2.5   Problem Solving

Five Steps for Problem Solving   ● ▪   Applying the Five Steps

Probably the most important use of algebra is as a tool for problem solving. In this section, we develop a problem-solving approach that is used throughout the remainder of the text.

Five Steps for Problem Solving

In Section 2.4, we solved several real-world problems. To solve them, we first *familiarized* ourselves with percent notation. We then *translated* each problem into an equation, *solved* the equation, *checked* the solution, and *stated* the answer.

> ### Five Steps for Problem Solving in Algebra
> 1. *Familiarize* yourself with the problem.
> 2. *Translate* to mathematical language. (This often means writing an equation.)
> 3. *Carry out* some mathematical manipulation. (This often means *solving* an equation.)
> 4. *Check* your possible answer in the original problem.
> 5. *State* the answer clearly, using a complete English sentence.

Of the five steps, the most important is probably the first one: becoming familiar with the problem. Here are some hints for familiarization.

> ### To Become Familiar with a Problem
> 1. Read the problem carefully. Try to visualize the problem.
> 2. Reread the problem, perhaps aloud. Make sure you understand all important words and any symbols or abbreviations.
> 3. List the information given and the question(s) to be answered. Choose a variable (or variables) to represent the unknown and specify exactly what the variable represents. For example, let $L =$ length in centimeters, $d =$ distance in miles, and so on.
> 4. Look for similarities between the problem and other problems you have already solved. Ask yourself what type of problem this is.
> 5. Find more information. Look up a formula in a book, at a library, or online. Consult a reference librarian or an expert in the field.
> 6. Make a table that uses all the information you have available. Look for patterns that may help in the translation.
> 7. Make a drawing and label it with known and unknown information, using specific units if given.
> 8. Think of a possible answer and check the guess. Note the manner in which the guess is checked.

Applying the Five Steps

EXAMPLE   1

Bicycling.   After finishing college, Nico spent a week touring Tuscany, Italy, by bicycle. He biked 260 km from Pisa through Siena to Florence. At Siena, he had biked three times as far from Pisa as he would then bike to Florence. How far had he biked, and how far did he have left to go?

SOLUTION

1. **Familiarize.** It is often helpful to make a drawing. In this case, we can use a map of Nico's trip.

To gain familiarity, let's suppose that Nico has 50 km to go. Then he would have traveled three times 50 km, or 150 km, already. Since 50 km + 150 km = 200 km and 200 km < 260 km, we see that our guess is too small. Rather than guess again, we let

d = the distance, in kilometers, from Siena to Florence

and

$3d$ = the distance, in kilometers, from Siena to Pisa.

(We could also let x = the distance to Pisa; then the distance to Florence would be $\frac{1}{3}x$.)

2. **Translate.** The lengths of the two parts of the trip must add up to 260 km. This leads to our translation.

Rewording: Distance to Florence   plus   distance to Pisa   is   260 km

Translating: d $+$ $3d$ $=$ 260

3. **Carry out.** We solve the equation:

$$d + 3d = 260$$
$$4d = 260 \qquad \text{Combining like terms}$$
$$d = 65. \qquad \text{Dividing both sides by 4}$$

4. **Check.** As predicted in the *Familiarize* step, d is greater than 50 km. If $d = 65$ km, then $3d = 195$ km. Since 65 km + 195 km = 260 km, we have a check.

5. **State.** At Siena, Nico had biked 195 km and had 65 km left to go to arrive in Florence.

TRY EXERCISE   9

Before we solve the next problem, we need to learn some additional terminology regarding integers.

The following are examples of **consecutive integers:** 16, 17, 18, 19, 20; and −31, −30, −29, −28. Note that consecutive integers can be represented in the form $x, x + 1, x + 2$, and so on.

The following are examples of **consecutive even integers:** 16, 18, 20, 22, 24; and $-52, -50, -48, -46$. Note that consecutive even integers can be represented in the form $x, x + 2, x + 4$, and so on.

