

The five steps for problem solving can be used for problems involving inequalities.

Translating to Inequalities

Before solving problems that involve inequalities, we list some important phrases to look for. Sample translations are listed as well.

STUDY SKILLS

Avoiding Temptation

Maximize the effectiveness of your study time by choosing the right location. For example, sit in a comfortable chair in a well-lit location, but stay away from a coffee shop where friends may stop by. Once you begin studying, let the answering machine answer the phone, shut off any cell phones, and do not check e-mail.

Important Words	Sample Sentence	Translation
is at least	Ming walks at least 2 mi a day.	$m \geq 2$
is at most	At most 5 students dropped the course.	$n \leq 5$
cannot exceed	The width cannot exceed 40 ft.	$w \leq 40$
must exceed	The speed must exceed 15 mph.	$s > 15$
is less than	Kamal's weight is less than 120 lb.	$w < 120$
is more than	Boston is more than 200 mi away.	$d > 200$
is between	The film was between 90 and 100 min long.	$90 < t < 100$
minimum	Ned drank a minimum of 5 glasses of water a day.	$w \geq 5$
maximum	The maximum penalty is \$100.	$p \leq 100$
no more than	Alan weighs no more than 90 lb.	$w \leq 90$
no less than	Mallory scored no less than 8.3.	$s \geq 8.3$

The following phrases deserve special attention.

Translating “at least” and “at most”

The quantity x is at least some amount q : $x \geq q$.
 (If x is *at least* q , it cannot be less than q .)

The quantity x is at most some amount q : $x \leq q$.
 (If x is *at most* q , it cannot be more than q .)

Solving Problems

EXAMPLE 1

Catering costs. To cater a party, Curtis’ Barbeque charges a \$50 setup fee plus \$15 per person. The cost of Hotel Pharmacy’s end-of-season softball party cannot exceed \$450. How many people can attend the party?

SOLUTION

- Familiarize.** Suppose that 20 people were to attend the party. The cost would then be $\$50 + \$15 \cdot 20$, or \$350. This shows that more than 20 people could attend without exceeding \$450. Instead of making another guess, we let n = the number of people in attendance.
- Translate.** The cost of the party will be \$50 for the setup fee plus \$15 times the number of people attending. We can reword as follows:

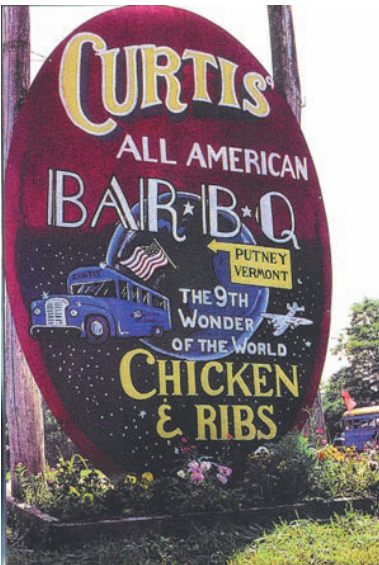
Rewording:	The setup fee	plus	the cost of the meals	cannot exceed	\$450.
	↓		↓	↓	↓
Translating:	50	+	$15 \cdot n$	\leq	450

- Carry out.** We solve for n :

$$\begin{aligned}
 50 + 15n &\leq 450 \\
 15n &\leq 400 && \text{Subtracting 50 from both sides} \\
 n &\leq \frac{400}{15} && \text{Dividing both sides by 15} \\
 n &\leq 26\frac{2}{3} && \text{Simplifying}
 \end{aligned}$$

- Check.** Although the solution set of the inequality is all numbers less than or equal to $26\frac{2}{3}$, since n represents the number of people in attendance, we round *down* to 26. If 26 people attend, the cost will be $\$50 + \$15 \cdot 26$, or \$440, and if 27 attend, the cost will exceed \$450.
- State.** At most 26 people can attend the party.

TRY EXERCISE 23



CAUTION! Solutions of problems should always be checked using the original wording of the problem. In some cases, answers might need to be whole numbers or integers or rounded off in a particular direction.

Some applications with inequalities involve *averages*, or *means*. You are already familiar with the concept of averages from grades in courses that you have taken.

Average, or Mean

To find the **average**, or **mean**, of a set of numbers, add the numbers and then divide by the number of addends.

EXAMPLE 2

Financial aid. Full-time students in a health-care education program can receive financial aid and employee benefits from Covenant Health System by working at Covenant while attending school and also agreeing to work there after graduation. Students who work an average of at least 16 hr per week receive extra pay and part-time employee benefits. For the first three weeks of September, Dina worked 20 hr, 12 hr, and 14 hr. How many hours must she work during the fourth week in order to average at least 16 hr per week for the month?

