

Percent problems arise so frequently in everyday life that most often we are not even aware of them. In this section, we will solve some real-world percent problems. Before doing so, however, we need to review a few basics.

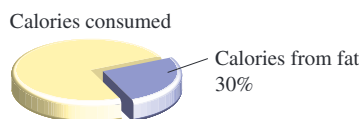
Converting Between Percent Notation and Decimal Notation

STUDY SKILLS

How Did They Get That!?

The *Student's Solutions Manual* is an excellent resource if you need additional help with an exercise in the exercise sets. It contains step-by-step solutions to the odd-numbered exercises in each exercise set.

Nutritionists recommend that no more than 30% of the calories in a person's diet come from fat. This means that of every 100 calories consumed, no more than 30 should come from fat. Thus, 30% is a ratio of 30 to 100.



The percent symbol % means “per hundred.” We can regard the percent symbol as part of a name for a number. For example,

30% is defined to mean $\frac{30}{100}$, or $30 \times \frac{1}{100}$, or 30×0.01 .

Percent Notation

$n\%$ means $\frac{n}{100}$, or $n \times \frac{1}{100}$, or $n \times 0.01$.

EXAMPLE 1

Convert to decimal notation: (a) 78%; (b) 1.3%.

SOLUTION

$$\begin{aligned} \text{a) } 78\% &= 78 \times 0.01 && \text{Replacing \% with } \times 0.01 \\ &= 0.78 \end{aligned}$$

$$\begin{aligned} \text{b) } 1.3\% &= 1.3 \times 0.01 && \text{Replacing \% with } \times 0.01 \\ &= 0.013 \end{aligned}$$

TRY EXERCISE 19

As shown above, multiplication by 0.01 simply moves the decimal point two places to the left.

To convert from percent notation to decimal notation, move the decimal point two places to the left and drop the percent symbol.

EXAMPLE 2

Convert the percent notation in the following sentence to decimal notation: Only 20% of teenagers get 8 hr of sleep a night.

Source: National Sleep Foundation

SOLUTION

$$20\% = 20.0\% \quad 0.20.0 \quad 20\% = 0.20, \text{ or simply } 0.2$$

Move the decimal point two places to the left.

TRY EXERCISE 11

The procedure used in Examples 1 and 2 can be reversed:

$$\begin{aligned} 0.38 &= 38 \times 0.01 \\ &= 38\%. && \text{Replacing } \times 0.01 \text{ with } \% \end{aligned}$$

To convert from decimal notation to percent notation, move the decimal point two places to the right and write a percent symbol.

EXAMPLE 3Convert to percent notation: (a) 1.27; (b) $\frac{1}{4}$; (c) 0.3.**SOLUTION**

a) We first move the decimal point two places to the right: 1.27.
and then write a % symbol: 127% This is the same as multiplying 1.27 by 100 and writing %.

b) Note that $\frac{1}{4} = 0.25$. We move the decimal point two places to the right: 0.25.
and then write a % symbol: 25% Multiplying by 100 and writing %

c) We first move the decimal point two places to the right (recall that $0.3 = 0.30$): 0.30.
and then write a % symbol: 30% Multiplying by 100 and writing %

TRY EXERCISE 33

Solving Percent Problems

In solving percent problems, we first *translate* the problem to an equation. Then we *solve* the equation using the techniques discussed in Sections 2.1–2.3. The key words in the translation are as follows.

Key Words in Percent Translations

“Of” translates to “ \cdot ” or “ \times ”. “Is” or “Was” translates to “ $=$ ”.
 “What” translates to a variable. “%” translates to “ $\times \frac{1}{100}$ ” or “ $\times 0.01$ ”.

EXAMPLE 4

What is 11% of 49?

SOLUTION

$$\begin{array}{cccccc} \text{Translate:} & \text{What} & \text{is} & 11\% & \text{of} & 49? \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & a & = & 0.11 & \cdot & 49 \end{array}$$

$$a = 5.39$$

“of” means multiply;
 11% = 0.11

STUDENT NOTES

A way of checking answers is by estimating as follows:

$$\begin{aligned} 11\% \times 49 &\approx 10\% \times 50 \\ &= 0.10 \times 50 = 5. \end{aligned}$$

Since 5 is close to 5.39, our answer is reasonable.

Thus, 5.39 is 11% of 49. The answer is 5.39.

TRY EXERCISE 51

EXAMPLE 5

3 is 16 percent of what?

SOLUTION

$$\begin{array}{cccccc} \text{Translate:} & 3 & \text{is} & 16 \text{ percent} & \text{of} & \text{what?} \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 3 & = & 0.16 & \cdot & y \end{array}$$

$$\frac{3}{0.16} = y \quad \text{Dividing both sides by 0.16}$$

$$18.75 = y$$

Thus, 3 is 16 percent of 18.75. The answer is 18.75.