The following are examples of **consecutive odd integers:** 21, 23, 25, 27, 29; and $-71, -69, -67, -65$. Note that consecutive odd integers can be also represented in the form $x, x + 2, x + 4$, and so on.
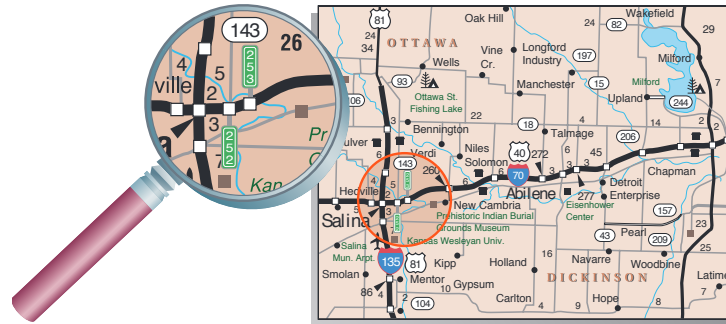
EXAMPLE **2**

Interstate mile markers. U.S. interstate highways post numbered markers at every mile to indicate location in case of an emergency. The sum of two consecutive mile markers on I-70 in Kansas is 559. Find the numbers on the markers.

Source: Federal Highway Administration, Ed Potalewski

x	x 1 1	Sum of x and x 1 1
114	115	229
252	253	505
302	303	605

SOLUTION

1. **Familiarize.** The numbers on the mile markers are consecutive positive integers. Thus if we let $x =$ the smaller number, then $x + 1 =$ the larger number.

To become familiar with the problem, we can make a table, as shown at left. First, we guess a value for x; then we find $x + 1$. Finally, we add the two numbers and check the sum.

From the table, we see that the first marker will be between 252 and 302. We could continue guessing and solve the problem this way, but let's work on developing our algebra skills.

2. **Translate.** We reword the problem and translate as follows.

Rewording: First integer plus second integer is 559.

Translating: x + $1x + 12$ = 559

3. **Carry out.** We solve the equation:

$$x + 1x + 12 = 559$$
$$2x + 1 = 559 \quad \text{Using an associative law and combining like terms}$$
$$2x = 558 \quad \text{Subtracting 1 from both sides}$$
$$x = 279. \quad \text{Dividing both sides by 2}$$

If x is 279, then $x + 1$ is 280.

4. **Check.** Our possible answers are 279 and 280. These are consecutive positive integers and $279 + 280 = 559$, so the answers check.

5. **State.** The mile markers are 279 and 280.

TRY EXERCISE 13

EXAMPLE 3

Color printers. Egads Computer Corporation rents a Xerox Phaser 8400 Color Laser Printer for $300 a month. A new art gallery is leasing a printer for a 2-month advertising campaign. The ink and paper for the brochures will cost an additional 21.5¢ per copy. If the gallery allots a budget of $3000, how many brochures can they print?

Source: egadscomputer.com

SOLUTION

1. **Familiarize.** Suppose that the art gallery prints 20,000 brochures. Then the cost is the monthly charges plus ink and paper cost, or

   2·$300   plus   cost per brochure   times   number of brochures
   $600   + $0.215   ·   20,000,

   which is $4900. Our guess of 20,000 is too large, but we have familiarized ourselves with the way in which a calculation is made. Note that we convert 21.5¢ to $0.215 so that all information is in the same unit, dollars. We let $c = $ the number of brochures that can be printed for $3000.

2. **Translate.** We reword the problem and translate as follows.

   Rewording: Monthly cost   plus   ink and paper cost   is $3000.

