p> $ax = c - b$ $x = \frac{c - b}{a}$
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EXAMPLE 3

Circumference of a circle. The formula $C = 2\pi r$ gives the *circumference* C of a circle with radius r . Solve for r .



Given a radius r , we can use this equation to find a circle's circumference C .

$$C = 2\pi r$$

We want this variable alone.

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

Dividing both sides by 2π

Given a circle's circumference C , we can use this equation to find the radius r .

$$\frac{C}{2\pi} = r$$

TRY EXERCISE 13

EXAMPLE 4

Solve for y : $3x - 4y = 10$.

SOLUTION There is one term that contains y , so we begin by isolating that term on one side of the equation.

$$3x - 4y = 10$$

We want this variable alone.

$$-4y = 10 - 3x$$

Subtracting $3x$ from both sides

$$-\frac{1}{4}(-4y) = -\frac{1}{4}(10 - 3x)$$

Multiplying both sides by $-\frac{1}{4}$

$$y = -\frac{10}{4} + \frac{3}{4}x$$

Multiplying using the distributive law

$$y = -\frac{5}{2} + \frac{3}{4}x$$

Simplifying the fraction

TRY EXERCISE 33

EXAMPLE 5

Nutrition. The number of calories K needed each day by a moderately active woman who weighs w pounds, is h inches tall, and is a years old, can be estimated using the formula

$$K = 917 + 6(w + h - a).^*$$

STUDY SKILLS**Pace Yourself**

Most instructors agree that it is better for a student to study for one hour four days in a week, than to study once a week for four hours. Of course, the total weekly study time will vary from student to student. It is common to expect an average of two hours of homework for each hour of class time.

Solve for w .

SOLUTION We reverse the order in which the operations occur on the right side:

$$K = 917 + 6(w + h - a)$$

We want w alone.

$$K - 917 = 6(w + h - a)$$

Subtracting 917 from both sides

$$\frac{K - 917}{6} = w + h - a$$

Dividing both sides by 6

$$\frac{K - 917}{6} + a - h = w.$$

Adding a and subtracting h on both sides

This formula can be used to estimate a woman's weight, if we know her age, height, and caloric needs.

TRY EXERCISE 43

*Based on information from M. Parker (ed.), *She Does Math!* (Washington, D.C.: Mathematical Association of America, 1995), p. 96.

The above steps are similar to those used in Section 2.2 to solve equations. We use the addition and multiplication principles just as before. An important difference that we will see in the next example is that we will sometimes need to factor.

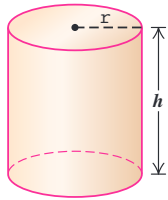
To Solve a Formula for a Given Variable

1. If the variable for which you are solving appears in a fraction, use the multiplication principle to clear fractions.
2. Isolate the term(s), with the variable for which you are solving on one side of the equation.
3. If two or more terms contain the variable for which you are solving, factor the variable out.
4. Multiply or divide to solve for the variable in question.

We can also solve for a letter that represents a constant.

EXAMPLE 6

Surface area of a right circular cylinder. The formula $A = 2\pi rh + 2\pi r^2$ gives the surface area A of a right circular cylinder of height h and radius r . Solve for π .



SOLUTION We have

$$\begin{aligned}
 A &= 2\pi rh + 2\pi r^2 && \text{We want this letter alone.} \\
 A &= \pi(2rh + 2r^2) && \text{Factoring} \\
 \frac{A}{2rh + 2r^2} &= \pi. && \text{Dividing both sides by } 2rh + 2r^2, \\
 &&& \text{or multiplying both sides by} \\
 &&& 1/(2rh + 2r^2)
 \end{aligned}$$

We can also write this as

$$\pi = \frac{A}{2rh + 2r^2}.$$

TRY EXERCISE 47

CAUTION! Had we performed the following steps in Example 6, we would not have solved for π :

$$\begin{aligned}
 A &= 2\pi rh + 2\pi r^2 && \text{We want } \pi \text{ alone.} \\
 A - 2\pi r^2 &= 2\pi rh && \text{Subtracting } 2\pi r^2 \text{ from both sides} \\
 \frac{A - 2\pi r^2}{2rh} &= \pi. && \text{Two occurrences of } \pi \\
 &&& \text{Dividing both sides by } 2rh
 \end{aligned}$$

The mathematics of each step is correct, but because π occurs on both sides of the formula, we have not solved the formula for π . Remember that the letter being solved for should be alone on one side of the equation, with no occurrence of that letter on the other side!

2.3

EXERCISE SET

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1. **Outdoor concerts.** The formula $d = 344t$ can be used to determine how far d , in meters, sound travels through room-temperature air in t seconds. At a large concert, fans near the back of the crowd experienced a 0.9-sec time lag between the time each word was pronounced on stage (as shown on large video monitors) and the time the sound reached their ears. How far were these fans from the stage?