Source: Covenant Health Systems

SOLUTION

1. **Familiarize.** Suppose Dina works 10 hr during the fourth week. Her average for the month would be

$$\frac{20 \text{ hr} + 12 \text{ hr} + 14 \text{ hr} + 10 \text{ hr}}{4} = 14 \text{ hr.} \quad \text{There are 4 addends, so we divide by 4.}$$

This shows that Dina must work more than 10 hr during the fourth week, if she is to average at least 16 hr of work per week. We let x represent the number of hours Dina works during the fourth week.

2. **Translate.** We reword the problem and translate as follows:

<i>Rewording:</i>	The average number of hours worked	should be at least	16 hr.
	↓	↓	↓
<i>Translating:</i>	$\frac{20 + 12 + 14 + x}{4}$	\geq	16

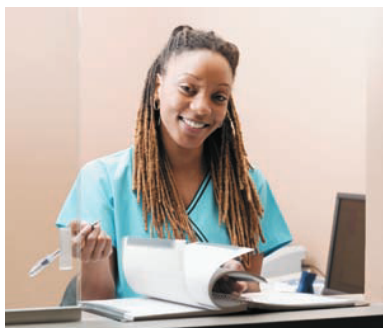
3. **Carry out.** Because of the fraction, it is convenient to use the multiplication principle first:

$$\begin{aligned} \frac{20 + 12 + 14 + x}{4} &\geq 16 \\ 4 \square \frac{20 + 12 + 14 + x}{4} \square &\geq 4 \cdot 16 && \text{Multiplying both sides by 4} \\ 20 + 12 + 14 + x &\geq 64 \\ 46 + x &\geq 64 && \text{Simplifying} \\ x &\geq 18. && \text{Subtracting 46 from both sides} \end{aligned}$$

4. **Check.** As a partial check, we show that if Dina works 18 hr, she will average at least 16 hr per week:

$$\frac{20 + 12 + 14 + 18}{4} = \frac{64}{4} = 16. \quad \text{Note that 16 is at least 16.}$$

5. **State.** Dina will average at least 16 hr of work per week for September if she works at least 18 hr during the fourth week.



Translating for Success

1. **Consecutive integers.** The sum of two consecutive even integers is 102. Find the integers.

2. **Salary increase.** After Susanna earned a 5% raise, her new salary was \$25,750. What was her former salary?

3. **Dimensions of a rectangle.** The length of a rectangle is 6 in. more than the width. The perimeter of the rectangle is 102 in. Find the length and the width.

4. **Population.** The population of Kelling Point is decreasing at a rate of 5% per year. The current population is 25,750. What was the population the previous year?

5. **Reading assignment.** Quinn has 6 days to complete a 150-page reading assignment. How many pages must he read the first day so that he has no more than 102 pages left to read on the 5 remaining days?

Translate each word problem to an equation or an inequality and select a correct translation from A–O.

- A. $0.05(25,750) = x$
- B. $x + 2x = 102$
- C. $2x + 2(x + 6) = 102$
- D. $150 - x \leq 102$
- E. $x - 0.05x = 25,750$
- F. $x + (x + 2) = 102$
- G. $x + (x + 6) > 102$
- H. $x + 5x = 150$
- I. $x + 0.05x = 25,750$
- J. $x + (2x + 6) = 102$
- K. $x + (x + 1) = 102$
- L. $102 + x > 150$
- M. $0.05x = 25,750$
- N. $102 + 5x > 150$
- O. $x + (x + 6) = 102$

Answers on page A-5

An additional, animated version of this activity appears in MyMathLab. To use MyMathLab, you need a course ID and a student access code. Contact your instructor for more information.

6. **Numerical relationship.** One number is 6 more than twice another. The sum of the numbers is 102. Find the numbers.

7. **DVD collections.** Together Mindy and Ken have 102 DVDs. If Ken has 6 more DVDs than Mindy, how many does each have?

8. **Sales commissions.** Kirk earns a commission of 5% on his sales. One year he earned commissions totaling \$25,750. What were his total sales for the year?

9. **Fencing.** Jess has 102 ft of fencing that he plans to use to enclose two dog runs. The perimeter of one run is to be twice the perimeter of the other. Into what lengths should the fencing be cut?

10. **Quiz scores.** Lupe has a total of 102 points on the first 6 quizzes in her sociology class. How many total points must she earn on the 5 remaining quizzes in order to have more than 150 points for the semester?