   Translating: 2·$300   +   1·$0.215·c = $3000

3. **Carry out.** We solve the equation:

   $$2(300) + 0.215c = 3000$$
   $$600 + 0.215c = 3000$$
   $$0.215c = 2400 \qquad \text{Subtracting 600 from both sides}$$
   $$c = \frac{2400}{0.215} \qquad \text{Dividing both sides by 0.215}$$
   $$c \approx 11{,}162. \qquad \text{We round } down \text{ to avoid going over the budget.}$$

4. **Check.** We check in the original problem. The cost for 11,162 brochures is 11,162·$0.215 = $2399.83. The rental for 2 months is 2·$300 = $600. The total cost is then $2399.83 + $600 = $2999.83, which is just under the amount that was allotted. Our answer is less than 20,000, as we expected from the *Familiarize* step.

5. **State.** The art gallery can make 11,162 brochures with the rental allotment of $3000.

TRY EXERCISE 37

EXAMPLE 4

Perimeter of NBA court. The perimeter of an NBA basketball court is 288 ft. The length is 44 ft longer than the width. Find the dimensions of the court.

Source: National Basketball Association

SOLUTION

1. **Familiarize.** Recall that the perimeter of a rectangle is twice the length plus twice the width. Suppose the court were 30 ft wide. The length would then be 30 + 44, or 74 ft, and the perimeter would be 2 · 30 ft + 2 · 74 ft, or 208 ft. This shows that in order for the perimeter to be 288 ft, the width must exceed 30 ft. Instead of guessing again, we let $w = $ the width of the court, in feet.

Since the court is "44 ft longer than it is wide," we let $w + 44 =$ the length of the court, in feet.
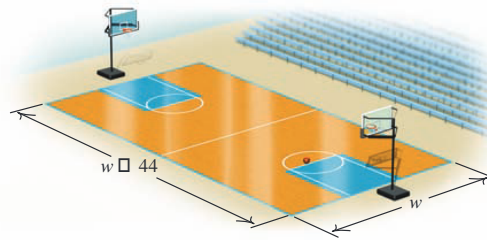
2. Translate. To translate, we use $w + 44$ as the length and 288 as the perimeter. To double the length, $w + 44$, parentheses are essential.

Rewording:	Twice the length	plus	twice the width	is	288 ft.
Translating:	$2(w + 44)$	$+$	$2w$	$=$	288

3. Carry out. We solve the equation:

$$2(w + 44) + 2w = 288$$

$2w + 88 + 2w = 288$ Using the distributive law

$4w + 88 = 288$ Combining like terms

$4w = 200$

$w = 50.$

The dimensions appear to be $w = 50$ ft, and $l = w + 44 = 94$ ft.

4. Check. If the width is 50 ft and the length is 94 ft, then the court is 44 ft longer than it is wide. The perimeter is $2(50 \text{ ft}) + 2(94 \text{ ft}) = 100 \text{ ft} + 188 \text{ ft}$, or 288 ft, as specified. We have a check.

5. State. An NBA court is 50 ft wide and 94 ft long.

TRY EXERCISE 25

> *CAUTION!*   Always be sure to answer the original problem completely. For instance, in Example 1 we needed to find *two* numbers: the distances from *each* city to Siena. Similarly, in Example 4 we needed to find two dimensions, not just the width. Be sure to label each answer with the proper unit.

EXAMPLE 5

Selling at an auction.   Jared is selling his collection of Transformers at an auction. He wants to be left with $1150 after paying a seller's premium of 8% on the final bid (hammer price) for the collection. What must the hammer price be in order for him to clear $1150?

SOLUTION

1. Familiarize. Suppose the collection sells for $1200. The 8% seller's premium can be determined by finding 8% of $1200:

$$8\% \text{ of } \$1200 = 0.08(\$1200) = \$96.$$

Subtracting this premium from $1200 would leave Jared with

$$\$1200 - \$96 = \$1104.$$

This shows that in order for Jared to clear $1150, the collection must sell for more than $1200. We let $x =$ the hammer price, in dollars. Jared then must pay a seller's premium of $0.08x$.

2. **Translate.** We reword the problem and translate as follows.

Rewording: Hammer price   less   seller's premium   is   amount remaining.

