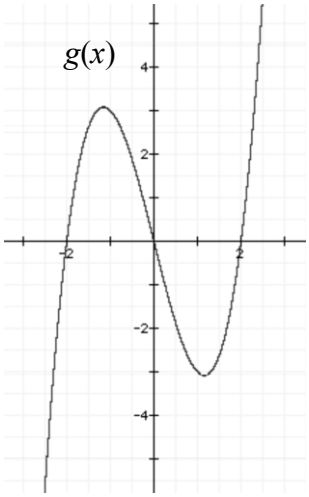
Math 176: Cumulative Semester Review
☺ A ton of Practice Problems ☺

1. Given the functions $f(x)$, $g(x)$ and $h(x)$:

$$f(x)=\frac{x+4}{x-2}$$



x	$h(x)$
–4	2
–1	–2
0	5
1	6
4	–5

Find the following:

a. Domain of $f(x)$
b. Range of $g(x)$
c. $f(7)$
d. $g(-2)$
e. $(f\circ h)(0)$
f. Is $g(x)$ 1–1?
g. $f(a+h)$
h. $h(-1)+g(-1)$
i. $h\big(g\big(h(-4)\big)\big)$
j. Estimate the interval(s) of increase of $g(x)$.
k. The average rate of change of $h(x)$, from $x=1$ to $x=4$

Sketch the graphs of the following functions. State the domain and range of each.

2. $f(x)=\begin{cases} -x, & \text{if } x<0 \\ x^2, & \text{if } 0\le x<2 \\ 1, & \text{if } x\ge 2 \end{cases}$

3. $g(x)=2^{x-1}$

4. $h(x)=-3+\ln x$

5. ~~The frequency of a vibrating string under constant tension is inversely proportional to its length. If a violin string 12 inches long vibrates 440 times per second, to what length must it be shortened to vibrate 660 times per second?~~

6. Find the inverse function of $k(x)=(x+1)^3+2$.

7. Express the quadratic function $f(x)=x^2+4x+1$ in standard form. Does *f* have a maximum or minimum value? Find it.

8. Suppose the graph of $g(x)$ is given. Describe how the graphs of the following function can be obtained from the graph of *g*.
   a. $y=g(x+2)$ b. $y=g(x)-3$ c. $y=-g(x)$ d. $g^{-1}(x)$

9. A stone is thrown upward from the top of a building. Its height (in feet) above the ground after *t* seconds is given by $h(t)=-16t^2+48t+32$. How high is the building? What maximum height does the stone reach? When does the stone hit the ground?

10. Determine if each function is one-to-one.

a. $k(x) = x^3 + 3$ b. $h(x) = \dfrac{1}{x^2}$

11. Given the polynomial $P(x) = x^5 - 3x^4 - x^3 + 11x^2 - 12x + 4$,

a. Determine the end behavior of $P(x)$.
b. How many complex zeros does $P(x)$ have (counting multiplicities)?
c. How many local extrema can $P(x)$ have? Use your graphing calculator to find all maximums and minimums. Round to 3 decimal places
d. Give the intervals of increase and decrease. Round to 3 decimal places.
e. List all the possible rational zeros of $P(x)$.
f. Graph $P(x)$ on your calculator. Can you determine any integer zeros?
g. Find the domain and range for $P(x)$.
h. Using the above information (and synthetic division), find **all** the zeros of $P(x)$.
i. Write the complete factorization of $P(x)$.
j. Sketch a graph of $P(x)$.

12. Divide: $\left(4x^3 - 2x - 10\right) \div \left(x + 3\right)$.

13. What is the remainder when $x^{102} + 4x$ is divided by $\left(x - 1\right)$?

14. Perform the indicated operations with complex numbers.

a. $\left(2 + 4i\right)\left(7i - 9\right)$ b. $\dfrac{\left(7 + 2i\right)}{\left(4 + 3i\right)}$

c. $\dfrac{i^{13} + i^{20}}{i^{24}}$ d. $\left(11 - i\right) - \left(7 - 2i\right)$

15. Graph the rational function $r(x) = \dfrac{x - 2}{x^2 - 2x - 8}$. Show clearly all x- and y-intercepts and asymptotes (vertical, horizontal, or slant). Finally, name the Domain and Range of $r(x)$.