2. **Furnace output.** Contractors in the Northeast use the formula $B = 30a$ to determine the minimum furnace output B , in British thermal units (Btu's), for a well-insulated house with a square feet of flooring. Determine the minimum furnace output for an 1800-ft² house that is well insulated.
Source: U.S. Department of Energy
3. **College enrollment.** At many colleges, the number of “full-time-equivalent” students f is given by

$$f = \frac{n}{15},$$

where n is the total number of credits for which students have enrolled in a given semester. Determine the number of full-time-equivalent students on a campus in which students registered for a total of 21,345 credits.

4. **Distance from a storm.** The formula $M = \frac{1}{5}t$ can be used to determine how far M , in miles, you are from lightning when its thunder takes t seconds to reach your ears. If it takes 10 sec for the sound of thunder to reach you after you have seen the lightning, how far away is the storm?
5. **Federal funds rate.** The Federal Reserve Board sets a target f for the federal funds rate, that is, the interest rate that banks charge each other for overnight borrowing of Federal funds. This target rate can be estimated by

$$f = 8.5 + 1.4(I - U),$$

where I is the core inflation rate over the previous 12 months and U is the seasonally adjusted unemployment rate. If core inflation is 0.025 and unemployment is 0.044, what should the federal funds rate be?

Source: Greg Mankiw, Harvard University, www.gregmankiw.blogspot.com/2006/06/what-would-alan-do.html

6. **Calorie density.** The calorie density D , in calories per ounce, of a food that contains c calories and weighs w ounces is given by

$$D = \frac{c}{w}.$$

Eight ounces of fat-free milk contains 84 calories. Find the calorie density of fat-free milk.

7. **Absorption of ibuprofen.** When 400 mg of the painkiller ibuprofen is swallowed, the number of milligrams n in the bloodstream t hours later (for $0 \leq t \leq 6$) is estimated by

$$n = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t.$$

How many milligrams of ibuprofen remain in the blood 1 hr after 400 mg has been swallowed?

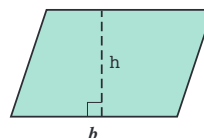
8. **Size of a league schedule.** When all n teams in a league play every other team twice, a total of N games are played, where

$$N = n^2 - n.$$

If a soccer league has 7 teams and all teams play each other twice, how many games are played?

In Exercises 9–48, solve each formula for the indicated letter.

9. $A = bh$, for b
(Area of parallelogram with base b and height h)



10. $A = bh$, for h
11. $d = rt$, for r
(A distance formula, where d is distance, r is speed, and t is time)

*Source: Nutrition Action Healthletter, March 2000, p. 9. Center for Science in the Public Interest, Suite 300; 1875 Connecticut Ave NW, Washington, D.C. 20008.

12. $d = rt$, for t

13. $I = Prt$, for P

(Simple-interest formula, where I is interest, P is principal, r is interest rate, and t is time)

14. $I = Prt$, for t

15. $H = 65 - m$, for m

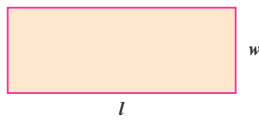
(To determine the number of heating degree days H for a day with m degrees Fahrenheit as the average temperature)

16. $d = h - 64$, for h

(To determine how many inches d above average an h -inch-tall woman is)

17. $P = 2l + 2w$, for l

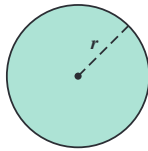
(Perimeter of a rectangle of length l and width w)



18. $P = 2l + 2w$, for w

19. $A = \pi r^2$, for π

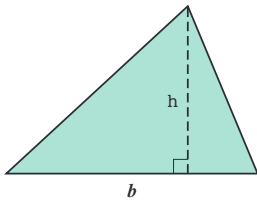
(Area of a circle with radius r)



20. $A = \pi r^2$, for r^2

21. $A = \frac{1}{2}bh$, for h

(Area of a triangle with base b and height h)



22. $A = \frac{1}{2}bh$, for b

23. $E = mc^2$, for c^2

(A relativity formula from physics)

24. $E = mc^2$, for m

25. $Q = \frac{c + d}{2}$, for d

26. $Q = \frac{p - q}{2}$, for p

27. $A = \frac{a + b + c}{3}$, for b

28. $A = \frac{a + b + c}{3}$, for c

29. $w = \frac{r}{f}$, for r

(To compute the wavelength w of a musical note with frequency f and speed of sound r)

30. $M = \frac{A}{s}$, for A

(To compute the Mach number M for speed A and speed of sound s)

31. $F = \frac{9}{5}C + 32$, for C

(To convert the Celsius temperature C to the Fahrenheit temperature F)

32. $M = \frac{5}{9}n + 18$, for n

33. $2x - y = 1$, for y

34. $3x - y = 7$, for y

35. $2x + 5y = 10$, for y

36. $3x + 2y = 12$, for y

37. $4x - 3y = 6$, for y

38. $5x - 4y = 8$, for y

39. $9x + 8y = 4$, for y

40. $x + 10y = 2$, for y

41. $3x - 5y = 8$, for y

42. $7x - 6y = 7$, for y

43. $z = 13 + 2(x + y)$, for x

44. $A = 115 + \frac{1}{2}(p + s)$, for s

45. $t = 27 - \frac{1}{4}(w - l)$, for l

46. $m = 19 - 5(x - n)$, for n

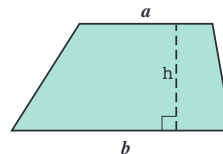
47. $A = at + bt$, for t

48. $S = rx + sx$, for x

49. **Area of a trapezoid.** The formula

$$A = \frac{1}{2}ah + \frac{1}{2}bh$$

can be used to find the area A of a trapezoid with bases a and b and height h . (Hint: First clear fractions.)