Translating: x $-$ $0.08x$ $=$ \$1150

3. **Carry out.** We solve the equation:

$$x - 0.08x = 1150$$
$$1x - 0.08x = 1150$$
$$0.92x = 1150$$

Combining like terms. Had we noted that after the premium has been paid, 92% remains, we could have begun with this equation.

$$x = \frac{1150}{0.92}$$

Dividing both sides by 0.92

$$x = 1250.$$

4. **Check.** To check, we first find 8% of \$1250:

8% of \$1250 $=$ 0.081\$12502 $=$ \$100.   This is the premium.

Next, we subtract the premium to find the remaining amount:

\$1250 $-$ \$100 $=$ \$1150.

Since, after Jared pays the seller's premium, he is left with \$1150, our answer checks. Note that the \$1250 hammer price is greater than \$1200, as predicted in the *Familiarize* step.

5. **State.** Jared's collection must sell for \$1250 in order for him to be left with \$1150.

TRY EXERCISE 7

EXAMPLE 6

Cross section of a roof.   In a triangular gable end of a roof, the angle of the peak is twice as large as the angle on the back side of the house. The measure of the angle on the front side is 20° greater than the angle on the back side. How large are the angles?

SOLUTION

1. **Familiarize.** We make a drawing. In this case, the measure of the back angle is x, the measure of the front angle is $x + 20$, and the measure of the peak angle is $2x$.
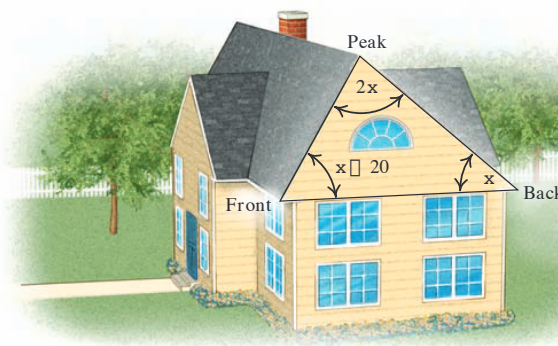
2. **Translate.** To translate, we need to recall that the sum of the measures of the angles in a triangle is 180°.

Rewording: Measure of <u>back angle</u> + measure of <u>front angle</u> + measure of <u>peak angle</u> is 180°.

Translating: x + $1x + 202$ + $2x$ =   180

3. **Carry out.** We solve:

$$x + 1x + 202 + 2x = 180$$
$$4x + 20 = 180$$
$$4x = 160$$
$$x = 40.$$

The measures for the angles appear to be:

Back angle: $x = 40°$,

Front angle: $x + 20 = 40 + 20 = 60°$,

Peak angle: $2x = 2(40) = 80°$.

4. **Check.** Consider 40°, 60°, and 80°, as listed above. The measure of the front angle is 20° greater than the measure of the back angle, the measure of the peak angle is twice the measure of the back angle, and the sum is 180°. These numbers check.

5. **State.** The measures of the angles are 40°, 60°, and 80°.   **TRY EXERCISE** 31

We close this section with some tips to aid you in problem solving.

> **Problem-Solving Tips**
> 1. The more problems you solve, the more your skills will improve.
> 2. Look for patterns when solving problems. Each time you study an example or solve an exercise, you may observe a pattern for problems found later.
> 3. Clearly define variables before translating to an equation.
> 4. Consider the dimensions of the variables and constants in the equation. The variables that represent length should all be in the same unit, those that represent money should all be in dollars or all in cents, and so on.
> 5. Make sure that units appear in the answer whenever appropriate and that you completely answer the original problem.

2.5   EXERCISE SET

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Solve. Even though you might find the answer quickly in some other way, practice using the five-step problem-solving process in order to build the skill of problem solving.

1. Three less than twice a number is 19. What is the number?

2. Two fewer than ten times a number is 78. What is the number?

3. Five times the sum of 3 and twice some number is 70. What is the number?

4. Twice the sum of 4 and three times some number is 34. What is the number?

5. *Price of an iPod.*   Kyle paid $120 for an iPod nano during a 20%-off sale. What was the regular price?

