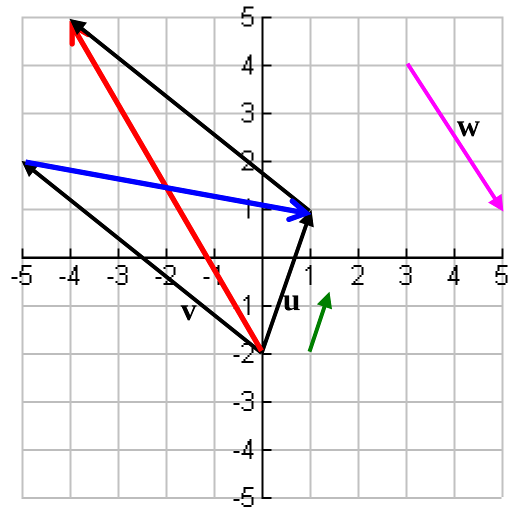
Chapter 9 & 10 Review Sheet: SOLUTIONS
Math 176, Precalculus, Vanden Eynden

1. a. From (5, 6) the vector points 5 right and 8 down, the terminal point is (5+5, 6–8)=(10, –2)
   b. *if* $\mathbf{u} = \langle a, b \rangle$ *then* $a = |\mathbf{u}|\cos\theta$, $b = |\mathbf{u}|\sin\theta$, so $\mathbf{u} = \langle 5\sqrt{3}, 5 \rangle$.

2 a. $\langle 3, 4 \rangle$ b. $\langle 19, -5 \rangle$ c. -54
   d. No, $\mathbf{u} \bullet \mathbf{v} = -18 \neq 0$, $\theta = \cos^{-1}\left(\frac{-18}{\sqrt{17}\sqrt{29}}\right) = 144.16°$

3. Using the dot product theorem: $\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$ $\theta = \cos^{-1}\left(\frac{45}{3\sqrt{229}}\right) = 7.6°$

For the vectors **u** and **v** graphed,



4. Graph the vector **u** + **v** **(in red)**

5. Graph the vector **u** – **v** **(in blue)**

6. Draw $\frac{1}{2}\mathbf{u}$ **(in green)**

7. Draw the vector $\mathbf{w} = 2\mathbf{i} - 3\mathbf{j}$ **(in pink)**

8. $\mathbf{u} = \langle 1, 3 \rangle$, $\mathbf{v} = \langle -5, 4 \rangle$

9. $|\mathbf{u}| = \sqrt{10}$, $|\mathbf{v}| = \sqrt{41}$

10. $\langle 1/2, 11 \rangle$

11. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = 0$

12. $\mathbf{u} \cdot \mathbf{v} = 7$

13. $\theta = \cos^{-1}\left(\frac{7}{\sqrt{10}\sqrt{41}}\right) = 69.8°$

14. (1, 3), (-3, -5)
   Substitution: $2x + 1 = -x^2 + 4$
   $$x^2 + 2x - 3 = 0$$
   $$(x-1)(x+3) = 0$$
   $$x = 1 \text{ or } x = -3$$
   $$x = 1 \to y = 2(1) + 1 = 3 \to (1, 3)$$
   $$x = -3 \to y = 2(-3) + 1 = -5 \to (-3, -5)$$

15. (3,2)

Gauss-Jordan Elimination (row-reduced echelon form):

$$\begin{bmatrix} 2 & 3 & 12 \\ 4 & -1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3/2 & 6 \\ 4 & -1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3/2 & 6 \\ 0 & -7 & -14 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3/2 & 6 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Matrix Inverse:

$$AX = B \qquad \text{solution:} \qquad X = A^{-1}B$$

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix} \qquad \frac{1}{-2-12}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{bmatrix}\begin{bmatrix} 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

16. (3,2,1)

Gaussian Elimination: (row echelon form):

$$\begin{bmatrix} 1 & -1 & -2 & -1 \\ 1 & 1 & 1 & 6 \\ 1 & 1 & -1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & 2 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1.5 & 3.5 \\ 0 & 2 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1.5 & 3.5 \\ 0 & 0 & -2 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1.5 & 3.5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\rightarrow \quad \begin{aligned} x - y - 2z &= -1 \\ y + 1.5z &= 3.5 \\ z &= 1 \end{aligned} \quad \rightarrow \quad \begin{aligned} x &= 3 \\ y &= 2 \\ z &= 1 \end{aligned}$$