2.7

EXERCISE SET

For Extra Help
MyMathLab

MathXL
PRACTICE

WATCH

DOWNLOAD

i Concept Reinforcement In each of Exercises 1–8, match the sentence with one of the following:

$$a < b; \quad a \leq b; \quad b < a; \quad b \leq a.$$

1. a is at least b .
2. a exceeds b .
3. a is at most b .
4. a is exceeded by b .
5. b is no more than a .
6. b is no less than a .
7. b is less than a .
8. b is more than a .

Translate to an inequality.

9. A number is less than 10.
10. A number is greater than or equal to 4.
11. The temperature is at most -3°C .
12. The average credit-card debt is more than \$8000.
13. To rent a car, a driver must have a minimum of 5 yr driving experience.
14. The Barringdean Shopping Center is no more than 20 mi away.
15. The age of the Mayan altar exceeds 1200 yr.
16. The maximum safe exposure limit of formaldehyde is 2 parts per million.
17. Tania earns between \$12 and \$15 an hour.
18. Leslie's test score was at least 85.
19. Wind speeds were greater than 50 mph.
20. The costs of production of that software cannot exceed \$12,500.
21. A room at Pine Tree Bed and Breakfast costs no more than \$120 a night.
22. The cost of gasoline was at most \$4 per gallon.

Use an inequality and the five-step process to solve each problem.

23. **Furnace repairs.** RJ's Plumbing and Heating charges \$55 plus \$40 per hour for emergency service. Gary remembers being billed over \$150 for an emergency call. How long was RJ's there?
24. **College tuition.** Karen's financial aid stipulates that her tuition not exceed \$1000. If her local community college charges a \$35 registration fee plus \$375 per course, what is the greatest number of courses for which Karen can register?
25. **Graduate school.** An unconditional acceptance into the Master of Business Administration (MBA) program at Arkansas State University will be given to students whose GMAT score plus 200 times the undergraduate grade point average is at least 950. Robbin's GMAT score was 500. What must her grade point average be in order to be unconditionally accepted into the program?
Source: graduateschool.astate.edu
26. **Car payments.** As a rule of thumb, debt payments (other than mortgages) should be less than 8% of a consumer's monthly gross income. Oliver makes \$54,000 a year and has a \$100 student-loan payment every month. What size car payment can he afford?
Source: money.cnn.com
27. **Quiz average.** Rod's quiz grades are 73, 75, 89, and 91. What scores on a fifth quiz will make his average quiz grade at least 85?
28. **Nutrition.** Following the guidelines of the U.S. Department of Agriculture, Dale tries to eat at least 5 half-cup servings of vegetables each day. For the first six days of one week, she had 4, 6, 7, 4, 6, and 4 servings. How many servings of vegetables should Dale eat on Saturday, in order to average at least 5 servings per day for the week?
29. **College course load.** To remain on financial aid, Millie needs to complete an average of at least 7 credits per quarter each year. In the first three quarters of 2008, Millie completed 5, 7, and 8 credits. How many credits of course work must Millie complete in the fourth quarter if she is to remain on financial aid?

30. **Music lessons.** Band members at Colchester Middle School are expected to average at least 20 min of practice time per day. One week Monroe practiced 15 min, 28 min, 30 min, 0 min, 15 min, and 25 min. How long must he practice on the seventh day if he is to meet expectations?



31. **Baseball.** In order to qualify for a batting title, a major league baseball player must average at least 3.1 plate appearances per game. For the first nine games of the season, a player had 5, 1, 4, 2, 3, 4, 4, 3, and 2 plate appearances. How many plate appearances must the player have in the tenth game in order to average at least 3.1 per game?
Source: Major League Baseball

32. **Education.** The Mecklenberg County Public Schools stipulate that a standard school day will average at least $5\frac{1}{2}$ hr, excluding meal breaks. For the first four days of one school week, bad weather resulted in school days of 4 hr, $6\frac{1}{2}$ hr, $3\frac{1}{2}$ hr, and $6\frac{1}{2}$ hr. How long must the Friday school day be in order to average at least $5\frac{1}{2}$ hr for the week?
Source: www.meck.k12.va.us

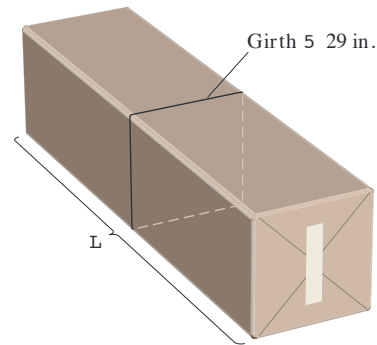
33. **Perimeter of a triangle.** One side of a triangle is 2 cm shorter than the base. The other side is 3 cm longer than the base. What lengths of the base will allow the perimeter to be greater than 19 cm?
34. **Perimeter of a sign.** The perimeter of a rectangular sign is not to exceed 50 ft. The length is to be twice the width. What widths will meet these conditions?
35. **Well drilling.** All Seasons Well Drilling offers two plans. Under the “pay-as-you-go” plan, they charge \$500 plus \$8 a foot for a well of any depth. Under their “guaranteed-water” plan, they charge a flat fee of \$4000 for a well that is guaranteed to provide adequate water for a household. For what depths would it save a customer money to use the pay-as-you-go plan?

