Practice Problems from Ch. 3 (3.1 – 3.9) **NAME__________________**
Math 180, Vanden Eynden

1. Differentiate the functions:

a. $f(x) = x^4 - \dfrac{2}{3x^2}$

b. $y = 4\pi^2$

c. $h(x) = \dfrac{2x-4}{x^3+2x+1}$

d. $F(x) = \sin x \cdot (1+\cos x)$

e. $s(x) = \sqrt{1-\tan x}$

f. $y = \ln(\cos(x^2))$

g. $y = 5\sec(3x)$

h. $n(x) = \pi^x$

i. $y = xe^{\cos x}$

j. $h(x) = \dfrac{3x-4}{5x+1}$

k. $f(x) = \ln(x^6+1)$

l. $y = (3x)^x$ (use logarithmic diff)

m. $g(x) = \csc(4x)$

n. $y = \sin^{-1}(6x)$

o. $k(x) = e^{x^2}\left(x^3 - 3x^2 + 5\right)$

p. $y = \arctan(e^x)$

q. $y = x\cos^{-1}(x^3)$

2. Let $h(x) = \sqrt{1-x^2} \cdot \arcsin(x)$. Find $\dfrac{dh}{dx}$ and **simplify** where possible.

3. Find an equation of a tangent line to $y^2 = x^3(2-y)$ at the point (1,1). Put your answer in the form y = mx + b

4. For what value(s) of x does the graph of $f(x) = 2e^{-x} + xe^{-x}$ have a horizontal tangent line?

5. At what point on the curve $y = [\ln(x+4)]^2$ is the tangent line horizontal?

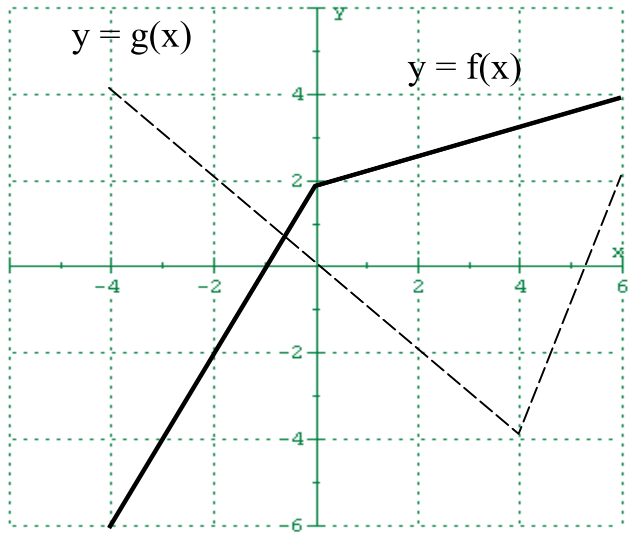
6. Find the equation of the tangent line to the curve $y = x - 4\cos(2x)$ at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$

7. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm? Note: The volume of a sphere is $V = \dfrac{4}{3}\pi r^3$

8. Find the equation of the tangent line to $y = \ln(e^x + e^{2x})$ at the point $(0, \ln 2)$.

9. a. Let $H(x) = f(g(x))$. Find $H'(2)$

b. Let $P(x) = \dfrac{f(x)}{g(x)}$. Find $P'(-2)$



10. Two people start from the same point. One walks east at 6 mi/h and the other walks northeast at 4 mi/h. How fast is the distance between the people changing after 30 minutes?

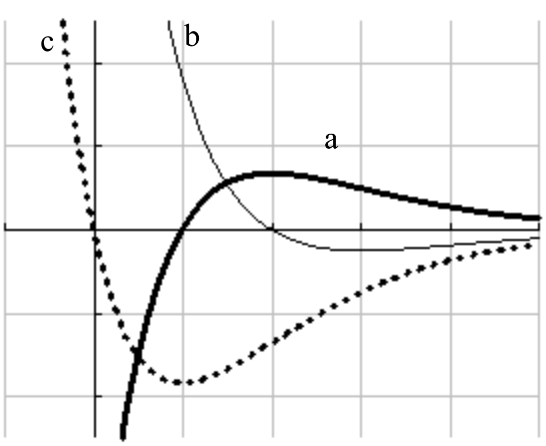
11. A particle moves on a vertical line so that its coordinate at time t is $s(t) = t^3 - 12t + 3, \quad t \geq 0$.

   a. Find the velocity and acceleration functions.

   b. When is the particle moving upward and when is it moving downward?

   c. When is the particle speeding up and when is it slowing down?

12. The figure shows the graphs of $f, f',$ and f''. Identify each curve and explain your choices in words.



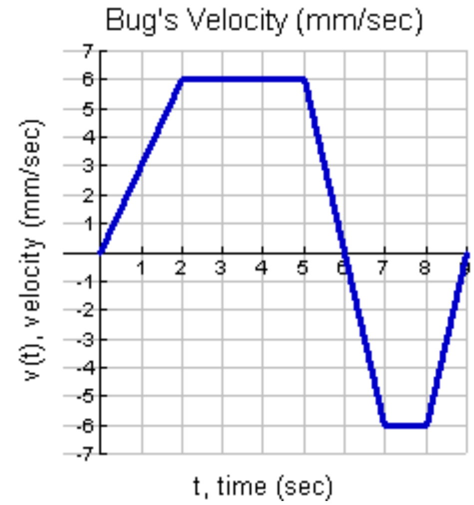
$f =$

$f' =$

$f'' =$

13. A bug begins to crawl up a vertical wire at time t = 0 seconds. The **velocity** v of the bug at time t, $0 \le t \le 9$, is given by the function whose graph is shown below. Velocity is given in mm/second.

a. At what value(s) of t does the bug **change direction**? Explain.



b. When is the bug moving **up** the wire?

c. When is the bug moving **down** the wire?

d. When is the bug moving the **fastest**?

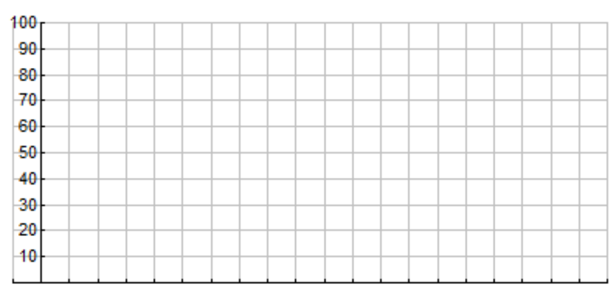
e. When is the bug **stopped**?

f. Say the bug started at the bottom of the wire. Will the bug return to the bottom at any time during the first 9 seconds? Explain.

14. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

15. A warm can of soda is placed in a cold refrigerator. Suppose the temperature T (in $^{\circ}$F) of the can of soda is a function of x, the time in the refrigerator (in minutes).

a. Sketch a likely graph of $T(x)$.



b. What is the meaning of $\frac{dT}{dx}$?

c. What are the units of $\frac{dT}{dx}$?

d. Is $\frac{dT}{dx}$ positive or negative?

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$