

Solutions of Inequalities ■ Graphs of Inequalities ■ Set-Builder and Interval Notation ■  
Solving Inequalities Using the Addition Principle ■ Solving Inequalities Using the Multiplication Principle ■  
Using the Principles Together

Many real-world situations translate to *inequalities*. For example, a student might need to register for *at least* 12 credits; an elevator might be designed to hold *at most* 2000 pounds; a tax credit might be allowable for families with incomes of *less than* \$25,000; and so on. Before solving applications of this type, we must adapt our equation-solving principles to the solving of inequalities.

### Solutions of Inequalities

Recall from Section 1.4 that an inequality is a number sentence containing  $>$  (is greater than),  $<$  (is less than),  $\geq$  (is greater than or equal to), or  $\leq$  (is less than or equal to). Inequalities like

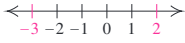
$$-7 > x, \quad t < 5, \quad 5x - 2 \geq 9, \quad \text{and} \quad -3y + 8 \leq -7$$

are true for some replacements of the variable and false for others.

Any value for the variable that makes an inequality true is called a **solution**. The set of all solutions is called the **solution set**. When all solutions of an inequality are found, we say that we have **solved** the inequality.

**EXAMPLE 1**

Determine whether the given number is a solution of  $x < 2$ : (a)  $-3$ ; (b)  $2$ .



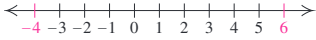
**SOLUTION**

- a) Since  $-3 < 2$  is true,  $-3$  is a solution.
- b) Since  $2 < 2$  is false,  $2$  is not a solution.

TRY EXERCISE 9

**EXAMPLE 2**

Determine whether the given number is a solution of  $y \geq 6$ : (a)  $6$ ; (b)  $-4$ .



**SOLUTION**

- a) Since  $6 \geq 6$  is true,  $6$  is a solution.
- b) Since  $-4 \geq 6$  is false,  $-4$  is not a solution.

TRY EXERCISE 11

### Graphs of Inequalities

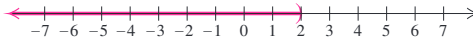
Because the solutions of inequalities like  $x < 2$  are too numerous to list, it is helpful to make a drawing that represents all the solutions. The **graph** of an inequality is such a drawing. Graphs of inequalities in one variable can be drawn on the number line by shading all points that are solutions. Parentheses are used to indicate endpoints that are *not* solutions and brackets to indicate endpoints that *are* solutions.\*

**EXAMPLE 3**

Graph each inequality: (a)  $x < 2$ ; (b)  $y \geq -3$ ; (c)  $-2 < x \leq 3$ .

**SOLUTION**

- a) The solutions of  $x < 2$  are those numbers less than 2. They are shown on the graph by shading all points to the left of 2. The parenthesis at 2 and the shading to its left indicate that 2 is *not* part of the graph, but numbers like 1.2 and 1.99 are.



- b) The solutions of  $y \geq -3$  are shown on the number line by shading the point for  $-3$  and all points to the right of  $-3$ . The bracket at  $-3$  indicates that  $-3$  is part of the graph.



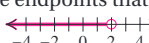
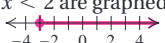
- c) The inequality  $-2 < x \leq 3$  is read “ $-2$  is less than  $x$  and  $x$  is less than or equal to 3,” or “ $x$  is greater than  $-2$  and less than or equal to 3.” To be a solution of  $-2 < x \leq 3$ , a number must be a solution of both  $-2 < x$  and  $x \leq 3$ . The number 1 is a solution, as are  $-0.5$ ,  $1.9$ , and  $3$ . The parenthesis indicates that  $-2$  is *not* a solution, whereas the bracket indicates that  $3$  is a solution. The other solutions are shaded.



TRY EXERCISE 17

**STUDENT NOTES**

Note that  $-2 < x < 3$  means  $-2 < x$  and  $x < 3$ . Because of this, statements like  $2 < x < 1$  make no sense—no number is both greater than 2 and less than 1.

\*An alternative notation uses open dots to indicate endpoints that are not solutions and closed dots to indicate endpoints that are solutions. Using this notation, the solutions of  $x < 2$  are graphed as  and the solutions of  $y \geq -3$  are graphed as .