16. Evaluate each expression without using a calculator.

a. $\log(1000)$ b. $\log_2 128$ c. $2^{\log_2 12}$ d. $\log_3 18 - \log_3 4 + \log_3 6$

e. $\log_4\left(\dfrac{1}{64}\right)$ f. $\log_{25} 1$ g. $\ln\sqrt{e^3}$ h. $\log_5\left(125^{10}\right)$

Solve for x. If necessary, round your answer to 3 decimal places.

17. $5^{4x} = 20$ 18. $4^{x+2} = 8^{3x-2}$ 19. $\log_2(x + 2) + \log_2(x - 1) = 2$

20. A sum of \$12,000 is invested at an interest rate of 3.5% per year, compounded monthly.

a. Find the amount of the investment after 3 years.
b. After what period of time will the investment amount to \$20,000?

21. Find the value of each trig function. If possible, give the EXACT value; otherwise, use a calculator to find an approximate value correct to 5 decimal places. You may need to use a half-angle, double angle, sum or difference formula. You may need to sketch an appropriate right triangle.

a. $\cos 225°$ b. $\sin\frac{2\pi}{3}$ c. $\sec\frac{11\pi}{6}$

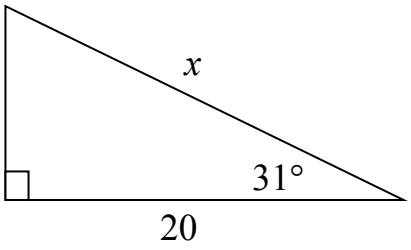
d. $\tan\left(-\frac{\pi}{4}\right)$ e. $\csc(3\pi)$ f. $\cot 123°$

g. $\sin 105°$ h. $\arcsin\left(-\frac{1}{2}\right)$ i. $\cot(-390°)$

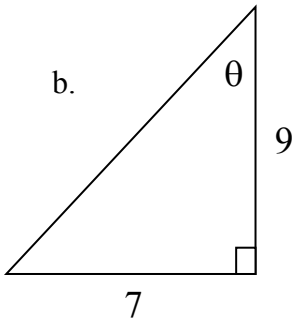
j. $\tan^{-1}(\tan\pi)$ k. $\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ l. $\cos\frac{\pi}{8}$

22. Solve for either x or θ in the following triangles. Round your answers to 2 decimals.

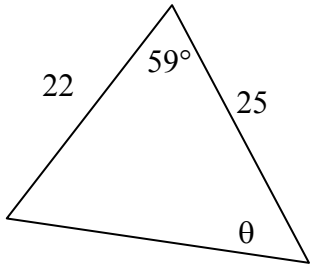
a.



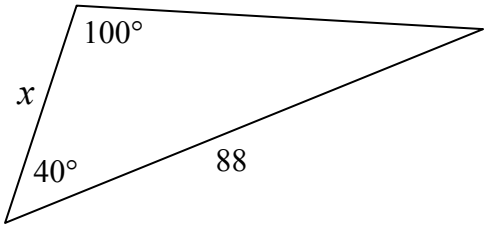
b.



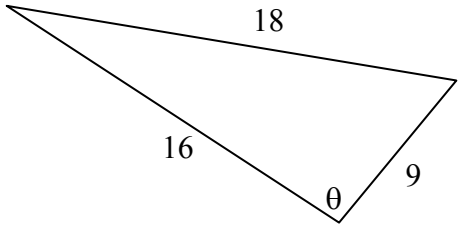
c.



d.



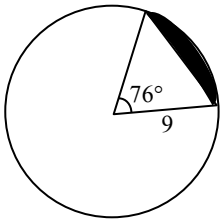
e.