6. *Price of sneakers.*   Amy paid $102 for a pair of New Balance 1122 running shoes during a 15%-off sale. What was the regular price?

7. *Price of a calculator.*   Kayla paid $137.80, including 6% tax, for her graphing calculator. How much did the calculator itself cost?

8. *Price of a printer.*   Laura paid $219.45, including 5% tax, for an all-in-one color printer. How much did the printer itself cost?

9. *Unicycling.*   In 2005, Ken Looi of New Zealand set a record by covering 235.3 mi in 24 hr on his unicycle. After 8 hr, he was approximately twice as far from the finish line as he was from the start. How far had he traveled?
Source: Guinness World Records 2007

10. *Sled-dog racing.*   The Iditarod sled-dog race extends for 1049 mi from Anchorage to Nome. If a musher is twice as far from Anchorage as from Nome, how many miles has the musher traveled?


The 1049-mile Iditarod race route
Nome   ALASKA
Anchorage
0   200
miles
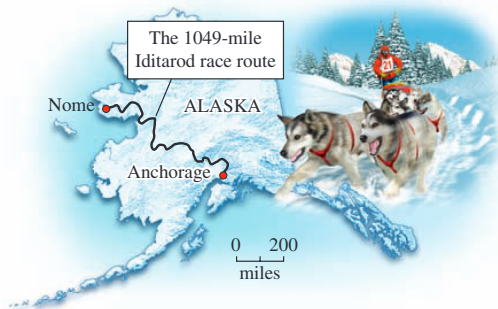
11. *Indy Car racing.*   In April 2008, Danica Patrick won the Indy Japan 300 with a time of 01:51:02.6739 for the 300-mi race. At one point, Patrick was 20 mi closer to the finish than to the start. How far had Patrick traveled at that point?

12. *NASCAR racing.*   In June 2007, Carl Edwards won the Michigan 400 with a time of 2:42:5 for the 400-mi race. At one point, Edwards was 80 mi closer to the finish than to the start. How far had Edwards traveled at that point?

13. *Apartment numbers.*   The apartments in Erica's apartment house are consecutively numbered on each floor. The sum of her number and her next-door neighbor's number is 2409. What are the two numbers?

14. *Apartment numbers.*   The apartments in Brian's apartment house are numbered consecutively on each floor. The sum of his number and his next-door neighbor's number is 1419. What are the two numbers?

15. *Street addresses.*   The houses on the west side of Lincoln Avenue are consecutive odd numbers. Sam and Colleen are next-door neighbors and the sum of their house numbers is 572. Find their house numbers.

16. *Street addresses.*   The houses on the south side of Elm Street are consecutive even numbers. Wanda and Larry are next-door neighbors and the sum of their house numbers is 794. Find their house numbers.

17. The sum of three consecutive page numbers is 99. Find the numbers.

18. The sum of three consecutive page numbers is 60. Find the numbers.

19. *Longest marriage.*   As half of the world's longest-married couple, the woman was 2 yr younger than her husband. Together, their ages totaled 204 yr. How old were the man and the woman?
Source: Guinness World Records 2007

20. *Oldest bride.*   The world's oldest bride was 19 yr older than her groom. Together, their ages totaled 185 yr. How old were the bride and the groom?
Source: Guinness World Records 2007

21. *e-mail.*   In 2006, approximately 125 billion e-mail messages were sent each day. The number of spam messages was about four times the number of non-spam messages. How many of each type of message were sent each day in 2006?
Source: Ferris Research

22. *Home remodeling.*   In 2005, Americans spent a total of $26 billion to remodel bathrooms and kitchens. They spent $5 billion more on kitchens than on bathrooms. How much was spent on each?
Source: Joint Center for Housing Studies, Harvard University

23. *Page numbers.*   The sum of the page numbers on the facing pages of a book is 281. What are the page numbers?

24. *Perimeter of a triangle.*   The perimeter of a triangle is 195 mm. If the lengths of the sides are consecutive odd integers, find the length of each side.