50. **Compounding interest.** The formula

$$A = P + Prt$$

is used to find the amount A in an account when simple interest is added to an investment of P dollars (see Exercise 13). Solve for P .

51. **Chess rating.** The formula

$$R = r + \frac{400(W - L)}{N}$$

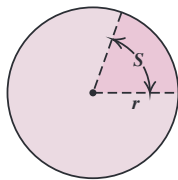
is used to establish a chess player's rating R after that player has played N games, won W of them, and lost L of them. Here r is the average rating of the opponents. Solve for L .

Source: The U.S. Chess Federation

52. **Angle measure.** The angle measure S of a sector of a circle is given by

$$S = \frac{360A}{\pi r^2},$$

where r is the radius, A is the area of the sector, and S is in degrees. Solve for r^2 .



53. Naomi has a formula that allows her to convert Celsius temperatures to Fahrenheit temperatures. She needs a formula for converting Fahrenheit temperatures to Celsius temperatures. What advice can you give her?
54. Under what circumstances would it be useful to solve $d = rt$ for r ? (See Exercise 11.)

Skill Review

Review simplifying expressions (Sections 1.6, 1.7, and 1.8).

Perform the indicated operations.

55. $-2 + 5 - (-4) - 17$ [1.6]
56. $-98 \div \frac{1}{2}$ [1.7]
- Aha! 57. $4.2(-11.75)(0)$ [1.7]
58. $(-2)^5$ [1.8]
- Simplify. [1.8]
59. $20 \div (-4) \cdot 2 - 3$
60. $5|8 - (2 - 7)|$

Synthesis

61. The equations

$$P = 2l + 2w \quad \text{and} \quad w = \frac{P}{2} - l$$

are equivalent formulas involving the perimeter P , length l , and width w of a rectangle. Devise a problem for which the second of the two formulas would be more useful.

62. While solving $2A = ah + bh$ for h , Lea writes

$$\frac{2A - ah}{b} = h. \text{ What is her mistake?}$$

63. The Harris–Benedict formula gives the number of calories K needed each day by a moderately active man who weighs w kilograms, is h centimeters tall, and is a years old as

$$K = 21.235w + 7.75h - 10.54a + 102.3.$$

If Janos is moderately active, weighs 80 kg, is 190 cm tall, and needs to consume 2852 calories a day, how old is he?

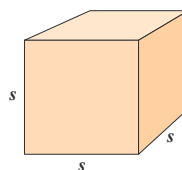
64. **Altitude and temperature.** Air temperature drops about 1° Celsius (C) for each 100-m rise above ground level, up to 12 km. If the ground level temperature is $t^\circ\text{C}$, find a formula for the temperature T at an elevation of h meters.

Source: *A Sourcebook of School Mathematics*, Mathematical Association of America, 1980

65. **Surface area of a cube.** The surface area A of a cube with side s is given by

$$A = 6s^2.$$

If a cube's surface area is 54 in^2 , find the volume of the cube.



66. **Weight of a fish.** An ancient fisherman's formula for estimating the weight of a fish is

$$w = \frac{lg^2}{800},$$

where w is the weight, in pounds, l is the length, in inches, and g is the girth (distance around the mid-section), in inches. Estimate the girth of a 700-lb yellow tuna that is 8 ft long.

- 67. Dosage size.** Clark's rule for determining the size of a particular child's medicine dosage c is

$$c = \frac{w}{a} \cdot d,$$

where w is the child's weight, in pounds, and d is the usual adult dosage for an adult weighing a pounds. Solve for a .

Source: Olsen, June Looby, et al., *Medical Dosage Calculations*. Redwood City, CA: Addison-Wesley, 1995

Solve each formula for the given letter.

68. $\frac{y}{z} \div \frac{z}{t} = 1$, for y

69. $ac = bc + d$, for c



70. $qt = r(s + t)$, for t

71. $3a = c - a(b + d)$, for a

- 72. Furnace output.** The formula

$$B = 50a$$

is used in New England to estimate the minimum furnace output B , in Btu's, for an old, poorly insulated house with a square feet of flooring. Find an equation for determining the number of Btu's saved by insulating an old house. (*Hint:* See Exercise 2.)

-  **73.** Revise the formula in Exercise 63 so that a man's weight in pounds ($2.2046 \text{ lb} = 1 \text{ kg}$) and his height in inches ($0.3937 \text{ in.} = 1 \text{ cm}$) are used.
-  **74.** Revise the formula in Example 5 so that a woman's weight in kilograms ($2.2046 \text{ lb} = 1 \text{ kg}$) and her height in centimeters ($0.3937 \text{ in.} = 1 \text{ cm}$) are used.