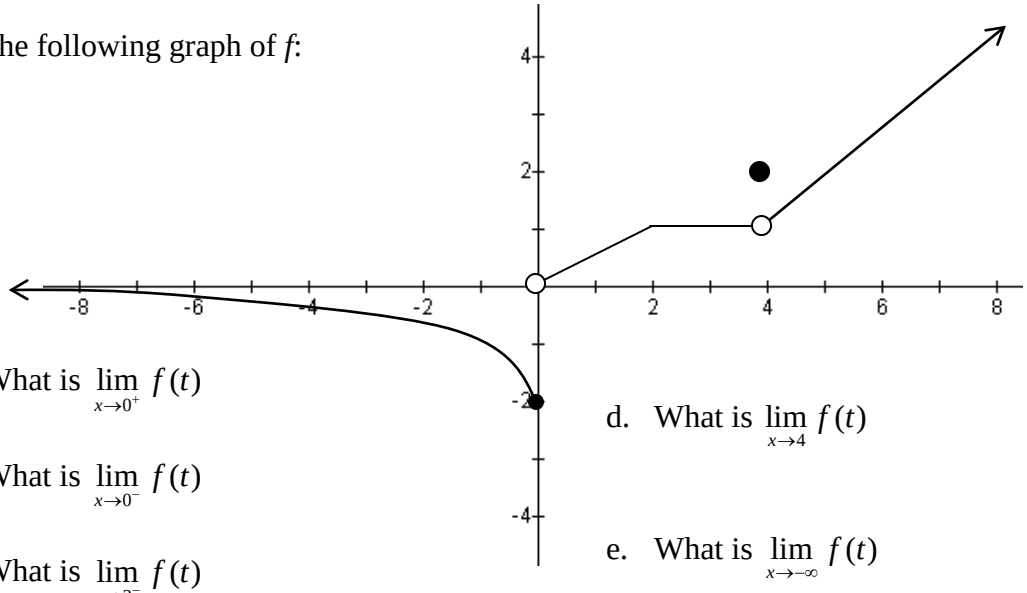
Chapter 2 REVIEW SHEET
Math 180, Vanden Eynden
NOTE: You must show all of your work on the exam for full credit.

1. Consider the following graph of *f*:



a. What is $\lim_{x\to 0^+} f(t)$

b. What is $\lim_{x\to 0^-} f(t)$

c. What is $\lim_{x\to 2^-} f(t)$

d. What is $\lim_{x\to 4} f(t)$

e. What is $\lim_{x\to -\infty} f(t)$

f. Where is the function discontinuous?

g. Where is the function not differentiable?

2. Find the limits, if they exist. Justify each answer with an algebraic argument (or a table or graph, if necessary). Be descriptive, if possible.

a. $\lim_{x\to -4} \dfrac{x+4}{x^2+3x-4}$

b. $\lim_{x\to 16} \dfrac{4-\sqrt{x}}{x-16}$

c. $\lim_{x\to 0} x^4 \sin\dfrac{10}{x}$ (Squeeze Thrm)

d. $\lim_{x\to 8^-} \dfrac{|x-8|}{x-8}$

e. $\lim_{x\to\infty} \dfrac{3x^8-4x+2}{13-9x^7+4x^8}$

f. $\lim_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$

g. $\lim_{x\to 0} \dfrac{\cos x-1}{x}$

h. $\lim_{x\to 1} \dfrac{x^2+x-2}{x-3}$

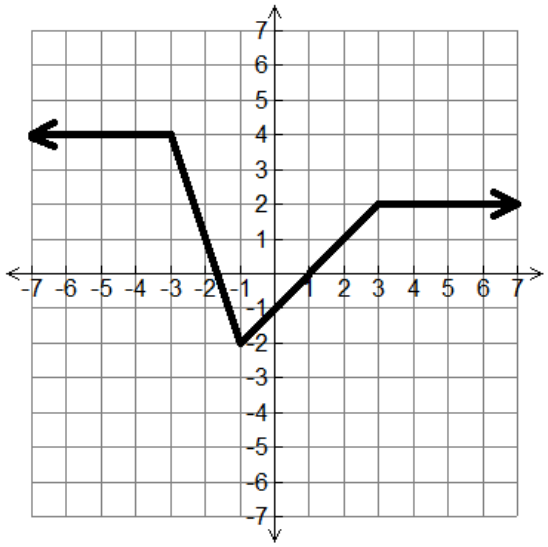
i. $\lim_{x\to 1} \dfrac{-2}{(1-x)^2}$

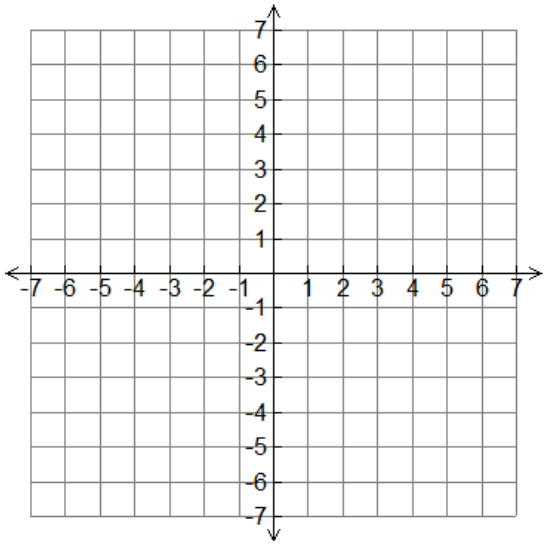
j. $\lim_{x\to -\infty} \dfrac{x^4+2x^3+1}{-2x^5+4x^2}$

k. $\lim_{x\to \pi} \dfrac{x}{\cos x}$

l. $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{x^3-3x^2}$

3. Let f be the function whose graph is given below. Accurately sketch a graph of f'





4. Given the function $g(x)=\frac{3}{x}$.

   a. Find $g'(x)$ using the limit definition.