23. If $\cos\theta = -\frac{6}{11}$ and θ is in quadrant II, find $\tan\theta$.

24. Find the amplitude, period, and phase shift of $y = 5\cos 3\left(x - \frac{\pi}{3}\right)$ and then sketch its graph.

25. Find the area of the shaded region:



26. The "Holiday Tree" in Rockefeller Center casts a shadow 68 ft long. Find the height of the tree if the angle of elevation of the sun is 47.4°.

27. Verify each identity.

a. $1+\tan x\tan\frac{x}{2}=\sec x$ b. $\sin^2 x\cot^2 x+\cos^2 x\tan^2 x=1$ c. $\frac{\sin 2x}{\sin x}-\frac{\cos 2x}{\cos x}=\sec x$

28. Find all solutions of the equation: $\sin x-2\sin^2 x=0$

29. Find all solutions of the equation $\sin x=\cos 2x$ in the interval $[0,2\pi]$.

30. Write $1+i\sqrt{3}$ in trigonometric form.

31. Find $(1+i)^7$

32. For $\mathbf{u}=\langle -8,12\rangle$, $\mathbf{v}=\langle 3,-2\rangle$, find **|u|, |v|,** $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $3\mathbf{u}$, $\frac{1}{4}\mathbf{u}-3\mathbf{v}$, and $\mathbf{u}\cdot\mathbf{v}$. Then find the angle θ between **u** and **v**. Round to the nearest degree.

33. Solve the system of nonlinear equations using the method of your choice. $\begin{cases} x-y=-2 \\ x^2+y^2=8 \end{cases}$

34. Solve the system of equations $\begin{cases} x+y+2z=6 \\ 2x+5z=12 \\ x+2y+3z=9 \end{cases}$

35. Given the following matrices, carry out the indicated algebraic operations BY HAND or EXPLAIN why it cannot be performed.

$A=\begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix}$ $B=\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ $C=\begin{bmatrix} 3 & 0 & -1 \\ 0 & 5 & -2 \end{bmatrix}$ $D=\begin{bmatrix} 6 & -9 & 1 \\ 2 & 3 & -6 \end{bmatrix}$ $E=\begin{bmatrix} 5 & 0 & -1 \\ -3 & 2 & 6 \\ 0 & 3 & 7 \end{bmatrix}$

a. $2A-3B$ b. CD c. $\det(A)$ d. B^{-1}

e. B^2 f. A^{-1} g. $\det(E)$ (expand by the row of your choice)

36. Write the partial fraction decomposition of $\frac{2x+5}{(x-2)(x+3)}$

37. Graph the solution of the system of inequalities. $\begin{cases} x^2+y^2\geq 4 \\ x-y>0 \end{cases}$

38. Find the focus and directrix of the parabola $x^2=-12y$, and sketch its graph.

39. Find the foci and eccentricity of the ellipse $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{25}=1$. Then sketch its graph.

40. Find the foci and asymptotes of the hyperbola $\frac{y^2}{4}-\frac{x^2}{16}=1$. Then sketch its graph.

41. Graph the conic section $2x^2-12x-y^2+6y+26=0$ by completing the square.

42. Sketch a graph of each polar equation

a. $r=2+\cos\theta$ b. $r=3$ c. $r=\sin 5\theta$ d. $\theta=\frac{\pi}{6}$

43. Sketch the parametric curve: $\begin{aligned} x &= 1-t^2 \\ y &= 1+t \end{aligned}$ Find a rectangular-coordinate equation for the curve by eliminating the parameter, t.

