********ANSWERS******* MATH 176 CHAPTER 7, 8, REVIEW ******ANSWERS*********

3. a. $\tan 75^\circ = \tan(45^\circ + 30^\circ) = \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \dfrac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

b. $\sin 18^\circ \cos 27^\circ + \cos 18^\circ \sin 27^\circ = \sin(18^\circ + 27^\circ) = \sin(45^\circ) = \dfrac{\sqrt{2}}{2}$

c. $\cos\dfrac{5\pi}{12} = \cos\left(\dfrac{1}{2}\cdot\dfrac{5\pi}{6}\right) = \sqrt{\dfrac{2-\sqrt{3}}{4}}$

d. $\cos\dfrac{x}{2} = -\sqrt{\dfrac{1+\cos x}{2}} = -\sqrt{\dfrac{1}{10}}$

4. a. $LHS = \cos\theta\left(\dfrac{1}{\cos\theta} - \cos\theta\right) = 1 - \cos^2\theta = \sin^2\theta = RHS$

b. $LHS = \sin 8x = \sin 2(4x) = 2\sin 4x \cos 4x = RHS$

5. a. $\sin x(\tan x + 1) = 0 \ \ so \ \ \sin x = 0 \ \ or \ \ \tan x = -1$

$x = \pi n \ \ or \ \ x = \dfrac{3\pi}{4} + \pi n$

b. $\cos 2x = -\dfrac{1}{2}$

$2x = \dfrac{2\pi}{3} + 2\pi n \qquad or \qquad 2x = \dfrac{4\pi}{3} + 2\pi n$

$x = \dfrac{\pi + 3\pi n}{3} \qquad or \qquad x = \dfrac{2\pi + 3\pi n}{3}$

$x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

6. a. $x = 3\sqrt{3}, \quad y = -3$ To find *r*: $r^2 = \left(3\sqrt{3}\right)^2 + (-3)^2 = 36, \ \ so \ \ r = 6$

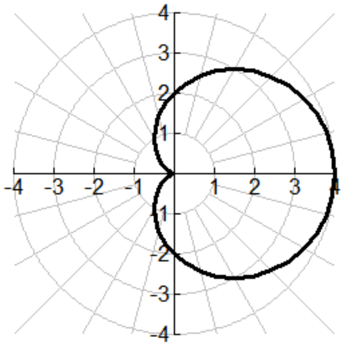
To find θ: $\tan\theta = \dfrac{y}{x} = \dfrac{-3}{3\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$, since $\left(3\sqrt{3}, -3\right)$ is in the 4th quadrant, $\theta = \dfrac{11\pi}{6}$

So $\left(3\sqrt{3}, -3\right) = \left(6, \dfrac{11\pi}{6}\right)$

6. b. $r = \sqrt{3}, \quad \theta = -\dfrac{5\pi}{3}$ To find *x*: $x = r\cos\theta = \sqrt{3}\cos\left(-\dfrac{5\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

To find *y*:: $y = r\sin\theta = \sqrt{3}\sin\left(-\dfrac{5\pi}{3}\right) = \dfrac{3}{2}$, So $\left(\sqrt{3}, -\dfrac{5\pi}{3}\right) = \left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)$

7. Graph $r = 2 + 2\cos\theta$
"Cardioid"



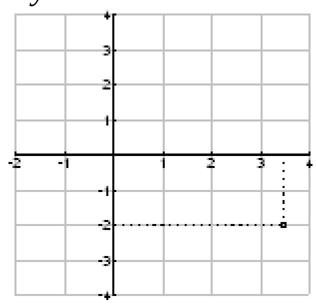
8. a. $x^2 - y^2 = (r\cos\theta)^2 - (r\sin\theta)^2 = r^2(\cos^2\theta - \sin^2\theta) = r^2(\cos 2\theta) = 1$

So $r^2 = \dfrac{1}{\cos 2\theta}$ or $r^2 = \sec 2\theta$

b. Multiply both sides by r to get: $r^2 = 6r\cos\theta$, and since $r^2 = x^2 + y^2$, and $x = r\cos\theta$

The equation in rectangular coordinates is: $x^2 + y^2 = 6x$.

9. Given the complex number $2\sqrt{3} - 2i$,



a. Graph the number in the complex plane →.
b. Convert the number to trig form.

$$r = \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2} = \sqrt{12+4} = 4$$

$$\tan\theta = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}\ ,\ \theta\, is\, in\, Quad\ IV, \theta = \frac{11\pi}{6}$$

$$2\sqrt{3} - 2i = r(\cos\theta + i\sin\theta) = 4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

10. Given $z_1 = -i\sqrt{2}$ and $z_2 = -3 - 3i\sqrt{3}$, find the following:

a.