## Set-Builder and Interval Notation

To write the solution set of  $x < 3$ , we can use **set-builder notation**:

$$\{x|x < 3\}.$$

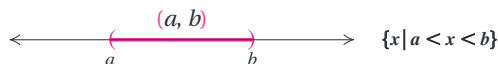
This is read “The set of all  $x$  such that  $x$  is less than 3.”

Another way to write solutions of an inequality in one variable is to use **interval notation**. Interval notation uses parentheses,  $( )$ , and brackets,  $[ ]$ .

If  $a$  and  $b$  are real numbers with  $a < b$ , we define the **open interval  $(a, b)$**  as the set of all numbers  $x$  for which  $a < x < b$ . Using set-builder notation, we write

$$(a, b) = \{x|a < x < b\}. \quad \text{Parentheses are used to exclude endpoints.}$$

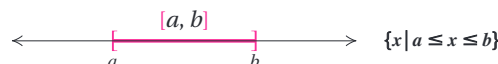
Its graph excludes the endpoints:



The **closed interval  $[a, b]$**  is defined as the set of all numbers  $x$  for which  $a \leq x \leq b$ . Thus,

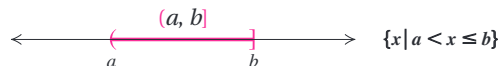
$$[a, b] = \{x|a \leq x \leq b\}. \quad \text{Brackets are used to include endpoints.}$$

Its graph includes the endpoints:



There are two kinds of **half-open intervals**, defined as follows:

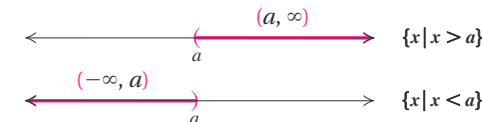
1.  $(a, b] = \{x|a < x \leq b\}$ . This is open on the left. Its graph is as follows:



2.  $[a, b) = \{x|a \leq x < b\}$ . This is open on the right. Its graph is as follows:



We use the symbols  $\infty$  and  $-\infty$  to represent positive infinity and negative infinity, respectively. Thus the notation  $(a, \infty)$  represents the set of all real numbers greater than  $a$ , and  $(-\infty, a)$  represents the set of all real numbers less than  $a$ .



The notation  $[a, \infty)$  or  $(-\infty, a]$  is used when we want to include the endpoint  $a$ .

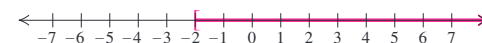
### EXAMPLE 4

Graph  $y \geq -2$  on a number line and write the solution set using both set-builder and interval notations.

**SOLUTION** Using set-builder notation, we write the solution set as  $\{y|y \geq -2\}$ .

Using interval notation, we write  $[-2, \infty)$ .

To graph the solution, we shade all numbers to the right of  $-2$  and use a bracket to indicate that  $-2$  is also a solution.



TRY EXERCISE 27

**CAUTION!** Do not confuse the *interval*  $(a, b)$  with the *ordered pair*  $(a, b)$ . The context in which the notation appears should make the meaning clear.

### STUDENT NOTES

You may have noticed which inequality signs in set-builder notation correspond to brackets and which correspond to parentheses. The relationship could be written informally as

$$\begin{array}{lll} \leq & \geq & [ ] \\ < & > & ( ) \end{array}$$

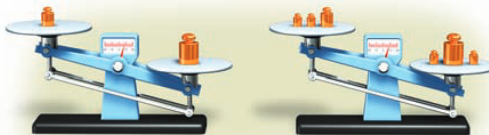
## STUDY SKILLS

## Sleep Well

Being well rested, alert, and focused is very important when studying math. Often, problems that may seem confusing to a sleepy person are easily understood after a good night's sleep. Using your time efficiently is always important, so you should be aware that an alert, wide-awake student can often accomplish more in 10 minutes than a sleepy student can accomplish in 30 minutes.

## Solving Inequalities Using the Addition Principle

Consider a balance similar to one that appears in Section 2.1. When one side of the balance holds more weight than the other, the balance tips in that direction. If equal amounts of weight are then added to or subtracted from both sides of the balance, the balance remains tipped in the same direction.