44. Find parametric equations for the line passing through (1, 3) and (4, 4).

45. Write the first 5 terms of the following sequences:

a. $a_n=\frac{(-1)^{n+1}}{n^2}$ b. $a_n=\frac{n!}{(n+1)!}$ c. $a_n=3a_{n-1}-5,\ a_1=5$

46. Determine whether the following sequences are arithmetic, geometric or neither. Find the 6th term. Find a formula for the nth term.

a. 4, 10, 16, 22, 28, …
b. 4, 5, 7, 10, 14, …
c. –2, 1, $-\frac{1}{2}$, $\frac{1}{4}$, $-\frac{1}{8}$, …
d. 3, 6*i*, –12, –24*i*, 48, …

47. Find each sum. (hint: check if the series is possibly arithmetic or geometric)

a. $\sum_{n=5}^{10}(n^2-1)$ b. $\sum_{k=1}^{20}(1+0.4k)$ c. $\sum_{i=1}^{10}5(-4)^i$

48. Use mathematical induction to prove that the formula is true for all natural numbers *n*.

$1+4+7+\ldots+(3n-2)=\frac{n(3n-1)}{2}$

49. Use Pascal's triangle to expand the expression: $(x+2y)^5$

50. Evaluate: a. $\binom{6}{4}$ b. $\sum_{k=0}^{4}\binom{4}{k}$ c. $\frac{n!}{2(n-2)!}$

*****ANSWERS******ANSWERS******ANSWERS******ANSWERS******ANSWERS****

Math 176: Cumulative Semester Review

☺ A ton of Practice Problems ☺

1.

a. $\mathbb{R}, x \neq 2$

b. $\mathbb{R}$

c. 11/5

d. 0

e. 3

f. No

g. $\dfrac{a+h+4}{a+h-2}$

h. 1

i. 5

j. $(-\infty,-1.2)\cup(1.2,\infty)$

k. –11/3

2. SEE GRAPH →

3. SEE GRAPH →

4. SEE GRAPH →

5. 8 in.

6. $k^{-1}(x)=\sqrt[3]{x-2}-1$

7. $f(x)=(x+2)^2-3$ has min of –3

8.