$$z_1 = \sqrt{2}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right);\ \ z_2 = 6\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

$$z_1 z_2 = 6\sqrt{2}\left(\cos\left(\frac{3\pi}{2} + \frac{4\pi}{3}\right) + i\sin\left(\frac{3\pi}{2} + \frac{4\pi}{3}\right)\right) = 6\sqrt{2}\left(\cos\frac{17\pi}{6} + i\sin\frac{17\pi}{6}\right)$$

$$= 6\sqrt{2}\left(-\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = -3\sqrt{6} + 3i\sqrt{2}$$

b.

$$\frac{z_1}{z_2} = \frac{\sqrt{2}}{6}\left(\cos\left(\frac{3\pi}{2} - \frac{4\pi}{3}\right) + i\sin\left(\frac{3\pi}{2} - \frac{4\pi}{3}\right)\right) = 6\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

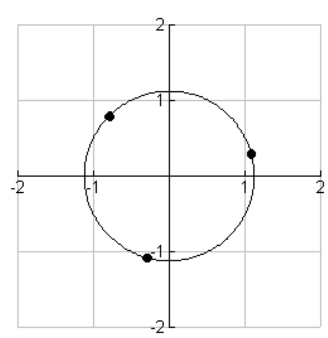
$$= \frac{\sqrt{2}}{6}\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \frac{\sqrt{6}}{12} + \frac{\sqrt{2}}{12}i$$

11.
$$(1-i)^8 = \left[\sqrt{2}\left(\cos(-45°) + i\sin(-45°)\right)\right]^8 = \left(\sqrt{2}\right)^8\left(\cos 8(-45°) + i\sin 8(-45°)\right).$$
$$= 16\left(\cos(-360°) + i\sin(-360°)\right) = 16(1+0i) = 16$$

12. Solve $z^3 = 1 + i$ means find the 3 cube roots of z. $z = 1 + i = \sqrt{2}(\cos 45° + i\sin 45°)$

The three cube roots w_0, w_1, w_2 are found with the Demoivre Theorem:

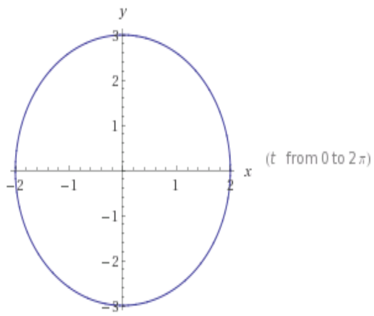
$$w_k = \left(\sqrt{2}\right)^{1/3}\left(\cos\left(\frac{45° + 360°k}{3}\right) + i\sin\left(\frac{45° + 360°k}{3}\right)\right),\ for\ k = 0,1,2$$



$$w_0=\left(\sqrt{2}\right)^{1/3}\left(\cos\left(\frac{45°}{3}\right)+i\sin\left(\frac{45°}{3}\right)\right)=\sqrt[6]{2}\left(\cos 15°+i\sin 15°\right)=1.08+.29i$$

$$w_1=\left(\sqrt{2}\right)^{1/3}\left(\cos\left(\frac{405°}{3}\right)+i\sin\left(\frac{405°}{3}\right)\right)=\sqrt[6]{2}\left(\cos 135°+i\sin 135°\right)=-.79+.79i$$

$$w_1=\left(\sqrt{2}\right)^{1/3}\left(\cos\left(\frac{765°}{3}\right)+i\sin\left(\frac{765°}{3}\right)\right)=\sqrt[6]{2}\left(\cos 255°+i\sin 255°\right)=-.29-1.08i$$

13.a. 

b.
$$\cos t=\frac{x}{2},\sin t=\frac{y}{3}$$
$$\cos^2 t+\sin^2 t=1$$
$$\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$$
$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

17. $\dfrac{-2\sqrt{42}+2}{15}$

18. $\dfrac{\sqrt{21}-4\sqrt{2}}{15}$

19. $\dfrac{-7}{9}$

20. $\tan^2 x-\sec^2 x=\dfrac{\sin^2 x}{\cos^2 x}-\dfrac{1}{\cos^2 x}=\dfrac{\sin^2 x-1}{\cos^2 x}=\dfrac{-\cos^2 x}{\cos^2 x}=-1$

21. $LHS=\dfrac{2\tan x}{1+\tan^2 x}=\dfrac{2\tan x}{\sec^2 x}=2\dfrac{\sin x}{\cos x}\cdot\dfrac{\cos^2 x}{1}=2\sin x\cos x=\sin 2x=RHS$

22. $LHS=\dfrac{\sec^2 x-\tan^2 x}{\cos^2 x}-1=\dfrac{1}{\cos^2 x}-\dfrac{\cos^2 x}{\cos^2 x}=\dfrac{1-\cos^2 x}{\cos^2 x}=\dfrac{\sin^2 x}{\cos^2 x}=\tan^2 x=RHS$

23.
$$2\sin\theta\cos\theta-2\cos^2\theta=0$$
$$2\cos\theta\left(\sin\theta-\cos\theta\right)=0$$
$$\cos\theta=0 \quad or \quad \sin\theta=\cos\theta$$
$$\theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{4},\frac{5\pi}{4}$$