The balance illustrates the idea that when a number, such as 2, is added to (or subtracted from) both sides of a true inequality, such as  $3 < 7$ , we get another true inequality:

$$3 + 2 < 7 + 2, \text{ or } 5 < 9.$$

Similarly, if we add  $-4$  to both sides of  $x + 4 < 10$ , we get an *equivalent* inequality:

$$x + 4 + (-4) < 10 + (-4), \text{ or } x < 6.$$

We say that  $x + 4 < 10$  and  $x < 6$  are **equivalent**, which means that both inequalities have the same solution set.

## The Addition Principle for Inequalities

For any real numbers  $a$ ,  $b$ , and  $c$ :

$$a < b \text{ is equivalent to } a + c < b + c;$$

$$a \leq b \text{ is equivalent to } a + c \leq b + c;$$

$$a > b \text{ is equivalent to } a + c > b + c;$$

$$a \geq b \text{ is equivalent to } a + c \geq b + c.$$

As with equations, our goal is to isolate the variable on one side.

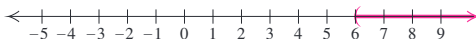
## EXAMPLE 5

Solve  $x + 2 > 8$  and then graph the solution.

**SOLUTION** We use the addition principle, subtracting 2 from both sides:

$$\begin{aligned} x + 2 - 2 &> 8 - 2 && \text{Subtracting 2 from, or adding } -2 \text{ to, both sides} \\ x &> 6. \end{aligned}$$

From the inequality  $x > 6$ , we can determine the solutions easily. Any number greater than 6 makes  $x > 6$  true and is a solution of that inequality as well as the inequality  $x + 2 > 8$ . Using set-builder notation, the solution set is  $\{x \mid x > 6\}$ . Using interval notation, the solution set is  $(6, \infty)$ . The graph is as follows:



Because most inequalities have an infinite number of solutions, we cannot possibly check them all. A partial check can be made using one of the possible solutions. For this example, we can substitute any number greater than 6—say, 6.1—into the original inequality:

$$\begin{array}{r} x + 2 > 8 \\ 6.1 + 2 \quad | \quad 8 \\ 8.1 \overset{?}{>} 8 \quad \text{TRUE} \quad 8.1 > 8 \text{ is a true statement.} \end{array}$$

Since  $8.1 > 8$  is true, 6.1 is a solution. Any number greater than 6 is a solution.

TRY EXERCISE 43

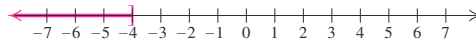
**EXAMPLE 6**

Solve  $3x - 1 \leq 2x - 5$  and then graph the solution.

**SOLUTION** We have

$$\begin{array}{ll} 3x - 1 \leq 2x - 5 & \\ 3x - 1 + 1 \leq 2x - 5 + 1 & \text{Adding 1 to both sides} \\ 3x \leq 2x - 4 & \text{Simplifying} \\ 3x - 2x \leq 2x - 4 - 2x & \text{Subtracting } 2x \text{ from both sides} \\ x \leq -4. & \text{Simplifying} \end{array}$$

The graph is as follows:



The student should check that any number less than or equal to  $-4$  is a solution. The solution set is  $\{x \mid x \leq -4\}$ , or  $(-\infty, -4]$ .

TRY EXERCISE 47

**TECHNOLOGY CONNECTION**

As a partial check of Example 5, we can let  $y_1 = 3x - 1$  and  $y_2 = 2x - 5$ . We set  $\text{TblStart} = -5$  and  $\Delta\text{Tbl} = 1$  in the **TABLESET** menu to get the following table. By scrolling up or down, you can note that for  $x \leq -4$ , we have  $y_1 \leq y_2$ .

X	Y <sub>1</sub>	Y <sub>2</sub>
2	-16	-15
5	-4	-13
	-13	-13
-3	-10	-11
-2	-7	-9
-1	-4	-7
0	-1	-5
1	2	-3

X = -5

**Solving Inequalities Using the Multiplication Principle**

There is a multiplication principle for inequalities similar to that for equations, but it must be modified when multiplying both sides by a negative number. Consider the true inequality

$$3 < 7.$$

If we multiply both sides by a *positive* number—say, 2—we get another true inequality:

$$3 \cdot 2 < 7 \cdot 2, \text{ or } 6 < 14. \quad \text{TRUE}$$

If we multiply both sides by a negative number—say,  $-2$ —we get a *false* inequality:

$$3 \cdot (-2) < 7 \cdot (-2), \text{ or } -6 < -14. \quad \text{FALSE}$$

The fact that  $6 < 14$  is true, but  $-6 < -14$  is false, stems from the fact that the negative numbers, in a sense, *mirror* the positive numbers. Whereas 14 is to the *right* of 6, the number  $-14$  is to the *left* of  $-6$ . Thus if we reverse the inequality symbol in  $-6 < -14$ , we get a true inequality:

$$-6 > -14. \quad \text{TRUE}$$



**The Multiplication Principle for Inequalities**

For any real numbers  $a$  and  $b$ , and for any *positive* number  $c$ :

$$a < b \text{ is equivalent to } ac < bc, \text{ and}$$

$$a > b \text{ is equivalent to } ac > bc.$$

For any real numbers  $a$  and  $b$ , and for any *negative* number  $c$ :

$$a < b \text{ is equivalent to } ac > bc, \text{ and}$$

$$a > b \text{ is equivalent to } ac < bc.$$

Similar statements hold for  $\leq$  and  $\geq$ .

**CAUTION!** When multiplying or dividing both sides of an inequality by a negative number, don't forget to reverse the inequality symbol!

**EXAMPLE 7**

Solve and graph each inequality: (a)  $\frac{1}{4}x < 7$ ; (b)  $-2y \leq 18$ .

**SOLUTION**

a)  $\frac{1}{4}x < 7$

$$4 \cdot \frac{1}{4}x < 4 \cdot 7 \quad \text{Multiplying both sides by 4, the reciprocal of } \frac{1}{4}$$

↑  
The symbol stays the same, since 4 is positive.

$$x < 28 \quad \text{Simplifying}$$



The solution set is  $\{x|x < 28\}$ , or  $(-\infty, 28)$ . The graph is shown at left.

b)  $-2y \leq 18$

$$\frac{-2y}{-2} \geq \frac{18}{-2} \quad \text{Multiplying both sides by } -\frac{1}{2}, \text{ or dividing both sides by } -2$$

↑  
At this step, we reverse the inequality, because  $-\frac{1}{2}$  is negative.

$$y \geq -9 \quad \text{Simplifying}$$

As a partial check, we substitute a number greater than  $-9$ , say  $-8$ , into the original inequality:

$$\frac{-2y \leq 18}{-2(-8) \quad | \quad 18}$$

$$16 \stackrel{?}{\leq} 18 \quad \text{TRUE} \quad 16 \leq 18 \text{ is a true statement.}$$



The solution set is  $\{y|y \geq -9\}$ , or  $[-9, \infty)$ . The graph is shown at left.

TRY EXERCISE 59

**Using the Principles Together**

We use the addition and multiplication principles together to solve inequalities much as we did when solving equations.

**EXAMPLE 8**

Solve: (a)  $6 - 5y > 7$ ; (b)  $2x - 9 < 7x + 1$ .

**SOLUTION**

a)  $6 - 5y > 7$

$$-6 + 6 - 5y > -6 + 7 \quad \text{Adding } -6 \text{ to both sides}$$

$$-5y > 1 \quad \text{Simplifying}$$

$$-\frac{1}{5} \cdot (-5y) < -\frac{1}{5} \cdot 1$$

Multiplying both sides by  $-\frac{1}{5}$ , or dividing both sides by  $-5$

$$y < -\frac{1}{5}$$

Remember to reverse the inequality symbol!  
Simplifying

As a partial check, we substitute a number smaller than  $-\frac{1}{5}$ , say  $-1$ , into the original inequality:

$$\begin{array}{r} 6 - 5y > 7 \\ 6 - 5(-1) \quad | \quad 7 \\ 6 - (-5) \quad | \\ 11 > 7 \quad \text{TRUE} \quad 11 > 7 \text{ is a true statement.} \end{array}$$

The solution set is  $\{y \mid y < -\frac{1}{5}\}$ , or  $(-\infty, -\frac{1}{5})$ . We show the graph in the margin for reference.



**b)**

$$2x - 9 < 7x + 1$$

$$2x - 9 - 1 < 7x + 1 - 1 \quad \text{Subtracting 1 from both sides}$$

$$2x - 10 < 7x \quad \text{Simplifying}$$

$$2x - 10 - 2x < 7x - 2x \quad \text{Subtracting 2x from both sides}$$

$$-10 < 5x \quad \text{Simplifying}$$

$$\frac{-10}{5} < \frac{5x}{5} \quad \text{Dividing both sides by 5}$$

$$-2 < x \quad \text{Simplifying}$$

The solution set is  $\{x \mid -2 < x\}$ , or  $\{x \mid x > -2\}$ , or  $(-2, \infty)$ .

