Math 176: Proving Trigonometric Identities

General Strategy: Start with one side of the identity (usually the more complicated side) and manipulate it to look like the other side of the equation.

Tools for manipulating:	Examples:
1. Use known trig identities	see back cover of textbook and yellow cheat sheet $\cos^2\theta + \sin^2\theta = 1$
2. Write all expressions in terms of sines and cosines.	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3. Add fractions by getting a common denominator.	$\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta}{\sin\theta\cos\theta} + \dfrac{\sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$
4. Separate one fraction into two fractions.	$\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$
5. Multiply a fraction by the conjugate of the denominator (or numerator)	$\dfrac{1}{1+\cos\theta} = \dfrac{1}{1+\cos\theta}\cdot\dfrac{1-\cos\theta}{1-\cos\theta} = \dfrac{1-\cos\theta}{1-\cos^2\theta}$
6. Factor out the greatest common factor.	$\tan^3\theta + \tan\theta = \tan\theta\left(\tan^2\theta + 1\right)$
7. Work on BOTH sides of the equation, working towards A = A.	$LHS = \dfrac{1+\cos\theta}{\cos\theta} = \dfrac{\tan^2\theta}{\sec\theta - 1} = RHS$ $LHS = \dfrac{1+\cos\theta}{\cos\theta} = \dfrac{1}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} = \sec\theta + 1$ $RHS = \dfrac{\tan^2\theta}{\sec\theta - 1} = \dfrac{\sec^2\theta - 1}{\sec\theta - 1} = \dfrac{(\sec\theta+1)(\sec\theta-1)}{\sec\theta - 1} = \sec\theta + 1$

First, let's get a little practice **simplifying** trig expressions:

1. $\sin x \cos x \sec x$

$= (\sin x)(\cos x)\left(\frac{1}{\cos x}\right)$

$= \sin x$

2. $\cos^3 x + \sin^2 x \cos x$

$= \cos x\,(\underbrace{\cos^2 x + \sin^2 x}_{1})$

$= \cos x$

3. $\dfrac{1+\cot\theta}{\csc\theta} = \dfrac{\left(1+\frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\sin\theta}} \cdot \dfrac{\sin\theta}{\sin\theta}$

$= \sin\theta + \cos\theta$

4. $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = \dfrac{\sin x}{\frac{1}{\sin x}} + \dfrac{\cos x}{\frac{1}{\cos x}}$

$= (\sin x)(\sin x) + (\cos x)(\cos x)$

$= 1$

Verify each trigonometric identity.

5. $\cos(-x) - \sin(-x) = \cos x + \sin x$

$$LHS = \cos(-x) - \sin(-x) = \cos x - (-\sin x)$$
$$= \cos x + \sin x = RHS$$

6. $\dfrac{\cos x}{\sec x \sin x} = \csc x - \sin x$

$$LHS = \frac{\cos x}{\sec x \sin x} = \frac{\cos x}{\frac{1}{\cos x} \cdot \sin x} = \frac{\cos^2 x}{\sin x} = \frac{1 - \sin^2 x}{\sin x}$$
$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \csc x - \sin x = RHS$$

7. $\dfrac{\sin w}{\sin w + \cos w} = \dfrac{\tan w}{1 + \tan w}$

$$RHS = \frac{\tan w}{1 + \tan w} = \left(\frac{\frac{\sin w}{\cos w}}{1 + \frac{\sin w}{\cos w}} \right) \cdot \frac{\cos w}{\cos w} = \frac{\frac{\sin w}{\cancel{\cos w}} \cdot \cancel{\cos w}}{\cos w + \frac{\sin w}{\cancel{\cos w}} \cdot \cancel{\cos w}}$$
$$= \frac{\sin w}{\cos w + \sin w} = LHS$$

8. $\dfrac{\sin A}{1 - \cos A} - \cot A = \csc A$

$$LHS = \frac{\sin A}{1 - \cos A} - \cot A = \frac{\sin A}{1 - \cos A} \cdot \frac{1 + \cos A}{1 + \cos A} - \cot A$$
$$= \frac{\sin A (1 + \cos A)}{1 - \cos^2 A} - \frac{\cos A}{\sin A} = \frac{\sin A (1 + \cos A)}{\sin^2 A} - \frac{\cos A}{\sin A}$$
$$= \frac{1 + \cos A}{\sin A} - \frac{\cos A}{\sin A} = \frac{1 + \cos A - \cos A}{\sin A} = \frac{1}{\sin A} = \csc A$$
$$= RHS$$

9. $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$LHS = \frac{\cos x}{1-\sin x} \cdot \frac{(1+\sin x)}{(1+\sin x)} = \frac{\cos x(1+\sin x)}{1-\sin^2 x} = \frac{\cos x(1+\sin x)}{\cos^2 x}$$

$$= \frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = RHS$$

10. $\dfrac{\cot x+1}{\cot x-1} = \dfrac{1+\tan x}{1-\tan x}$

$$LHS = \frac{\cot x+1}{\cot x-1} = \frac{\left(\frac{1}{\tan x}+1\right)}{\left(\frac{1}{\tan x}-1\right)} \cdot \frac{\tan x}{\tan x} = \frac{1+\tan x}{1-\tan x}$$

$$= RHS$$

11. $(\tan x + \cot x)\sin x \cos x = 1$

$$LHS = (\tan x+\cot x)\sin x\cos x = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\sin x\cos x$$

$$= \frac{\sin x \sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{\cos x \cancel{\sin x} \cos x}{\cancel{\sin x}}$$

$$= \sin^2 x + \cos^2 x = 1 = RHS$$

12. $(\sin x + \cos x)^4 = (1 + 2\sin x \cos x)^2$

$$LHS = (\sin x + \cos x)^4 = \left((\sin x+\cos x)^2\right)^2$$

$$= \left(\sin^2 x + 2\sin x\cos x + \cos^2 x\right)^2$$

$$= (1 + 2\sin x\cos x)^2 = RHS$$