Chapter 9 & 10 Review Sheet
Math 176, Precalculus, Vanden Eynden

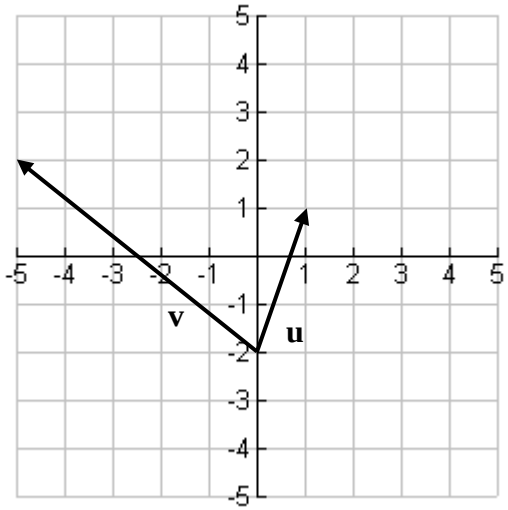
CHAPTER 9

1. Graph and write vectors (component form, using unit vectors **i** and **j**, using sin and cos – vertical and horizontal components of a vector).
    a. If the vector $5\mathbf{i}-8\mathbf{j}$ is placed in the plane with its initial point at $P(5,6)$, find its terminal point.
    b. Find the vector u having length $|\mathbf{u}|=10$ and direction $\theta=30^\circ$

2. Perform vector operations graphically and algebraically (scalar multiplication, vector addition/subtraction, dot product) – know when the result is a scalar vs. a vector. Also find angles between 2 vectors - know when vectors are perpendicular (orthogonal vectors).
    Given **u** = <4,1> and **v** = <–5,2>, find the following:
    a. $2\mathbf{u}+\mathbf{v}$
    b. $\mathbf{u}-3\mathbf{v}$
    c. $3\mathbf{u}\cdot\mathbf{v}$
    d. Are **u** and **v** orthogonal? If not, find the angle between the given vectors to the nearest tenth of a degree.

3. Calculate the angle between two vectors, **u** and **v**.
    Given $\mathbf{u}=\langle 0,3\rangle$ and $\mathbf{v}=\langle 2,15\rangle$, find the angle θ between **u** and **v**

For the vectors **u** and **v** graphed:



4. Graph the vector $\mathbf{u}+\mathbf{v}$

5. Graph the vector $\mathbf{u}-\mathbf{v}$

6. Draw $\frac{1}{2}\mathbf{u}$

7. Draw the vector $\mathbf{w}=2\mathbf{i}-3\mathbf{j}$

8. Find **u** and **v** in component form.

9. Find the magnitudes, $|\mathbf{u}|$ and $|\mathbf{v}|$.

10. Find $3\mathbf{u}+\frac{1}{2}\mathbf{v}$

11. Given $\mathbf{w}=2\mathbf{i}-3\mathbf{j}$, find $\mathbf{u}\cdot(\mathbf{v}+\mathbf{w})$

12. Find $\mathbf{u}\cdot\mathbf{v}$ (dot product)

13. Find the angle θ between **u** and v

CHAPTER 10

Know how to solve a system of linear equations using each of the following methods. You should be able to use these methods BY HAND and with the aid of your calculator. Be able to determine when a system has one solution, infinitely many solutions (dependent system), and no solution (inconsistent system.)

a. Graphing
b. Substitution
c. Elimination
d. Gaussian elimination (transforming matrix to row echelon form)
e. Gauss-Jordan elimination (transforming matrix to reduced row echelon form)
f. Matrix Inverse (matrix equation $AX = B$ has solution $X = A^{-1}B$)
g. Cramer's Rule

14. Solve $\begin{cases} y = -x^2 + 4 \\ y = 2x + 1 \end{cases}$ using substitution or elimination.

15. Solve $\begin{cases} 2x + 3y = 12 \\ 4x - y = 10 \end{cases}$ using Gauss-Jordan elimination and Matrix Inverse

16. Solve $\begin{cases} x - y - 2z = -1 \\ x + y + z = 6 \\ x + y - z = 4 \end{cases}$ using Gaussian elimination and Cramer's Rule

17. Sketch the graphs of 3 different systems of linear equations in 2 variables: One system that is inconsistent, one system that is dependent, and one system that is both consistent and independent.