TRY EXERCISE 69

All of the equation-solving techniques used in Sections 2.1 and 2.2 can be used with inequalities provided we remember to reverse the inequality symbol when multiplying or dividing both sides by a negative number.

**EXAMPLE 9**

Solve: **(a)**  $16.3 - 7.2p \leq -8.18$ ; **(b)**  $3(x - 9) - 1 \leq 2 - 5(x + 6)$ .

**SOLUTION**

**a)** The greatest number of decimal places in any one number is *two*. Multiplying both sides by 100 will clear decimals. Then we proceed as before.

$$16.3 - 7.2p \leq -8.18$$

$$100(16.3 - 7.2p) \leq 100(-8.18) \quad \text{Multiplying both sides by 100}$$

$$100(16.3) - 100(7.2p) \leq 100(-8.18) \quad \text{Using the distributive law}$$

$$1630 - 720p \leq -818 \quad \text{Simplifying}$$

$$-720p \leq -818 - 1630 \quad \text{Subtracting 1630 from both sides}$$

$$-720p \leq -2448 \quad \text{Simplifying; } -818 - 1630 = -2448$$

$$p \geq \frac{-2448}{-720} \quad \text{Dividing both sides by } -720$$

Remember to reverse the symbol!

$$p \geq 3.4$$



The solution set is  $\{p \mid p \geq 3.4\}$ , or  $[3.4, \infty)$ .

$$\begin{aligned} \text{b) } 3(x - 9) - 1 &\leq 2 - 5(x + 6) \\ 3x - 27 - 1 &\leq 2 - 5x - 30 && \text{Using the distributive law to remove} \\ &&& \text{parentheses} \\ 3x - 28 &\leq -5x - 28 && \text{Simplifying} \\ 3x - 28 + 28 &\leq -5x - 28 + 28 && \text{Adding 28 to both sides} \\ 3x &\leq -5x \\ 3x + 5x &\leq -5x + 5x && \text{Adding 5x to both sides} \\ 8x &\leq 0 \\ x &\leq 0 && \text{Dividing both sides by 8} \end{aligned}$$



The solution set is  $\{x|x \leq 0\}$ , or  $(-\infty, 0]$ .

TRY EXERCISE 83

## 2.6

## EXERCISE SET

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**i Concept Reinforcement** Insert the symbol  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , or  $\neq$  to make each pair of inequalities equivalent.

1.  $-5x \leq 30$ ;  $x \blacksquare -6$       2.  $-7t \geq 56$ ;  $t \blacksquare -8$   
3.  $-2t > -14$ ;  $t \blacksquare 7$       4.  $-3x < -15$ ;  $x \blacksquare 5$

Classify each pair of inequalities as “equivalent” or “not equivalent.”

5.  $x < -2$ ;  $-2 > x$       6.  $t > -1$ ;  $-1 < t$   
7.  $-4x - 1 \leq 15$ ;  $-4x \leq 16$       8.  $-2t + 3 \geq 11$ ;  $-2t \geq 14$

Determine whether each number is a solution of the given inequality.

9.  $x > -4$   
a) 4      b)  $-6$       c)  $-4$   
10.  $t < 3$   
a)  $-3$       b) 3      c)  $2\frac{19}{20}$   
11.  $y \leq 19$   
a) 18.99      b) 19.01      c) 19  
12.  $n \geq -4$   
a) 0      b)  $-4.1$       c)  $-3.9$   
13.  $c \geq -7$   
a) 0      b)  $-5.4$       c) 7.1

14.  $a > 6$   
a) 6      b)  $-6.7$       c) 0  
15.  $z < -3$   
a) 0      b)  $-3\frac{1}{3}$       c) 1  
16.  $m \leq -2$   
a)  $-1\frac{9}{10}$       b) 0      c)  $-2\frac{1}{3}$

Graph on a number line.

