Chapter 7, 8 Review Sheet
Math 176, Precalculus, Vanden Eynden

1. Be familiar with the identities and all the formulas introduced in chapter 7 and how and when to use them. Although you will be given a formula sheet (last page), you need to be efficient when using it.

2. **Memorize** the "Fundamental Identities" including the reciprocal, ratio, Pythagorean (with variations) and the odd/even identities. Memorize the unit circle. Memorize the domains and ranges of the inverse trig functions ($\sin^{-1} x,\ \cos^{-1} x,\ \tan^{-1} x$).

Reciprocal Identities

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta} \qquad \cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = csc^2\theta$$

Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Even/Odd Functions

$\cos(-\theta) = \cos\theta$ Even

$\sin(-\theta) = -\sin\theta$, $\tan(-\theta) = -\tan\theta$ Odd

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3. Find exact values of trig expressions using trig identities and formulas.
   Find the exact value of the following:
   a. $\tan 75^\circ$.
   b. $\sin 18^\circ \cos 27^\circ + \cos 18^\circ \sin 27^\circ$
   c. $\cos\frac{5\pi}{12}$.
   d. $\cos\frac{x}{2}$ given $\cos x = -\frac{4}{5}, 180^\circ < x < 270^\circ$

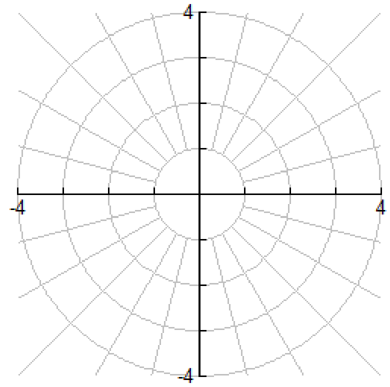
4. Know how to prove trig identities by working with one side (usually the more complicated side) or both sides, using known identities and formulas, and algebra techniques (factoring, finding the LCD, or multiplying by the "conjugate").
   Prove the following identities:
   a. $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$
   b. $\sin 8x = 2\sin 4x \cos 4x$

5. Know how to solve trig equations – finding all solutions vs. solutions in a given interval. Also know how to deal with equations involving multiple angles.
   a. Find all solutions of the equation $\tan x \sin x + \sin x = 0$.
   b. Find all solutions of the equation $2\cos 2x + 1 = 0$ in the interval $[0, 2\pi)$.

6. Know how to plot points given in polar coordinates. Memorize the formulas that relate polar to rectangular coordinates (pg 543). They are: $\begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix}$ and $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$
   a. Convert: $(3\sqrt{3}, -3)$ to polar
   b. Convert: $\left(\sqrt{3}, -\frac{5\pi}{3}\right)$ to rectangular

7. Be familiar with the different polar curves (circles and spiral, limacons, roses, lemniscates, pg 552.) Be able to graph a polar curve by plotting points and with the help of a graphing calculator.

   Sketch the graph of the polar equation: $r = 2 + 2\cos\theta$



8. Convert from polar to rectangular coordinates and vice versa.
   a. Convert $x^2 - y^2 = 1$ to polar form.
   b. Convert the polar equation $r = 6\cos\theta$ to rectangular coordinates.

9. Convert complex numbers in the form $a + bi$ into polar/trig form of $r(\cos\theta + i\sin\theta)$. Know how to find the modulus r and the argument θ and be careful where the complex number is located in the complex plane (determine the correct argument θ in the correct quadrant).

   Given the complex number $2\sqrt{3} - 2i$,
   a. Graph the number in the complex plane.
   b. Convert the number to trig form.

10. Perform operations on complex numbers in polar/trig form $r(\cos\theta + i\sin\theta)$.

    Given $z_1 = -i\sqrt{2}$ and $z_2 = -3 - 3i\sqrt{3}$, Convert each to polar form, then find the following:

    a. $z_1 z_2$ b. $\dfrac{z_1}{z_2}$

11. Use Demoivre's Theorem to find powers of complex numbers.

    Find $(1-i)^8$.

12. Finding nth roots of complex numbers.

    Solve $z^3 = 1 + i$ and graph the solutions in the complex plane.

13. Sketch curves represented by parametric equations and be able to convert from parametric equations to rectangular-coordinate equations.