18. Solve $\begin{cases} x^2 + y^2 = 4 \\ y = 2x - 2 \end{cases}$ using the method of your choice.

19. Solve $\begin{cases} x^2 + y = 9 \\ x - y + 3 = 0 \end{cases}$ using any method.

20. The sum of the digits of a two-digit number is 7. When the digits are reversed, the number is increased by 27. Set up a system of 2 linear equations that represents this problem. Find the number.

21. Graph the solution of the system of inequalities. $\begin{cases} x > 0 \\ y > 0 \\ x + y < 10 \\ x^2 + y^2 > 9 \end{cases}$

Know how to perform the following matrix operations, manipulations and procedures, BY HAND and with the aid of your calculator.

a. Transform a matrix into row echelon form or reduced row echelon form using the 3 elementary row operations.

b. Addition, subtraction and multiplication of matrices (also know when these operations can NOT be performed.)

c. Compute the inner product of a row and a column.

d. Find the inverse of an n x n matrix. Know when a matrix has no inverse.

e. If a matrix has an inverse, be able to verify that ir is really the inverse of the matrix (verify $AA^{-1} = A^{-1}A = I_n$).

f. Find the determinant of an n x n matrix.

g. Transform a system of equations into a matrix equation ($AX = B$)

h. Know how to use Cramer's Rule to solve a system of equations.

22. Given the following matrices, carry out the indicated algebraic operations or explain why it cannot be performed.

$$A = \begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & \frac{1}{2} & 5 \\ 1 & -1 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -\frac{5}{2} & 0 \\ 0 & 2 & -3 \end{bmatrix}$$

a. $5A$ b. AC c. CA d. $2C + A$ e. $2B - 3C$

23. Find the inverse of the following matrices (if it exists) and verify (by matrix multiplication) that this is really the inverse:

a. $\begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 3 & -1 \\ 1 & 4 & 0 \\ -1 & -3 & 2 \end{bmatrix}$ c. $\begin{bmatrix} -2 & 3 \\ -6 & 9 \end{bmatrix}$

24. Find the determinants of the following matrices:

a. $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ c. $\begin{bmatrix} -2 & 3 \\ -6 & 9 \end{bmatrix}$

25. Find the partial fraction decomposition of the rational expression $\dfrac{7x-3}{x^3+2x^2-3x}$.

26. Find the partial fraction decomposition of the rational expression $\dfrac{3x+1}{x^2-4x+4}$.

CHAPTER 11

Know the following definitions, terminology and equations of the conic sections in 11.1 thru 11.4.

a. The geometric definitions and resulting equations for parabolas, ellipses and hyperbolas.
b. Parabolas: Given the equation, name the focus, vertex and directrix and use to sketch the graph. Given the vertex and focus (or directrix) find the equation of the parabola.
c. Ellipses: Given the equation, accurately sketch the graph of an ellipse, finding the vertices, foci and eccentricity. Given the graph, find the equation of the ellipse.
d. Hyperbolas: Given the equation, accurately sketch the graph of a hyperbola along the x-axis or y-axis. Find the vertices, asymptotes and foci.
e. Shifted conic sections: Given the equation, determine which type of conic section (by possibly completing the squares), then accurately sketch the graph of the conic.

27. Find the focus and directrix of the parabola $y^2 = 3x$, then sketch it's graph.

28. Which parabola is wider? $x^2 = 2y$ or $x^2 = 4y$

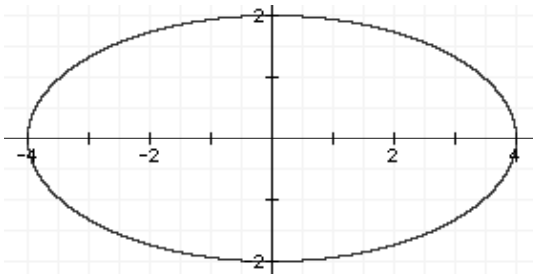
29. Find the equation for the parabola that has its vertex at the origin and has directrix $x = -6$.

30. What is a latus rectum, and do we each have one? Why is it useful?

31. Sketch the graph of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$. Find its foci and eccentricity.

32. What does the eccentricity of an ellipse measure?

33. Find the equation of the ellipse graphed below.



34. Find the vertices, foci and asymptotes of the hyperbola. Then sketch its graph.

a. $\frac{x^2}{5} - \frac{y^2}{16} = 1$ b. $9y^2 - 4x^2 = 36$

35. Graph the following shifted conics.

a. $\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$ b. $2x^2 + y^2 = 2y + 1$

(complete the square first)