Chapter 3 & 4 Review Sheet
Math 176, Precalculus, Vanden Eynden

Sketch the graph of the function by transforming the graph of the "parent" function. Describe each transformation in words.

	Transformation:	Parent function
1.	$T(x) = -x^4 + 2$	$P(x) = x^4$
2.	$T(x) = \dfrac{1}{2x}$	$P(x) = \dfrac{1}{x}$
3.	$T(x) = \dfrac{1}{x+4}$	$P(x) = \dfrac{1}{x}$
4.	$T(x) = 3 + 5^{(-x)}$	$P(x) = 5^x$
5.	$T(x) = \ln(x+2) - 3$	$P(x) = \ln x$

6. Given the polynomial $P(x) = x^4 + x^2 - 6x + 4$,
   a. Determine the end behavior of $P(x)$.
   b. How many complex zeros (real or imaginary) does $P(x)$ have (counting multiplicities)?
   c. How many local extrema can $P(x)$ have?
   d. List all the possible rational zeros of $P(x)$.
   e. Graph $P(x)$ on your calculator. Can you determine any integer zeros?
   f. Find the domain and range for $P(x)$.
   g. Using the above information (and synthetic division), find **all** the zeros of $P(x)$.
   h. Write the complete factorization of $P(x)$.
   i. Sketch a graph of $P(x)$.

7. Divide. a. $(2x^3 + x^2 - 8x + 15) \div (x^2 + 2x)$. b. $(3x^3 - 5x - 4) \div (x - 2)$.

8. Find a polynomial of degree 4 having integer coefficients and zeros $3i$ and 4, with 4 having multiplicity of 2.

9. Does there exist a polynomial of degree 4 with integer coefficients that has zeros i, $2i$, $3i$, $4i$? If so, find it. If not, explain why.

10. The remainder of $\dfrac{P(x)}{x-5}$ is 17. What can you say about $P(5)$?

11. What is the remainder when the polynomial $P(x) = x^{500} + 6x^{203} - 5x + 32$ is divided by $x - 1$?

12. If a polynomial $p(x)$ has a zero at $x = c$ of multiplicity 2, what does the graph of $p(x)$ look like at c?

13. Graph the following rational functions. Show clearly all x- and y-intercepts and asymptotes (vertical, horizontal, or slant). Finally, name the Domain and Range for each function.
   a. $r(x) = \dfrac{3x-1}{2x+4}$ b. $r(x) = \dfrac{2x^2 - 6x - 7}{x-4}$ c. $r(x) = \dfrac{1}{(x-2)^2}$

14. Solve each inequality.

a. $x^3 - 3x^2 < 4x - 12$

b. $\dfrac{3x+1}{x+2} \geq \dfrac{2}{3}$

15. If $f(x) = 5^x$ *then* $f^{-1}(x) =$

16. If $k(x) = \log_4 x$ *then* $k^{-1}(x) =$

17. Evaluate each expression without using a calculator.

a. $\ln(e^6)$
b. $\log_4 8$
c. $e^{2\ln 7}$
d. $\log_8 6 - \log_8 3 + \log_8 2$
e. $\log\left(\log\left(10^{10}\right)\right)$
f. $\log_3 1$
g. $\ln\sqrt{e}$
h. $\log_3\left(\dfrac{1}{3}\right)$

Use the laws of logarithms to rewrite the following expressions in “expanded form”.

18. $\log\left(x^2\sqrt{y}\right)$

19. $\log_2\left(\dfrac{x-1}{x+1}\right)^2$

Rewrite as a single logarithm and simplify, if possible.

20. $\ln x + \ln\left(x^2 y\right) + 3\ln y$

21. $\dfrac{3}{2}\log(x-y) - 2\log\left(x^2 - y^2\right)$

Solve:

22. $e^{3x} = 10$

23. $2^{1-x} = 3^{2x+5}$

24. log (log x) = 1

25. $\log x + \log(x+1) = \log 12$

26. $\log_2 x - 3\log_2 5 = 2\log_2 10$

27. A rancher with 800 ft of fencing wants to enclose a rectangular area and then divide it into 3 pens with fencing parallel to one side of the rectangle.
   a. Find a function $A(x)$, that gives the total area in terms of x.
   b. State the domain of $A(x)$ in the context of this problem.
   c. Graph this function on its domain.
   d. What are the dimensions that yield the maximum area?

28. A sum of $5000 is invested at an interest rate of 8.5% per year, compounded semi-annually.
   a. Find the amount of the investment after 1.5 years.
   b. After what period of time will the investment amount to $7000?

29. An open box (it has no top) with a square base is to have a volume of 12 cubic feet.
   a. Find a function that models the surface area of the box.
   b. Find the box dimesions that minimize the amount of material used.

30. Find the rate of interest on an account in which a $2500 investment has grown to $2790 in 1 year if the interest on the account is compounded continuously.

31. Express the quadratic function $g(x) = -2x^2 + 12x - 13$ in standard form, $g(x) = a(x-h)^2 + k$. Does g have a maximum or minimum? What is it?