25. *Hancock Building dimensions.*   The top of the John Hancock Building in Chicago is a rectangle whose length is 60 ft more than the width. The perimeter is 520 ft. Find the width and the length of the rectangle. Find the area of the rectangle.

26. *Dimensions of a state.*   The perimeter of the state of Wyoming is 1280 mi. The width is 90 mi less than the length. Find the width and the length.

27. A rectangular community garden is to be enclosed with 92 m of fencing. In order to allow for compost storage, the garden must be 4 m longer than it is wide. Determine the dimensions of the garden.
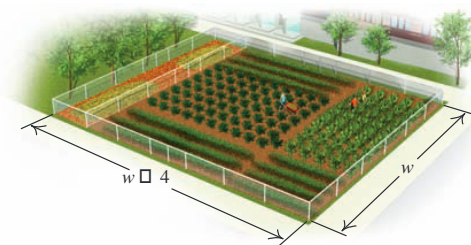

28. *Perimeter of a high school basketball court.*   The perimeter of a standard high school basketball court is 268 ft. The length is 34 ft longer than the width. Find the dimensions of the court.
Source: Indiana High School Athletic Association

29. *Two-by-four.*   The perimeter of a cross section of a "two-by-four" piece of lumber is $10\frac{1}{2}$ in. The length is twice the width. Find the actual dimensions of the cross section of a two-by-four.
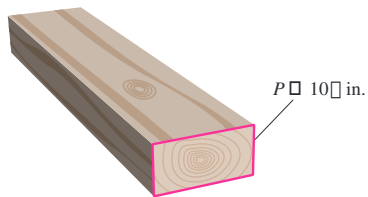
30. *Standard billboard sign.*   A standard rectangular highway billboard sign has a perimeter of 124 ft. The length is 6 ft more than three times the width. Find the dimensions of the sign.


31. *Angles of a triangle.*   The second angle of an architect's triangle is three times as large as the first. The third angle is 30° more than the first. Find the measure of each angle.

32. *Angles of a triangle.*   The second angle of a triangular garden is four times as large as the first. The third angle is 45° less than the sum of the other two angles. Find the measure of each angle.

33. *Angles of a triangle.*   The second angle of a triangular kite is four times as large as the first. The third angle is 5° more than the sum of the other two angles. Find the measure of the second angle.

34. *Angles of a triangle.*   The second angle of a triangular building lot is three times as large as the first. The third angle is 10° more than the sum of the other two angles. Find the measure of the third angle.

35. *Rocket sections.*   A rocket is divided into three sections: the payload and navigation section in the top, the fuel section in the middle, and the rocket engine section in the bottom. The top section is one-sixth the length of the bottom section. The middle section is one-half the length of the bottom section. The total length is 240 ft. Find the length of each section.
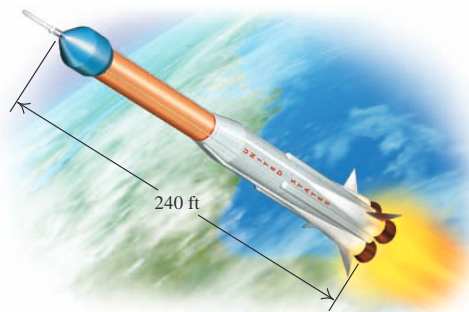
36. *Gourmet sandwiches.*   Jenny, Demi, and Drew buy an 18-in. long gourmet sandwich and take it back to their apartment. Since they have different appetites, Jenny cuts the sandwich so that Demi gets half of what Jenny gets and Drew gets three-fourths of what Jenny gets. Find the length of each person's sandwich.