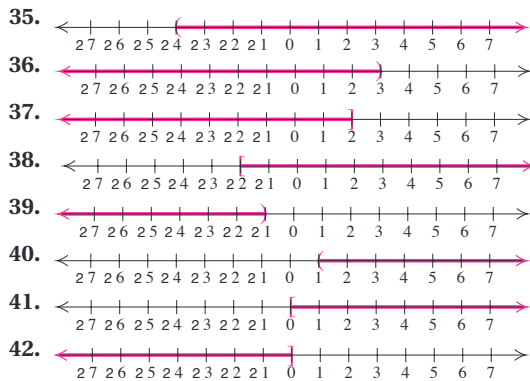
17.  $y < 2$       18.  $x \leq 7$   
19.  $x \geq -1$       20.  $t > -2$   
21.  $0 \leq t$       22.  $1 \leq m$   
23.  $-5 \leq x < 2$   
24.  $-3 < x \leq 5$   
25.  $-4 < x < 0$   
26.  $0 \leq x \leq 5$

Graph each inequality, and write the solution set using both set-builder notation and interval notation.

27.  $y < 6$       28.  $x > 4$   
29.  $x \geq -4$       30.  $t \leq 6$   
31.  $t > -3$       32.  $y < -3$   
33.  $x \leq -7$       34.  $x \geq -6$



Describe each graph using set-builder notation and interval notation.



Solve using the addition principle. Graph and write set-builder notation and interval notation for each answer.

43.  $y + 6 > 9$                       44.  $x + 8 \leq -10$   
 45.  $n - 6 < 11$                       46.  $n - 4 > -3$   
 47.  $2x \leq x - 9$                       48.  $3x \leq 2x + 7$   
 49.  $y + \frac{1}{3} \leq \frac{5}{6}$                       50.  $x + \frac{1}{4} \leq \frac{1}{2}$   
 51.  $t - \frac{1}{8} > \frac{1}{2}$                       52.  $y - \frac{1}{3} > \frac{1}{4}$   
 53.  $-9x + 17 > 17 - 8x$   
 54.  $-8n + 12 > 12 - 7n$

- Aha!** 55.  $-23 < -t$                       56.  $19 < -x$   
 57.  $10 - y \leq -12$                       58.  $3 - y \geq -6$

Solve using the multiplication principle. Graph and write set-builder notation and interval notation for each answer.

59.  $4x < 28$                       60.  $3x \geq 24$   
 61.  $-24 > 8t$                       62.  $-16x < -64$   
 63.  $1.8 \geq -1.2n$                       64.  $9 \leq -2.5a$   
 65.  $-2y \leq \frac{1}{5}$                       66.  $-2x \geq \frac{1}{5}$   
 67.  $-\frac{8}{5} > 2x$                       68.  $-\frac{5}{8} < -10y$

Solve using the addition and multiplication principles.

69.  $2 + 3x < 20$                       70.  $7 + 4y < 31$   
 71.  $4t - 5 \leq 23$                       72.  $15x - 7 \leq -7$   
 73.  $39 > 3 - 9x$                       74.  $5 > 5 - 7y$   
 75.  $5 - 6y > 25$                       76.  $8 - 2y > 9$   
 77.  $-3 < 8x + 7 - 7x$                       78.  $-5 < 9x + 8 - 8x$   
 79.  $6 - 4y > 6 - 3y$                       80.  $7 - 8y > 5 - 7y$   
 81.  $7 - 9y \leq 4 - 7y$                       82.  $6 - 13y \leq 4 - 12y$

83.  $2.1x + 43.2 > 1.2 - 8.4x$   
 84.  $0.96y - 0.79 \leq 0.21y + 0.46$   
 85.  $1.7t + 8 - 1.62t < 0.4t - 0.32 + 8$   
 86.  $0.7n - 15 + n \geq 2n - 8 - 0.4n$

87.  $\frac{x}{3} + 4 \leq 1$

88.  $\frac{2}{3} - \frac{x}{5} < \frac{4}{15}$

89.  $3 < 5 - \frac{t}{7}$

90.  $2 > 9 - \frac{x}{5}$

91.  $4(2y - 3) \leq -44$

92.  $3(2y - 3) > 21$

93.  $8(2t + 1) > 4(7t + 7)$

94.  $3(t - 2) \geq 9(t + 2)$

95.  $3(r - 6) + 2 < 4(r + 2) - 21$


96.  $5(t + 3) + 9 \geq 3(t - 2) - 10$


97.  $\frac{4}{5}(3x + 4) \leq 20$

98.  $\frac{2}{3}(2x - 1) \geq 10$

99.  $\frac{2}{3}(\frac{7}{8} - 4x) - \frac{5}{8} < \frac{3}{8}$

100.  $\frac{3}{4}(3x - \frac{1}{2}) - \frac{2}{3} < \frac{1}{3}$

 101. Are the inequalities  $x > -3$  and  $x \geq -2$  equivalent? Why or why not?

 102. Are the inequalities  $t < -7$  and  $t \leq -8$  equivalent? Why or why not?

## Skill Review

Review simplifying expressions (Section 1.8).