TRY EXERCISE 47

EXAMPLE 6

What percent of \$50 is \$34?

SOLUTION

$$\begin{array}{cccccc} \text{Translate:} & \text{What percent} & \text{of} & \$50 & \text{is} & \$34? \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & n & \cdot & 50 & = & 34 \end{array}$$

$$n = \frac{34}{50} \quad \text{Dividing both sides by 50}$$

$$n = 0.68 = 68\% \quad \text{Converting to percent notation}$$

Thus, \$34 is 68% of \$50. The answer is 68%.

TRY EXERCISE 43

Examples 4–6 represent the three basic types of percent problems. Note that in all the problems, the following quantities are present:

- a percent, expressed in decimal notation in the translation,
- a base amount, indicated by “of” in the problem, and
- a percentage of the base, found by multiplying the base times the percent.

- b) Since the original price of \$20,830 represents 100% of the MSRP, the sale price represents a discount of $1100 - 912\%$, or 9%.

Alternatively, we could find the amount of discount and then calculate the percent of discount:

$$\text{Amount of discount: } \$20,830 - \$18,955 = \$1875.$$

Rewording: What percent of 20,830 is 1875?

$$\text{Translating: } n \cdot 20,830 = 1875$$

$$n = \frac{1875}{20,830}$$

Dividing both sides by 20,830

$$n \approx 0.09 = 9\%$$

Converting to percent notation

Again we find that the percent of discount is 9%.

TRY EXERCISE 69

2.4

EXERCISE SET

For Extra Help
MyMathLab

MathXL
PRACTICE

WATCH

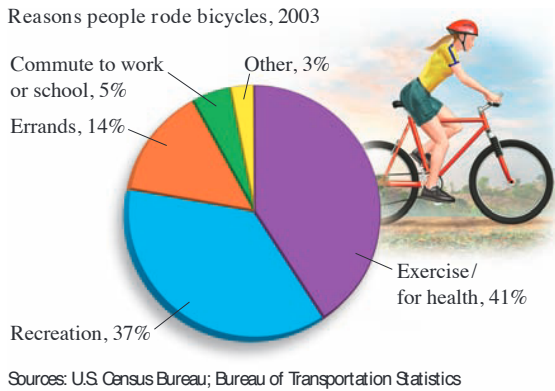
DOWNLOAD

i *Concept Reinforcement* In each of Exercises 1–10, match the question with the most appropriate translation from the column on the right. Some choices are used more than once.

- | | |
|-----------------------------------|----------------------|
| 1. ___ What percent of 57 is 23? | a) $a = 10.57223$ |
| 2. ___ What percent of 23 is 57? | b) $57 = 0.23y$ |
| 3. ___ 23 is 57% of what number? | c) $n \cdot 23 = 57$ |
| 4. ___ 57 is 23% of what number? | d) $n \cdot 57 = 23$ |
| 5. ___ 57 is what percent of 23? | e) $23 = 0.57y$ |
| 6. ___ 23 is what percent of 57? | f) $a = 10.23257$ |
| 7. ___ What is 23% of 57? | |
| 8. ___ What is 57% of 23? | |
| 9. ___ 23% of what number is 57? | |
| 10. ___ 57% of what number is 23? | |

- 55. What is 2% of 40?
- 56. What is 40% of 2?
- Aha!** 57. 25 is what percent of 50?
- 58. 0.8 is 2% of what number?
- 59. What percent of 69 is 23?
- 60. What percent of 40 is 9?

Riding bicycles. *There are 57 million Americans who ride a bicycle at least occasionally. The following circle graph shows the reasons people ride. In each of Exercises 61–64, determine the number of Americans who ride a bicycle for the given reason.*



- 61. Commute to school or work
- 62. Run errands
- 63. Exercise for health
- 64. Recreation
- 65. **College graduation.** To obtain his bachelor's degree in nursing, Cody must complete 125 credit hours of instruction. If he has completed 60% of his requirement, how many credits did Cody complete?
- 66. **College graduation.** To obtain her bachelor's degree in journalism, Addy must complete 125 credit hours of instruction. If 20% of Addy's credit hours remain to be completed, how many credits does she still need to take?
- 67. **Batting average.** In the 2007 season, Magglio Ordonez of the Detroit Tigers had 216 hits. His batting average was 0.363, the highest in major league baseball for that season. This means that of the total number of at bats, 36.3% were hits. How many at bats did he have?
Source: ESPN
- 68. **Pass completions.** At one point in a recent season, Peyton Manning of the Indianapolis Colts had com-

pleted 357 passes. This was 62.5% of his attempts. How many attempts did he make?
Source: National Football League