37. *Taxi rates.*   In Chicago, a taxi ride costs $2.25 plus $1.80 for each mile traveled. Debbie has budgeted $18 for a taxi ride (excluding tip). How far can she travel on her $18 budget?
Source: City of Chicago

38. *Taxi fares.*   In New York City, taxis charge $2.50 plus $2.00 per mile for off-peak fares. How far can Ralph travel for $17.50 (assuming an off-peak fare)?
Source: New York City Taxi and Limousine Commission

39. *Truck rentals.*   Truck-Rite Rentals rents trucks at a daily rate of $49.95 plus 39¢ per mile. Concert Productions has budgeted $100 for renting a truck to haul equipment to an upcoming concert. How far can they travel in one day and stay within their budget?

40. *Truck rentals.*   Fine Line Trucks rents an 18-ft truck for $42 plus 35¢ per mile. Judy needs a truck for one day to deliver a shipment of plants. How far can she drive and stay within a budget of $70?

41. *Complementary angles.*   The sum of the measures of two *complementary* angles is 90°. If one angle measures 15° more than twice the measure of its complement, find the measure of each angle.

Complementary angles
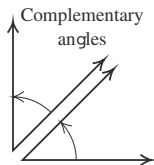
42. *Complementary angles.*   Two angles are complementary. (See Exercise 41.) The measure of one angle is $1\frac{1}{2}$ times the measure of the other. Find the measure of each angle.

43. *Supplementary angles.*   The sum of the measures of two *supplementary* angles is 180°. If the measure of one angle is $3\frac{1}{2}$ times the measure of the other, find the measure of each angle.

Supplementary angles
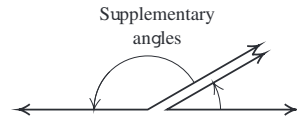
44. *Supplementary angles.*   Two angles are supplementary. (See Exercise 43.) If one angle measures 45° less than twice the measure of its supplement, find the measure of each angle.

45. *Copier paper.*   The perimeter of standard-size copier paper is 99 cm. The width is 6.3 cm less than the length. Find the length and the width.

46. *Stock prices.*   Sarah's investment in Jet Blue stock grew 28% to $448. How much did she originally invest?

47. *Savings interest.*   Janeka invested money in a savings account at a rate of 6% simple interest. After 1 yr, she has $6996 in the account. How much did Janeka originally invest?

48. *Credit cards.*   The balance in Will's Mastercard® account grew 2%, to $870, in one month. What was his balance at the beginning of the month?

49. *Scrabble®.*   In a single game on October 12, 2006, Michael Cresta and Wayne Yorra set three North American Scrabble records: the most points in one game by one player, the most total points in the game, and the most points on a single turn. Cresta scored 340 points more than Yorra, and together they scored 1320 points. What was the winning score?
Source: www.slate.com

50. *Color printers.*   The art gallery in Example 3 decides to raise its budget to $5000 for the 2-month period. How many brochures can they print for $5000?

51. *Selling a home.*   The Brannons are planning to sell their home. If they want to be left with $117,500 after paying 6% of the selling price to a realtor as a commission, for how much must they sell the house?

52. *Budget overruns.*   The massive roadworks project in Boston known as The Big Dig cost approximately $14.6 billion. This cost was 484% more than the original estimate. What was the original estimate of the cost of The Big Dig?
Sources: Taxpayers for Common Sense; www.msnbc.cmsn.com

53. *Cricket chirps and temperature.*   The equation $T = \frac{1}{4}N + 40$ can be used to determine the temperature T, in degrees Fahrenheit, given the number of times N a cricket chirps per minute. Determine the number of chirps per minute for a temperature of 80°F.

54. *Race time.*   The equation $R = -0.028t + 20.8$ can be used to predict the world record in the 200-m dash, where R is the record in seconds and t is the number of years since 1920. In what year will the record be 18.0 sec?

55. Sean claims he can solve most of the problems in this section by guessing. Is there anything wrong with this approach? Why or why not?

56. When solving Exercise 20, Beth used a to represent the bride's age and Ben used a to represent the groom's age. Is one of these approaches preferable to the other? Why or why not?

Skill Review

To prepare for Section 2.6, review inequalities (Section 1.4).