Simplify. [1.8]

103.  $5x - 2(3 - 6x)$

104.  $8m - n - 3(2m + 5n)$


105.  $x - 2[4y + 3(8 - x) - 1]$


106.  $5 - 3t - 4[6 + 5(2t - 1) + t]$

107.  $3[5(2a - b) + 1] - 5[4 - (a - b)]$

108.  $9x - 2\{4 - 5[6 - 2(x + 1) - x]\}$

## Synthesis

 **109.** Explain how it is possible for the graph of an inequality to consist of just one number. (*Hint:* See Example 3c.)

 **110.** The statements of the addition and multiplication principles begin with *conditions* set for the variables. Explain the conditions given for each principle.

*Solve.*

**Aha!** **111.**  $x < x + 1$

**112.**  $6[4 - 2(6 + 3t)] > 5[3(7 - t) - 4(8 + 2t)] - 20$

**113.**  $27 - 4[2(4x - 3) + 7] \geq 2[4 - 2(3 - x)] - 3$

*Solve for x.*

**114.**  $\frac{1}{2}(2x + 2b) > \frac{1}{3}(21 + 3b)$

**115.**  $-(x + 5) \geq 4a - 5$

**116.**  $y < ax + b$  (Assume  $a < 0$ .)

**117.**  $y < ax + b$  (Assume  $a > 0$ .)

**118.** Graph the solutions of  $|x| < 3$  on a number line.

**Aha!** **119.** Determine the solution set of  $|x| > -3$ .

**120.** Determine the solution set of  $|x| < 0$ .

## CONNECTING the CONCEPTS

The procedure for solving inequalities is very similar to that used to solve equations. There are, however, two important differences.

- The multiplication principle for inequalities differs from the multiplication principle for equations: When we multiply or divide on both sides of an inequality by a *negative* number, we must *reverse* the direction of the inequality.
- The solution set of an equation like those we solved in this chapter typically consists of one number. The solution set of an inequality typically consists of a set of numbers and is written using set-builder notation.

Compare the following solutions.

Solve:  $2 - 3x = x + 10$ .

**SOLUTION**

$$2 - 3x = x + 10$$

$$-3x = x + 8$$

Subtracting 2 from both sides

$$-4x = 8$$

Subtracting  $x$  from both sides

$$x = -2$$

Dividing both sides by  $-4$

The solution is  $-2$ .

Solve:  $2 - 3x > x + 10$ .

**SOLUTION**

$$2 - 3x > x + 10$$

$$-3x > x + 8$$

Subtracting 2 from both sides

$$-4x > 8$$

Subtracting  $x$  from both sides

$$x < -2$$

Dividing both sides by  $-4$  and reversing the direction of the inequality symbol

The solution is  $\{x|x < -2\}$ , or  $(-\infty, -2)$ .

**MIXED REVIEW**

Solve.

1.  $x - 6 = 15$

2.  $x - 6 \square 15$

3.  $3x = -18$

4.  $3x - 7 = 18$

5.  $-3x - 7 = 18$

6.  $5x + 2 = 17$

7.  $7 - 3x = 8$

8.  $4y - 7 = 5$

9.  $3 - t \square 19$

10.  $2 + 3n = 5n - 9$

11.  $3 - 5a = 7a + 9$

12.  $1.2x - 3.4 = 0.4x + 5.2$

13.  $\frac{2}{3}x + 52 \square -4$

14.  $\frac{n}{5} - 6 = 15$

15.  $0.5x - 2.7 = 3x + 7.9$

16.  $516 - t = -45$

17.  $8 - \frac{y}{3} \square 7$

18.  $\frac{1}{3}x - \frac{5}{6} = \frac{3}{2} - \frac{1}{6}x$

19.  $-15 = 7 - 5x$

20.  $10 \square -21a - 52$