36. **Cost of road service.** Rick’s Automotive charges \$50 plus \$15 for each (15-min) unit of time when making a road call. Twin City Repair charges \$70 plus \$10 for each unit of time. Under what circumstances would it be more economical for a motorist to call Rick’s?

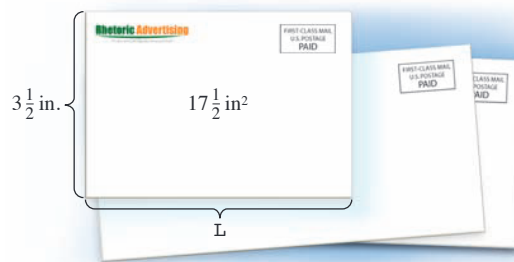
37. **Insurance-covered repairs.** Most insurance companies will replace a vehicle if an estimated repair exceeds 80% of the “blue-book” value of the vehicle. Michele’s insurance company paid \$8500 for repairs to her Subaru after an accident. What can be concluded about the blue-book value of the car?

38. **Insurance-covered repairs.** Following an accident, Jeff’s Ford pickup was replaced by his insurance company because the damage was so extensive. Before the damage, the blue-book value of the truck was \$21,000. How much would it have cost to repair the truck? (See Exercise 37.)

39. **Sizes of packages.** The U.S. Postal Service defines a “package” as a parcel for which the sum of the length and the girth is less than 84 in. (Length is the longest side of a package and girth is the distance around the other two sides of the package.) A box has a fixed girth of 29 in. Determine (in terms of an inequality) those lengths for which the box is considered a “package.”



40. **Sizes of envelopes.** Rhetoric Advertising is a direct-mail company. It determines that for a particular campaign, it can use any envelope with a fixed width of $3\frac{1}{2}$ in. and an area of at least $17\frac{1}{2}$ in². Determine (in terms of an inequality) those lengths that will satisfy the company constraints.



41. **Body temperature.** A person is considered to be feverish when his or her temperature is higher than 98.6°F . The formula $F = \frac{9}{5}C + 32$ can be used to

convert Celsius temperatures C to Fahrenheit temperatures F . For which Celsius temperatures is a person considered feverish?

42. **Gold temperatures.** Gold stays solid at Fahrenheit temperatures below 1945.4° . Determine (in terms of an inequality) those Celsius temperatures for which gold stays solid. Use the formula given in Exercise 41.



43. **Area of a triangular sign.** Zoning laws in Harrington prohibit displaying signs with areas exceeding 12 ft^2 . If Flo's Marina is ordering a triangular sign with an 8-ft base, how tall can the sign be?



44. **Area of a triangular flag.** As part of an outdoor education course, Trisha needs to make a bright-colored triangular flag with an area of at least 3 ft^2 . What heights can the triangle be if the base is $1\frac{1}{2} \text{ ft}$?



45. **Fat content in foods.** Reduced Fat Skippy[®] peanut butter contains 12 g of fat per serving. In order for a food to be labeled "reduced fat," it must have at least 25% less fat than the regular item. What can you conclude about the number of grams of fat in a serving of the regular Skippy peanut butter?

Source: Best Foods

46. **Fat content in foods.** Reduced Fat Chips Ahoy![®] cookies contain 5 g of fat per serving. What can you conclude about the number of grams of fat in regular Chips Ahoy! cookies (see Exercise 45)?

Source: Nabisco Brands, Inc.

47. **Weight gain.** In the last weeks before the yearly Topsfield Weigh In, heavyweight pumpkins gain about 26 lb per day. Charlotte's heaviest pumpkin weighs 532 lb on September 5. For what dates will its weight exceed 818 lb?

Source: Based on a story in the Burlington Free Press

48. **Pond depth.** On July 1, Garrett's Pond was 25 ft deep. Since that date, the water level has dropped $\frac{2}{3} \text{ ft}$ per week. For what dates will the water level not exceed 21 ft?