Write a true sentence using either $<$ or $>$. [1.4]

57. $-8 \ \blacksquare \ 1$

58. $-2 \ \blacksquare \ -5$

59. $\frac{1}{2} \ \blacksquare \ 0$

60. $-3 \ \blacksquare \ -1$

Write a second inequality with the same meaning. [1.4]

61. $x \geq -4$

62. $x < 5$

63. $5 > y$

64. $-10 \leq t$

Synthesis

65. Write a problem for a classmate to solve. Devise it so that the problem can be translated to the equation $x + 1x + 22 + 1x + 42 = 375$.

66. Write a problem for a classmate to solve. Devise it so that the solution is "Audrey can drive the rental truck for 50 mi without exceeding her budget."

67. *Discounted dinners.*   Kate's "Dining Card" entitles her to $10 off the price of a meal after a 15% tip has been added to the cost of the meal. If, after the discount, the bill is $32.55, how much did the meal originally cost?

68. *Test scores.*   Pam scored 78 on a test that had 4 fill-in questions worth 7 points each and 24 multiple-choice questions worth 3 points each. She had one fill-in question wrong. How many multiple-choice questions did Pam get right?

69. *Gettysburg Address.*   Abraham Lincoln's 1863 Gettysburg Address refers to the year 1776 as "four *score* and seven years ago." Determine what a score is.

70. One number is 25% of another. The larger number is 12 more than the smaller. What are the numbers?

71. A storekeeper goes to the bank to get $10 worth of change. She requests twice as many quarters as half dollars, twice as many dimes as quarters, three times as many nickels as dimes, and no pennies or dollars. How many of each coin did the storekeeper get?

72. *Perimeter of a rectangle.*   The width of a rectangle is three fourths of the length. The perimeter of the rectangle becomes 50 cm when the length and the width are each increased by 2 cm. Find the length and the width.

73. *Discounts.*   In exchange for opening a new credit account, Macy's Department Stores® subtracts 10% from all purchases made the day the account is established. Julio is opening an account and has a coupon for which he receives 10% off the first day's reduced price of a camera. If Julio's final price is $77.75, what was the price of the camera before the two discounts?

74. *Sharing fruit.*   Apples are collected in a basket for six people. One third, one fourth, one eighth, and one fifth of the apples are given to four people, respectively. The fifth person gets ten apples, and one apple remains for the sixth person. Find the original number of apples in the basket.

75. *eBay purchases.*   An eBay seller charges $9.99 for the first DVD purchased and $6.99 for all others. For shipping and handling, he charges the full shipping fee of $3 for the first DVD, one half of the shipping charge for the second item, and one third of the shipping charge per item for all remaining items. The total cost of a shipment (excluding tax) was $45.45. How many DVDs were in the shipment?

76. *Winning percentage.*   In a basketball league, the Falcons won 15 of their first 20 games. In order to win 60% of the total number of games, how many more games will they have to play, assuming they win only half of the remaining games?

77. *Taxi fares.*   In New York City, a taxi ride costs $2.50 plus 40¢ per $\frac{1}{5}$ mile and 40¢ per minute stopped in traffic. Due to traffic, Glenda's taxi took 20 min to complete what is usually a 10-min drive. If she is charged $18.50 for the ride, how far did Glenda travel?
Source: New York City Taxi and Limousine Commission

78. *Test scores.*   Ella has an average score of 82 on three tests. Her average score on the first two tests is 85. What was the score on the third test?

79. A school purchases a piano and must choose between paying $2000 at the time of purchase or $2150 at the end of one year. Which option should the school select and why?

80. Annette claims the following problem has no solution: "The sum of the page numbers on facing pages is 191. Find the page numbers." Is she correct? Why or why not?

Aha!

81. The perimeter of a rectangle is 101.74 cm. If the length is 4.25 cm longer than the width, find the dimensions of the rectangle.

82. The second side of a triangle is 3.25 cm longer than the first side. The third side is 4.35 cm longer than the second side. If the perimeter of the triangle is 26.87 cm, find the length of each side.