- 69. **Tipping.** Trent left a \$4 tip for a meal that cost \$25.
 - a) What percent of the cost of the meal was the tip?
 - b) What was the total cost of the meal including the tip?
- 70. **Tipping.** Selena left a \$12.76 tip for a meal that cost \$58.
 - a) What percent of the cost of the meal was the tip?
 - b) What was the total cost of the meal including the tip?
- 71. **Crude oil imports.** In April 2007, crude oil imports to the United States averaged 10.2 million barrels per day. Of this total, 3.4 million came from Canada and Mexico. What percent of crude oil imports came from Canada and Mexico? What percent came from the rest of the world?
Source: Energy Information Administration
- 72. **Alternative-fuel vehicles.** Of the 550,000 alternative-fuel vehicles produced in the United States in 2004, 150,000 were E85 flexible-fuel vehicles. What percent of alternative-fuel vehicles used E85? What percent used other alternative fuels?
Source: Energy Information Administration
- 73. **Student loans.** Glenn takes out a subsidized federal Stafford loan for \$2400. After a year, Glenn decides to pay off the interest, which is 7% of \$2400. How much will he pay?
- 74. **Student loans.** To finance her community college education, LaTonya takes out a Stafford loan for \$3500. After a year, LaTonya decides to pay off the interest, which is 8% of \$3500. How much will she pay?
- 75. **Infant health.** In a study of 300 pregnant women with "good-to-excellent" diets, 95% had babies in good or excellent health. How many women in this group had babies in good or excellent health?
- 76. **Infant health.** In a study of 300 pregnant women with "poor" diets, 8% had babies in good or excellent health. How many women in this group had babies in good or excellent health?
- 77. **Cost of self-employment.** Because of additional taxes and fewer benefits, it has been estimated that a self-employed person must earn 20% more than a non-self-employed person performing the same task(s). If Tia earns \$16 an hour working for Village

Copy, how much would she need to earn on her own for a comparable income?

78. Refer to Exercise 77. Rik earns \$18 an hour working for Round Edge stairbuilders. How much would Rik need to earn on his own for a comparable income?
79. **Budget overruns.** The Indianapolis Central Library expansion, begun in 2002, was expected to cost \$103 million. By 2006, library officials estimated the cost would be \$45 million over budget. By what percent did the actual cost exceed the initial estimate?
Source: *The Indianapolis Star*, 5/23/06



80. **Fastest swimmer.** In 1990, Tom Jager of the United States set a world record by swimming 50 m at a rate of 2.29 m/s. Previously, the fastest swimming rate on record was 2.26 m/s, set in 1975 by David Holmes Edgar, also of the United States. Calculate the percentage by which the rate increased.
Source: *Guinness Book of World Records* 1975 and 1998
81. A bill at Officeland totaled \$47.70. How much did the merchandise cost if the sales tax is 6%?
82. Marta's checkbook shows that she wrote a check for \$987 for building materials. What was the price of the materials if the sales tax is 5%?
83. **Deducting sales tax.** A tax-exempt school group received a bill of \$157.41 for educational software. The bill incorrectly included sales tax of 6%. How much should the school group pay?
84. **Deducting sales tax.** A tax-exempt charity received a bill of \$145.90 for a sump pump. The bill incorrectly included sales tax of 5%. How much does the charity owe?
85. **Body fat.** One author of this text exercises regularly at a local YMCA that recently offered a body-fat percentage test to its members. The device used

measures the passage of a very low voltage of electricity through the body. The author's body-fat percentage was found to be 16.5% and he weighs 191 lb. What part, in pounds, of his body weight is fat?

86. **Areas of Alaska and Arizona.** The area of Arizona is 19% of the area of Alaska. The area of Alaska is 586,400 mi². What is the area of Arizona?



87. **Direct mail.** Only 2.15% of mailed ads lead to a sale or a response from customers. In 2006, businesses sent out 114 billion pieces of direct mail (catalogs, coupons, and so on). How many pieces of mail led to a response from customers?
Sources: Direct Marketing Association; U.S. Postal Service
88. **Kissing and colds.** In a medical study, it was determined that if 800 people kiss someone else who has a cold, only 56 will actually catch the cold. What percent is this?
89. **Calorie content.** Pepperidge Farm Light Style 7 Grain Bread® has 140 calories in a 3-slice serving. This is 15% less than the number of calories in a serving of regular bread. How many calories are in a serving of regular bread?
90. **Fat content.** Peek Freans Shortbread Reduced Fat Cookies® contain 35 calories of fat in each serving. This is 40% less than the fat content in the leading imported shortbread cookie. How many calories of fat are in a serving of the leading shortbread cookie?
91. Campus Bookbuyers pays \$30 for a book and sells it for \$60. Is this a 100% markup or a 50% markup? Explain.
92. If Julian leaves a \$12 tip for a \$90 dinner, is he being generous, stingy, or neither? Explain.