49. **Cell-phone budget.** Liam has budgeted \$60 a month for his cell phone. For his service, he pays a monthly fee of \$39.95, plus taxes of \$6.65, plus 10¢ for each text message sent or received. How many text messages can he send or receive and not exceed his budget?

50. **Banquet costs.** The women's volleyball team can spend at most \$700 for its awards banquet at a local restaurant. If the restaurant charges a \$100 setup fee plus \$24 per person, at most how many can attend?

51. **World records in the mile run.** The formula

$$R = -0.0065t + 4.3259$$

can be used to predict the world record, in minutes, for the 1-mi run t years after 1900. Determine (in terms of an inequality) those years for which the world record will be less than 3.6 min.

Source: Based on information from Information Please Database 2007, Pearson Education, Inc.

52. **Women's records in the women's 1500-m run.** The formula

$$R = -0.0026t + 4.0807$$

can be used to predict the world record, in minutes, for the 1500-m run t years after 1900. Determine (in terms of an inequality) those years for which the world record will be less than 3.8 min.


Source: Based on information from Track and Field

-  **53. Toll charges.** The equation

$$y = 0.06x + 0.50$$

can be used to determine the approximate cost y , in dollars, of driving x miles on the Pennsylvania Turnpike. For what mileages x will the cost be at most \$14?





-  **54. Price of a movie ticket.** The average price of a movie ticket can be estimated by the equation

$$P = 0.169Y - 333.04,$$

where Y is the year and P is the average price, in dollars. For what years will the average price of a movie ticket be at least \$7? (Include the year in which the \$7 ticket first occurs.)

Source: National Association of Theatre Owners

-  **55.** If f represents Fran's age and t represents Todd's age, write a sentence that would translate to $t + 3 < f$.
-  **56.** Explain how the meanings of "Five more than a number" and "Five is more than a number" differ.

Skill Review

Review operations with real numbers (Sections 1.5–1.8).

Simplify.

57. $-2 + (-5) - 7$ [1.6]

58. $\frac{1}{2} \div \square - \frac{3}{4} \square$ [1.7]

59. $3 \cdot (-10) \cdot (-1) \cdot (-2)$ [1.7]

60. $-6.3 + (-4.8)$ [1.5]





61. $(3 - 7) - (4 - 8)$ [1.8]

62. $3 - 2 + 5 \cdot 10 \div 5^2 \cdot 2$ [1.8]

63. $\frac{-2 - (-6)}{8 - 10}$ [1.8]

64. $\frac{1 - (-7)}{-3 - 5}$ [1.8]

Synthesis

-  **65.** Write a problem for a classmate to solve. Devise the problem so the answer is "At most 18 passengers can go on the boat." Design the problem so that at least one number in the solution must be rounded down.
-  **66.** Write a problem for a classmate to solve. Devise the problem so the answer is "The Rothmans can drive 90 mi without exceeding their truck rental budget."
- 67. Ski wax.** Green ski wax works best between 5° and 15° Fahrenheit. Determine those Celsius temperatures for which green ski wax works best. (See Exercise 41.)
- 68. Parking fees.** Mack's Parking Garage charges \$4.00 for the first hour and \$2.50 for each additional hour. For how long has a car been parked when the charge exceeds \$16.50?
- Aha!** **69.** The area of a square can be no more than 64 cm^2 . What lengths of a side will allow this?
- Aha!** **70.** The sum of two consecutive odd integers is less than 100. What is the largest pair of such integers?
- 71. Nutritional standards.** In order for a food to be labeled "lowfat," it must have fewer than 3 g of fat per serving. Reduced-fat tortilla chips contain 60% less fat than regular nacho cheese tortilla chips, but still cannot be labeled lowfat. What can you conclude about the fat content of a serving of nacho cheese tortilla chips?
- 72. Parking fees.** When asked how much the parking charge is for a certain car (see Exercise 68), Mack replies, "between 14 and 24 dollars." For how long has the car been parked?
- 73. Frequent buyer bonus.** Alice's Books allows customers to select one free book for every 10 books purchased. The price of that book cannot exceed the average cost of the 10 books. Neoma has bought 9 books whose average cost is \$12 per book. How much should her tenth book cost if she wants to select a \$15 book for free?
-  **74. Grading.** After 9 quizzes, Blythe's average is 84. Is it possible for Blythe to improve her average by two points with the next quiz? Why or why not?
-  **75. Discount card.** Barnes & Noble offers a member card for \$25 a year. This card entitles a customer to a 40% discount off list price on hardcover bestsellers, a 20% discount on adult hardcovers, and a 10% discount on other purchases. Describe two sets of circumstances for which an individual would save money by becoming a member.
Source: Barnes & Noble