24.
$$3\sin 2\theta-2\sin\theta=0$$
$$3\left(2\sin\theta\cos\theta\right)-2\sin\theta=0$$
$$6\sin\theta\cos\theta-2\sin\theta=0$$
$$2\sin\theta\left(3\cos\theta-1\right)=0$$
$$\sin\theta=0 \quad or \quad \cos\theta=\frac{1}{3}$$
$$\theta=\pi k,k\in\mathbb{Z}$$
$$\theta=1.231rad+2\pi k,k\in\mathbb{Z}$$
$$\theta=-1.231rad+2\pi k,k\in\mathbb{Z}$$

25.
$$3x=\frac{5\pi}{6}+2\pi n \quad or \quad 3x=\frac{7\pi}{6}+2\pi n$$
$$x=\frac{5\pi}{18}+\frac{2\pi n}{3} \quad or \quad x=\frac{7\pi}{18}+\frac{2\pi n}{3}$$
$$x=\frac{5\pi}{18}+\frac{12\pi n}{18} \quad or \quad x=\frac{7\pi}{18}+\frac{12\pi n}{18}$$
$$n=0 \quad x=\frac{5\pi}{18},\frac{7\pi}{18}$$
$$n=1 \quad x=\frac{17\pi}{18},\frac{19\pi}{18}$$
$$n=2 \quad x=\frac{29\pi}{18},\frac{31\pi}{18}$$

26. $\tan x=-\dfrac{4}{3},\ \cos x=-\dfrac{3}{5},\ \sin x=\dfrac{4}{5}$

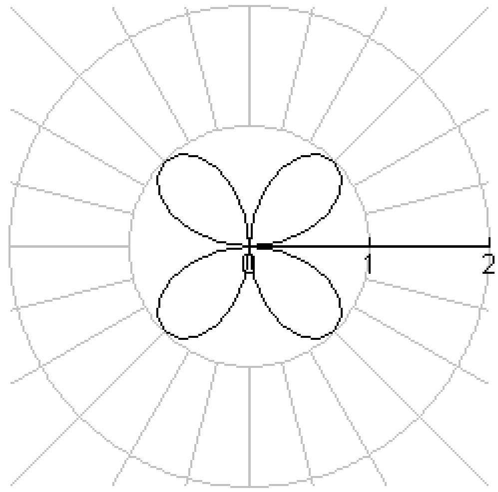
$$\sin 2x=2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)=\frac{-24}{25},\quad \cos 2x=\left(\frac{-3}{5}\right)^2-\left(\frac{4}{5}\right)^2=\frac{-7}{25},\quad \tan 2x=\frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2}=\frac{24}{7}$$

27. $\tan 15^\circ = \tan\frac{30^\circ}{2} = 2-\sqrt{3}$

28. $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$

29. $\cos\frac{\pi}{12} = \cos\left(\frac{1}{2}\cdot\frac{\pi}{6}\right) = \sqrt{\frac{2+\sqrt{3}}{4}}$

30. see graph →



31. $-\sqrt{3}+i\sqrt{3} = \sqrt{6}\left(\cos 135° + i\sin 135°\right)$

32. $6\sqrt{6}\left(\cos\left(135^\circ + 300^\circ\right) + i\sin\left(135^\circ + 300^\circ\right)\right) = 6\sqrt{6}\left(\cos 75^\circ + i\sin 75^\circ\right)$

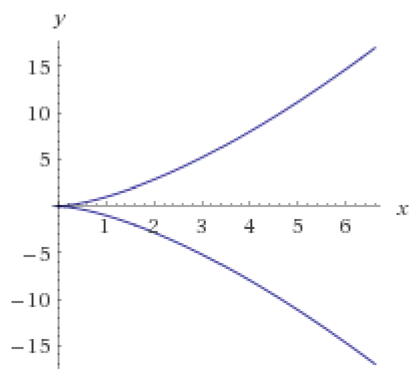
33. $\frac{\sqrt{6}\left(\cos\left(135^\circ\right) + i\sin\left(135^\circ\right)\right)}{6(\cos 300^\circ + i\sin 300^\circ)} = \frac{\sqrt{6}}{6}\left(\cos\left(-165^\circ\right) + i\sin\left(-165^\circ\right)\right)$

34.

$$\left(1-i\sqrt{3}\right)^4 = \left[2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)\right]^4 = 2^4\left(\cos\left(4\cdot\frac{5\pi}{3}\right) + i\sin\left(4\cdot\frac{5\pi}{3}\right)\right) = 16\left(\cos\left(\frac{20\pi}{3}\right) + i\sin\left(\frac{20\pi}{3}\right)\right)$$

$$= 16\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) = 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -8 + 8i\sqrt{3}$$

35.a



b. $y^2 = 64t^6 = (4t^2)^3$

$so, y^2 = x^3$