a. shift left 2

b. shift down 3

c. reflected about the x-axis

d. reflected about y = x

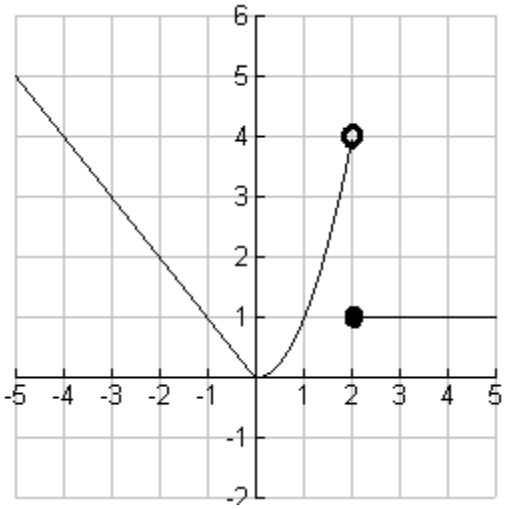
9. building: h(0)=32 ft

max height = h(1.5)= 68 ft

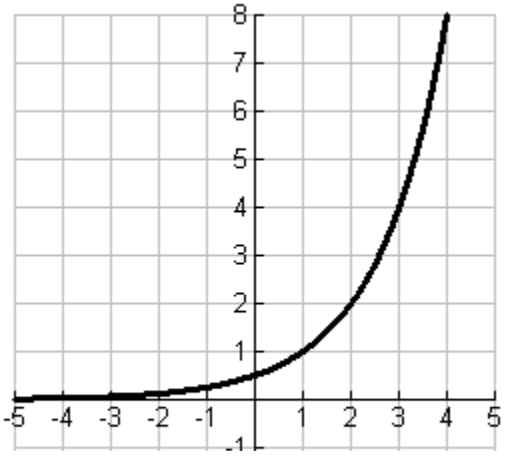
hits ground at approx. 3.56 sec

10. Graph and use vertical and horizontal line test. a. yes b. no

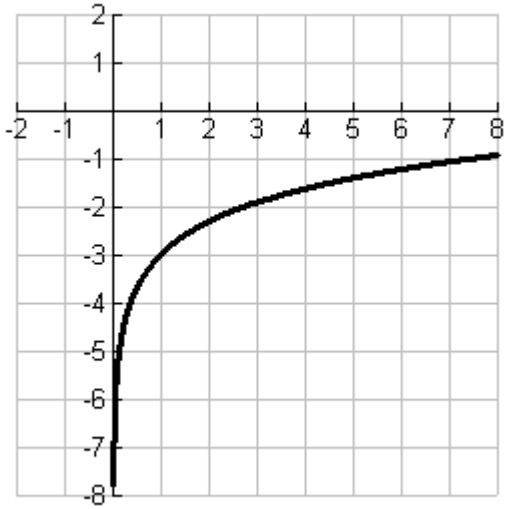
2.



3.



4.



a. end behavior like x^5, $as\ x \to \infty, P(x) \to \infty$
$as\ x \to -\infty, P(x) \to -\infty$

b. 5

c. Up to 4 (P(x) has only 2). Max= 28.266 @ x= –1.362, Min= –0.396 @ x = 1.762

d. Increasing: $(-\infty,-1.362) \cup (1.762,\infty)$
Decreasing (–1.362, 1.762)

e. $\pm 1, \pm 2, \pm 4$

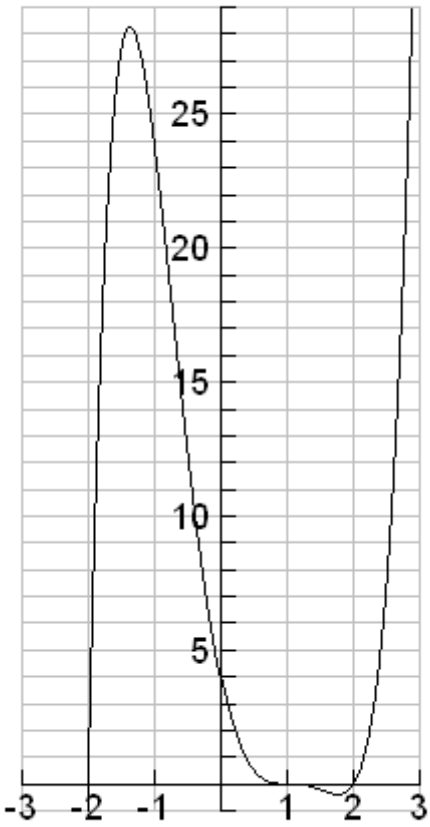
f. Zeros @ x = –2, 1 (mult. 3), 2

g. D: $\mathbb{R}$ R: $\mathbb{R}$

h. Zeros @ x = –2, 1 (mult. 3), 2

i. $P(x) = (x+2)(x-1)^3(x-2)$

j. SEE GRAPH →



12. $4x^2 - 12x + 34 + \dfrac{-112}{x+3}$

13. $P(1) = 5$

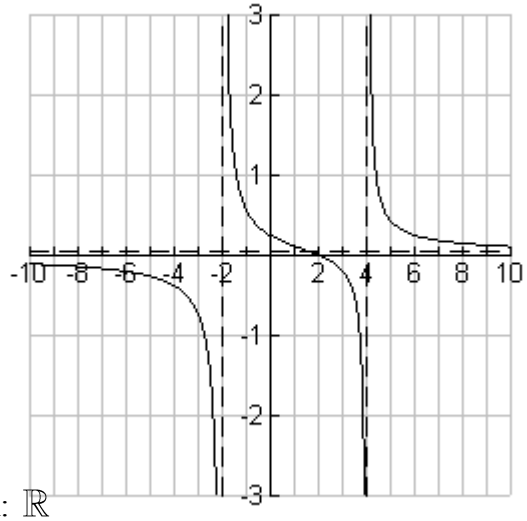
14.