   b. Find the equation of the tangent line to the curve at the point (1, 3)

5. Find a function f and a number *a* such that

$$\lim_{h\to 0}\frac{(2+h)^6-64}{h}=f'(a)$$

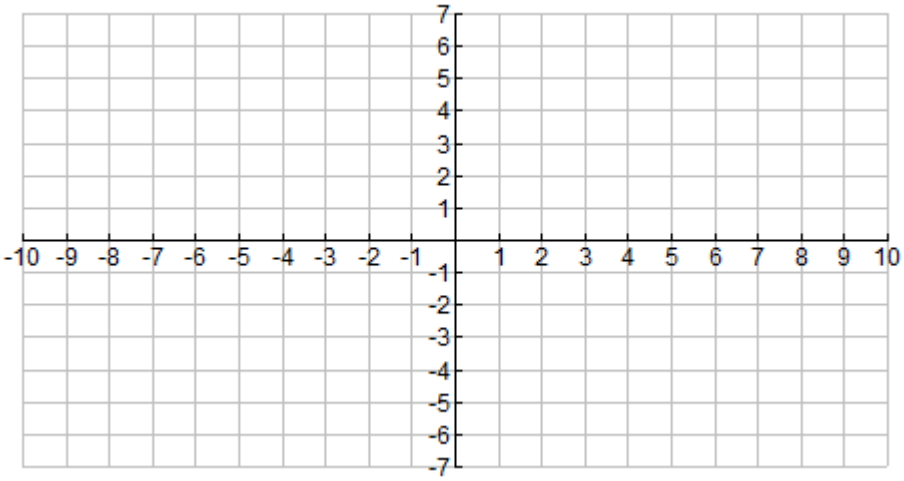
a =

f(*x*)=

6. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$f(0)=0,\quad f'(0)=3,\quad f(3)=2,\quad \lim_{x\to\infty}f(x)=4,\quad \lim_{x\to 3}f(x)=5,$$

$$\lim_{x\to -2}f(x)=-\infty,\quad \lim_{x\to -\infty}f(x)=0$$



7. If a ball is thrown into the air with a velocity of 28 ft/s, its height (in feet) after t seconds is given by $y = 28t - 16t^2$.

   a. Find the velocity of the ball when $t = a$.

   b. Find the velocity of the ball after 1 second.

   c. When does the ball hit the ground?

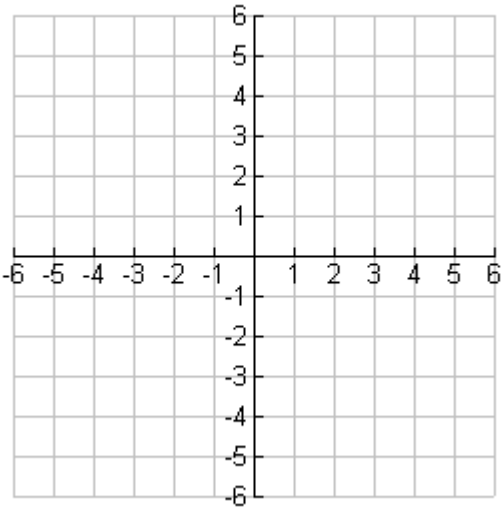
   d. What is the velocity when the ball hits the ground (impact velocity).

8. Find the vertical and horizontal asymptotes for the following function. Don't use your calculator!

$$h(x) = \frac{3x - 3}{x^2 + x - 2}$$

9. Graph the following piecewise defined function accurately.

$$h(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$



   a. Find the following limits, if they exist.
   (if necessary, be descriptive when the limit is ∞ or $-\infty$)

$\lim_{x \to -4} h(x) =$ $\lim_{x \to 3} h(x) =$ $\lim_{x \to 0^+} h(x) =$

$\lim_{x \to 0^-} h(x) =$ $\lim_{x \to 0} h(x) =$ $\lim_{x \to \infty} h(x) =$

   b. At what values of x is $h(x)$ discontinuous?

   c. At what values of x is $h(x)$ not differentiable?

10. Use the Intermediate Value Theorem to show that the function $f(x) = x^4 - 2x^2 + 3x$ has a zero in the interval [–2, –1].

11. Use the formal limit definition of the derivative to find the equation of the tangent line to $g(x) = \sqrt{x} + 1$ at $x = 3$.

12. Given that $\lim_{x \to 3} f(x) = 5$, $\lim_{x \to 3} g(x) = 0$, and $\lim_{x \to 3} h(x) = -8$, find the following limits, if they exist. If the limit does not exist, explain why.

    a. $\lim_{x \to 3} \left(f(x) + h(x) \right)$

    b. $\lim_{x \to 3} x^2 f(x)$

    c. $\lim_{x \to 3} \left(f(x) \right)^2$

    d. $\lim_{x \to 3} \dfrac{h(x)}{2 f(x)}$

    e. $\lim_{x \to 3} \dfrac{f(x)}{g(x)}$

    f. $\lim_{x \to 3} \sqrt[3]{h(x)}$

13. Motion on Mars. A projectile is launched vertically upward from the surface of Mars. The table below gives the height of the object at the indicated time following the launch.

Time in seconds	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6
Height in feet	0.0	18.2	34.3	48.4	60.4	70.4	78.3	84.2	88.1	90.0	89.6	87.3	82.9	75.5	63.1

Use the data to compute the average velocity of the projectile on the following time intervals:

    a. [0.0, 4.0]

    b. [0.8, 3.2]

    c. [1.2, 2.8]

    d. Approximate the instantaneous velocity of the projectile at 2.0 seconds