Skill Review

To prepare for Section 2.5, review translating to algebraic expressions and equations (Section 1.1).

Translate to an algebraic expression or equation. [1.1]

93. Twice the length plus twice the width
94. 5% of \$180
95. 5 fewer than the number of points Tino scored
96. 15 plus the product of 1.5 and x
97. The product of 10 and half of a
98. 10 more than three times a number
99. The width is 2 in. less than the length.
100. A number is four times as large as a second number.

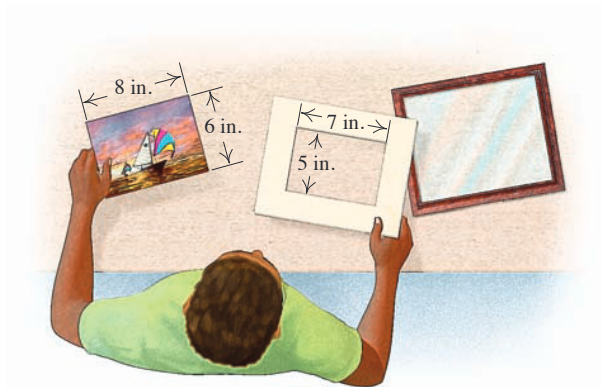
Synthesis

101. How is the use of statistics in the following misleading?
 - a) A business explaining new restrictions on sick leave cited a recent survey indicating that 40% of all sick days were taken on Monday or Friday.
 - b) An advertisement urging summer installation of a security system quoted FBI statistics stating that over 26% of home burglaries occur between Memorial Day and Labor Day.
102. Erin is returning a tent that she bought during a 25%-off storewide sale that has ended. She is offered store credit for 125% of what she paid (not to be used on sale items). Is this fair to Erin? Why or why not?
103. The community of Bardville has 1332 left-handed females. If 48% of the community is female and 15% of all females are left-handed, how many people are in the community?
104. It has been determined that at the age of 10, a girl has reached 84.4% of her final adult height. Dana is 4 ft 8 in. at the age of 10. What will her final adult height be?

105. It has been determined that at the age of 15, a boy has reached 96.1% of his final adult height. Jaraan is 6 ft 4 in. at the age of 15. What will his final adult height be?

106. **Dropout rate.** Between 2002 and 2004, the high school dropout rate in the United States decreased from 105 to 103 per thousand. Calculate the percent by which the dropout rate decreased and use that percentage to estimate dropout rates for the United States in 2005 and in 2006.
Source: www.childrendsdatabank.org

107. **Photography.** A 6-in. by 8-in. photo is framed using a mat meant for a 5-in. by 7-in. photo. What percentage of the photo will be hidden by the mat?



108. **Aha!** Would it be better to receive a 5% raise and then, a year later, an 8% raise or the other way around? Why?
109. Jorge is in the 30% tax bracket. This means that 30¢ of each dollar earned goes to taxes. Which would cost him the least: contributing \$50 that is tax-deductible or contributing \$40 that is not tax-deductible? Explain.

CORNER

Sales and Discounts

Focus: Applications and models using percent

Time: 15 minutes

Group size: 3

Materials: Calculators are optional.

Often a store will reduce the price of an item by a fixed percentage. When the sale ends, the items are returned to their original prices. Suppose a department store reduces all sporting goods 20%, all clothing 25%, and all electronics 10%.

ACTIVITY

- Each group member should select one of the following items: a \$50 basketball, an \$80 jacket, or a \$200 MP3 player. Fill in the first three columns of the first three rows of the chart below.
- Apply the appropriate discount and determine the sale price of your item. Fill in the fourth column of the chart.
- Next, find a multiplier that can be used to convert the sale price back to the original price and fill in the remaining column of the chart. Does this multiplier depend on the price of the item?
- Working as a group, compare the results of part (3) for all three items. Then develop a formula for a multiplier that will restore a sale price to its original price, p , after a discount r has been applied. Complete the fourth row of the table and check that your formula will duplicate the results of part (3).
- Use the formula from part (4) to find the multiplier that a store would use to return an item to its original price after a “30% off” sale expires. Fill in the last line on the chart.
- Inspect the last column of your chart. How can these multipliers be used to determine the percentage by which a sale price is increased when a sale ends?

Original Price, p	Discount, r	$1 - r$	Sale Price	Multiplier to convert back to p
p	r	$1 - r$		
	0.30			