a. $-46 - 22i$

b. $\dfrac{34}{25} - \dfrac{13}{25}i$

c. $1 + i$

d. $4 + i$



15. SEE GRAPH →
D: $\mathbb{R}, x \neq 4, -2$ R: $\mathbb{R}$

16.

a. 3
b. 7
c. 12
d. 3
e. –3
f. 0
g. 3/2
h. 30

17. $x = \dfrac{\ln 20}{4 \ln 5} \approx 0.465$

18. $x = \frac{10}{7} \approx 1.429$

19. $x = 2$

20. a. $A(t) = P\left(1+\frac{r}{n}\right)^{nt} = 12{,}000\left(1+\frac{.035}{12}\right)^{12t}$

$A(3) = \$13{,}326.49$

b. $20{,}000 = 12{,}000\left(1+\frac{.035}{12}\right)^{12t}$

$t = 14.6$ *years*

21.

a. $-\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $\frac{2\sqrt{3}}{3}$ d. –1 e. Undefined

f. –0.64941 g. $\sin(60°+45°) = \frac{\sqrt{6}+\sqrt{2}}{4}$ h. $-\frac{\pi}{6}$ i. $-\sqrt{3}$

j. 0 k. $\frac{1}{2}$ l. $\cos\left(\frac{1}{2}\cdot\frac{\pi}{4}\right) = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

22.

a. 23.09 c. 23.29 e. 87.41°

b. 37.87° d. 57.44

23. $-\frac{\sqrt{85}}{6}$

24. amp = 5, period = $\frac{2\pi}{3}$, phase shift: right $\frac{\pi}{3}$

25. $Area_{\text{sector}} - Area_{\text{triangle}} = \frac{1}{2}(9^2)\left(76\cdot\frac{\pi}{180}\right) - \frac{1}{2}(9\cdot 9)\sin 76° = 14.4 \ \ sq.units$

26. 73.9 feet

27. a. $LHS = 1+\tan x\tan\frac{x}{2} = 1+\frac{\sin x}{\cos x}\cdot\frac{1-\cos x}{\sin x} = 1+\frac{1-\cos x}{\cos x} = 1+\frac{1}{\cos x}+\frac{-\cos x}{\cos x}$

$= 1+\sec x-1 = \sec x = RHS$

b. **Error! Objects cannot be created from editing field codes.**

c. $LHS = \frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{2\sin x\cos x}{\sin x} - \frac{2\cos^2 x-1}{\cos x} = 2\cos x - \frac{2\cos^2 x}{\cos x} + \frac{1}{\cos x}$

$= 2\cos x - 2\cos x + \sec x = \sec x = RHS$

28. $x = \pi n,\ \frac{\pi}{6} + 2\pi n,\ \frac{5\pi}{6} + 2\pi n$

29. $x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{3\pi}{2}$

30. $1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

31. $8 - 8i$

32. $|\mathbf{u}| = \sqrt{208}$, $|\mathbf{v}| = \sqrt{13}$, $\mathbf{u} + \mathbf{v} = \langle -5, 10\rangle$, $\mathbf{u} - \mathbf{v} = \langle -11, 14\rangle$, $3\mathbf{u} = \langle -24, 36\rangle$, $\frac{1}{4}\mathbf{u} - 3\mathbf{v} = \langle -11, 9\rangle$, and $\mathbf{u}\cdot\mathbf{v} = -48$. $\theta = 157°$

33. $\left(-1+\sqrt{3}, 1+\sqrt{3}\right)$ *and* $\left(-1-\sqrt{3}, 1-\sqrt{3}\right)$