Cramer's Rule:

$$\det D = |D| = \begin{vmatrix} 1 & -1 & -2 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -4$$

$$\det D_x = |D_x| = \begin{vmatrix} -1 & -1 & -2 \\ 6 & 1 & 1 \\ 4 & 1 & -1 \end{vmatrix} = -12$$

$$\det D_y = |D_y| = \begin{vmatrix} 1 & -1 & -2 \\ 1 & 6 & 1 \\ 1 & 4 & -1 \end{vmatrix} = -8$$

$$\det D_z = |D_z| = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & 6 \\ 1 & 1 & 4 \end{vmatrix} = -4$$

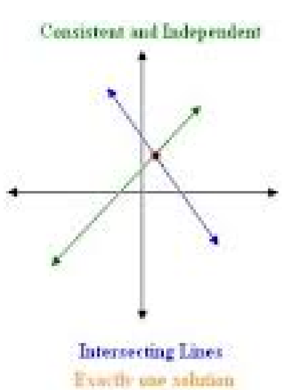
$$x = \frac{|D_x|}{|D|} = \frac{-12}{-4} = 3$$

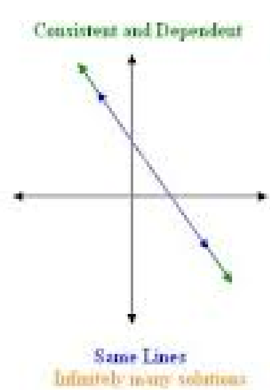
$$y = \frac{|D_y|}{|D|} = \frac{-8}{-4} = 2$$

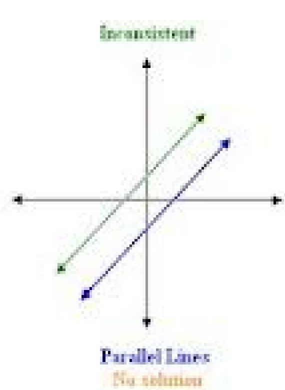
$$z = \frac{|D_z|}{|D|} = \frac{-4}{-4} = 1$$

answer: (3,2,1)

17. Inconsistent = parallel lines, dependent = same line, neither = intersecting lines (one solution)







18. $(0,-2), \quad (8/5, 6/5)$

Substitution: $x^2 + (2x-2)^2 = 4$

$5x^2 - 8x = 0$

$x(5x-8) = 0$

$x = 0 \text{ or } x = 8/5$

$x = 0 \to y = 2(0) - 2 = -2 \to (0,-2)$

$x = 8/5 \to y = 2(8/5) - 2 = 6/5 \to (8/5, 6/5)$

19. (2, 5) , (–3, 0)

elimination:

$$\begin{array}{r} x^2 + y = 9 \\ x - y = -3 \\ \hline x^2 + x = 6 \end{array}$$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

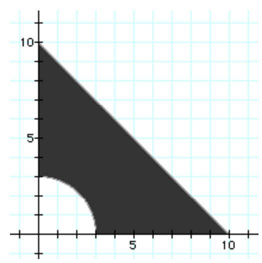
$x = 2 \text{ or } x = -3$

$x = 2 \to 2 - y = -3 \quad y = 5 \quad \to (2,5)$

$x = -3 \to -3 - y = -3 \quad y = 0 \quad \to (-3,0)$

20. $x = tens\ digit$, $y = ones\ digit$ $\quad \begin{cases} x + y = 7 \\ 10x + y + 27 = 10y + x \end{cases}$ $\quad$ (2,5) → 25 is the number.

21. See graph. All boundaries are “dashed lines”



22. a. $\begin{bmatrix} 10 & -25 \\ 0 & 35 \end{bmatrix}$ b. $\begin{bmatrix} 4 & -15 & 15 \\ 0 & 14 & -21 \end{bmatrix}$ c. DNE, # cols of C ≠ # rows of A

d. DNE, dimensions are not the same e. $\begin{bmatrix} 0 & \frac{17}{2} & 10 \\ 2 & -8 & 15 \end{bmatrix}$

23. a. $\begin{bmatrix} 7/2 & -3/2 \\ 2 & -1 \end{bmatrix}$ b. $\begin{bmatrix} 8 & -3 & 4 \\ -2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ c. DNE, det = 0

24. a. 14 b. 0 c. 0

25. $\frac{1}{x} - \frac{2}{x+3} + \frac{1}{x-1}$

26. $\frac{3}{x-2} + \frac{7}{(x-2)^2}$