    Given the following parametric equations, $x = 2\cos t, y = 3\sin t, 0 \le t \le 2\pi$
    a. Sketch its curve.
    b. Find a rectangular-coordinate equation for the curve by eliminating the parameter.

(MORE) PRACTICE PROBLEMS TO REVIEW

CHAPTER 7

Given $\sin y = \frac{-2}{5}$, $\cos x = \frac{-1}{3}$ with $\frac{\pi}{2} < x < \pi$ and $\pi < y < \frac{3\pi}{2}$, give the exact values for the following:

17. $\sin(x + y)$ 18. $\cos(x - y)$ 19. $\cos 2x$

20. Express $\tan^2 x - \sec^2 x$ in terms of sin *x* and cos *x*, and simplify.

Verify that each equation is an identity.

21. $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$ 22. $\frac{\sec^2 x - \tan^2 x}{\cos^2 x} - 1 = \tan^2 x$

Solve each equation over the indicated interval.

23. $\sin 2\theta = 2\cos^2\theta$, $[0, 2\pi]$ 24. $3\sin 2\theta - 2\sin\theta = 0$, all $\mathbb{R}$ 25. $\cos 3x = -\frac{\sqrt{3}}{2}$, $[0, 2\pi)$

Find sin 2x, cos 2x, and tan 2x from the given information

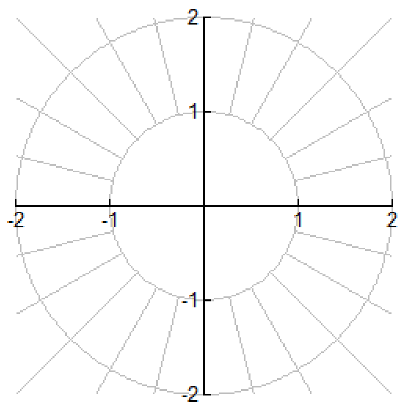
26. $\tan x = -\frac{4}{3}$, x *in quadrant II*

Use an appropriate half-angle, double angle, sum or difference formula to find the exact value of the expression.

27. $\tan 15^\circ$ 28. $\sin 75^\circ$ 29. $\cos\frac{\pi}{12}$

CHAPTER 8

30. Sketch the graph of the polar equation: $r = \sin 2\theta$



31. Write $-\sqrt{3} + i\sqrt{3}$ in polar/trig form.

32. Find the product $(-\sqrt{3} + i\sqrt{3})(6(\cos 300^\circ + i\sin 300^\circ))$ in polar/trig form.

33. Find the quotient $\frac{-\sqrt{3} + i\sqrt{3}}{6(\cos 300^\circ + i\sin 300^\circ)}$ in polar/trig form.

34. Find $\left(1 - i\sqrt{3}\right)^4$ and write your answer as a complex number.

35. Given the following parametric equations, $x = 4t^2$, $y = 8t^3$
    a. Sketch its curve.
    b. Find a rectangular-coordinate equation for the curve by eliminating the parameter.

Trigonometric Identities and Formulas for Exam#4

Cofunction Identities

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right)$$

Addition and Subtraction Formulas

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Power-Reducing Formulas

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$

$$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$$

Double-Angle Formulas

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$ (1st form)

$$= 2\cos^2 A - 1$$ (2nd form)

$$= 1 - 2\sin^2 A$$ (3rd form)

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Half-Angle Formulas

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

Products and Quotients in Trig Form

If $z_1 = r_1\left(\cos\theta_1 + i\sin\theta_1\right)$ and $z_2 = r_2\left(\cos\theta_2 + i\sin\theta_2\right)$, then

$$z_1 z_2 = r_1 r_2\left[\cos\left(\theta_1+\theta_2\right) + i\sin\left(\theta_1+\theta_2\right)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos\left(\theta_1-\theta_2\right) + i\sin\left(\theta_1-\theta_2\right)\right]$$

DeMoivre's Theorem

If $z = r\left(\cos\theta + i\sin\theta\right)$ is a complex number in trigonometric form and *n* is an integer, then

$$z^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

*n*th Roots of Complex Numbers

If $z = r\left(\cos\theta + i\sin\theta\right)$ is a complex number in trigonometric form and *n* is a positive integer, then *z* has *n* distinct *n*th roots

$$w_k = r^{1/n}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right] \qquad \text{for } k = 0,1,2,\ldots,n-1.$$