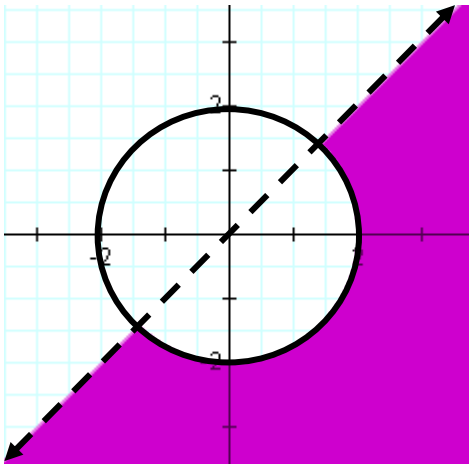
34. (1, 1, 2)

35. a. $\begin{bmatrix} 15 & 18 \\ 12 & -1 \end{bmatrix}$ b. DNE c. 0 d. $\begin{bmatrix} 1 & 2/3 \\ 0 & 1/3 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$

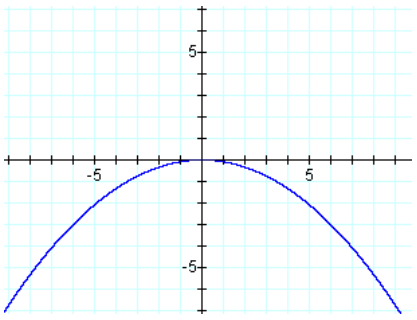
f. DNE g. –11

36. $\frac{9}{5(x-2)} + \frac{1}{5(x+3)}$

37. SEE GRAPH→ (solution is shaded region)

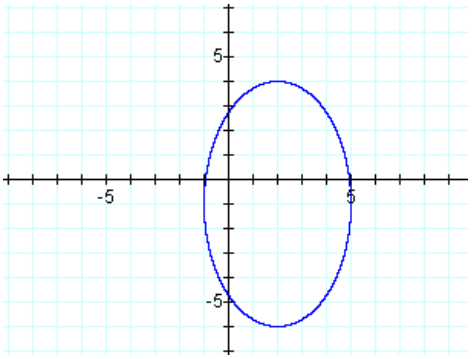


38. focus: (0,–3) directrix: y = 3 SEE GRAPH→

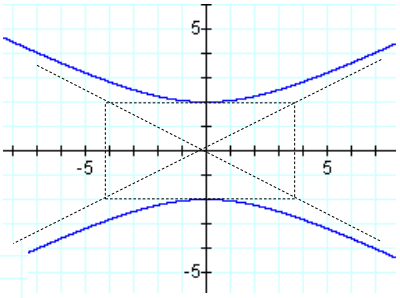


39. foci: (2,3) & (2,–5) eccentricity: 0.8

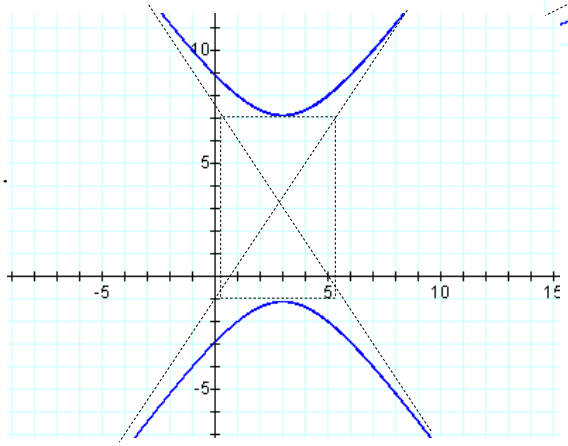
SEE GRAPH→



40. foci: $\left(0,\pm\sqrt{20}\right)$ asymptotes: $y=\pm\frac{1}{2}x$ SEE GRAPH→



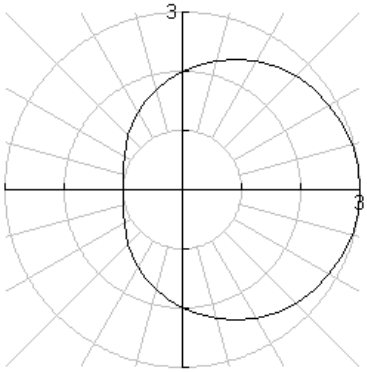
41. SEE GRAPH→

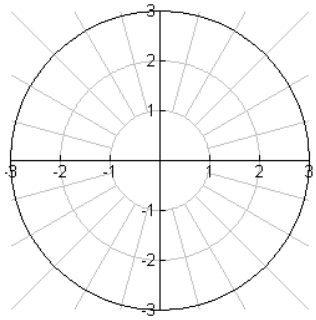


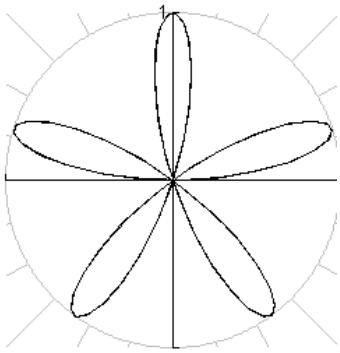
This equation is not pretty..

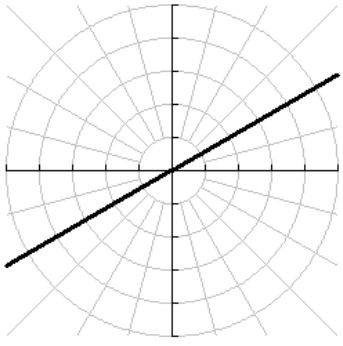
$$\frac{(y-3)^2}{17}-\frac{(x-3)^2}{(17/2)}=1$$

42. a. b. c. d.

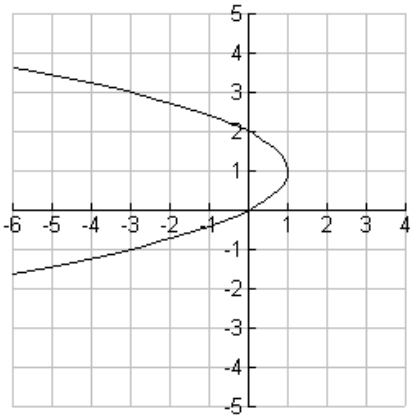








43. rectangular-coordinate equation: $x = 1-(y-1)^2$ SEE GRAPH→



44. $\begin{aligned} x &= 1+3t \\ y &= 3+t \end{aligned}$

45. a. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$ b. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$ c. $5, 10, 25, 70, 115, \ldots$

46. a. $arith: a_n = 4+(n-1)6,\ a_6 = 34$

b. $neither: a_n = a_{n-1} + n - 1,\ a_6 = 19$

c. $geom: a_n = -2\left(\frac{-1}{2}\right)^{n-1},\ a_6 = \frac{1}{16}$

d. $geom: a_n = 3(3i)^{n-1},\ a_6 = 144i$

47. a. 349 b. 104 c. 4,194,300

48. $n=1:\ 1 = \frac{1(3(1)-1)}{2} = 1$

Assume $n = k: 1+4+7+\ldots+(3k-2) = \frac{k(3k-1)}{2}$

Prove $n = k+1: 1+4+7+\ldots+(3k-2)+(3(k+1)-2) = \frac{(k+1)(3(k+1)-1)}{2}$

$\underbrace{1+4+7+\ldots+(3k-2)}_{\frac{k(3k-1)}{2}} + (3(k+1)-2) = \frac{k(3k-1)}{2} + (3(k+1)-2)$

$= \frac{3k^2-k}{2} + \frac{2(3k+1)}{2} = \frac{3k^2-k+6k+2}{2} = \frac{3k^2+5k+2}{2} = \frac{(3k+2)(k+1)}{2} = \frac{(k+1)(3(k+1)-1)}{2}$

so, by mathematical induction, the statement is true for all natural numbers, n.

49. $(x+2y)^5 = x^5 + 5x^4(2y) + 10x^3(2y)^2 + 10x^2(2y)^3 + 5x(2y)^4 + (2y)^5$
$= x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

50. a. 15 b. 16 c. $\frac{n(n-1)}{2}$