Solutions to Practice Problems from Ch. 3
Math 180 , Vanden Eynden

1. Differentiate the functions:

a. $f'(x) = 4x^3 + \dfrac{4}{3x^3}$

b. $y' = 0$

c. $h'(x) = \dfrac{-4x^3 + 12x^2 + 10}{(x^3 + 2x + 1)^2}$

d. $F'(x) = \cos x + \cos^2 x - \sin^2 x$

e. $s'(x) = \dfrac{-\sec^2 x}{2\sqrt{1-\tan x}}$

f. $y' = -2x\tan(x^2)$

g. $y' = 15\sec(3x)\tan(3x)$

h. $n'(x) = \pi^x \ln \pi$

i. $y' = e^{\cos x}(1 - x\sin x)$

j. $h'(x) = \dfrac{23}{(5x+1)^2}$

k. $f'(x) = \dfrac{6x^5}{x^6 + 1}$

l. $y' = (3x)^x(\ln(3x) + 1)$

m. $g'(x) = -4\csc(4x)\cot(4x)$

n. $y' = \dfrac{6}{\sqrt{1-36x^2}}$

o. $k'(x) = e^{x^2}\left(2x^4 - 6x^3 + 3x^2 + 4x\right)$

p. $y' = \dfrac{e^x}{1+e^{2x}}$

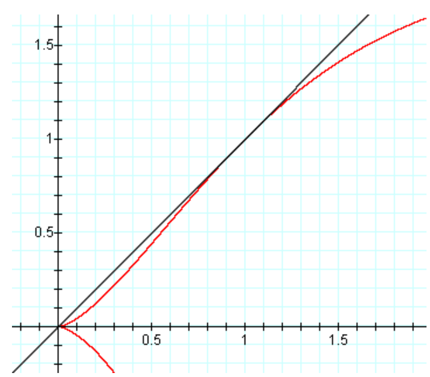
q. $y' = \cos^{-1}(x^3) - \dfrac{3x^3}{\sqrt{1-x^6}}$

2. $\dfrac{dh}{dx} = 1 - \dfrac{x\sin^{-1}(x)}{\sqrt{1-x^2}}$

3. Differentiate implicitly: $\dfrac{dy}{dx} = \dfrac{3x^2(2-y)}{2y+x^3}$, so m = 1

Tangent line at (1, 1): $y = x$

See graph to verify.

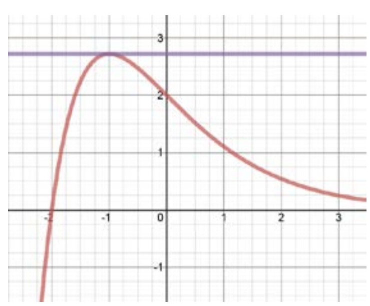


4. $f'(x) = -e^{-x} - xe^{-x} = 0$

$-e^{-x}(1-x) = 0$

So tangent is horizontal when $x = -1$

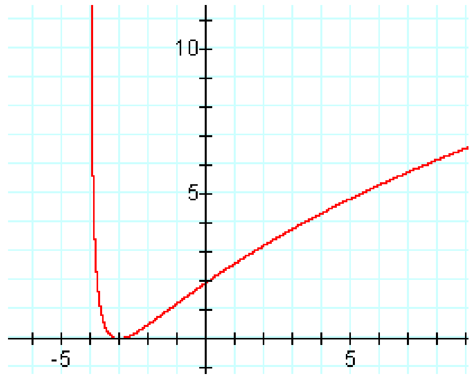
See graph to verify.



5. Set $y' = \frac{2\ln(x+4)}{x+4} = 0$ and solve for x. when $x = -3$

So the tangent line is horizontal at (–3, 0).

See graph to verify.



6. $y' = 1 + 8\sin(2x)$

$y'(\pi/4) = 9$

So tangent line: $y = 9x - 2\pi$

7. Implicitly differentiate $V = \frac{4}{3}\pi r^3$ with respect to *t*: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

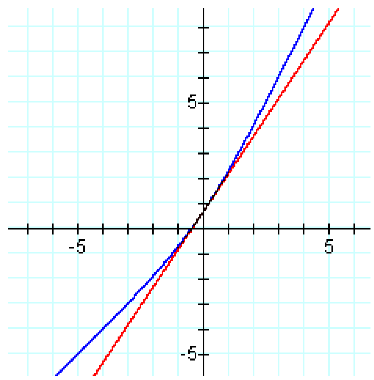
$\frac{dV}{dt} = 100$, $r = 25$ Plug in and solve for $\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{25\pi}\text{cm/s} \approx 0.0127\text{cm/s}$

8. Slope = $y'(x) = \frac{e^x + 2e^{2x}}{e^x + e^{2x}}$ at x = 0, $y'(0) = \frac{3}{2}$

So equation of tangent line is $y = \frac{3}{2}x + \ln 2$.

See graph to verify.



9. a. By chain rule:

$H'(x) = f'(g(x))g'(x)$, so

From the graph $g(2) = -2, g'(2) = -1$, so we have

$f'(-2) = 2$, so that gives

So $H'(2) = -2$

$H'(2) = f'(g(2))g'(2)$

$H'(2) = \left(f'(-2)\right)\cdot -1$

$H'(2) = (2)\cdot -1.$

b. By Quotient rule:

$P'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, so

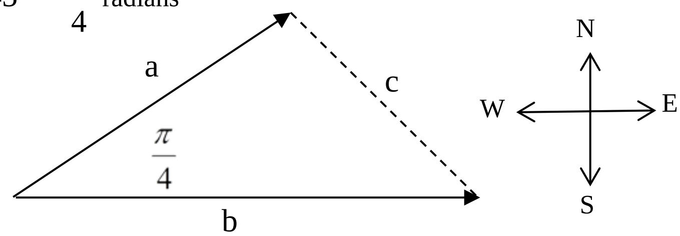
$P'(-2) = \frac{g(-2)f'(-2) - f(-2)g'(-2)}{[g(-2)]^2}$

$P'(-2) = \frac{(2)(2) - (-2)(-1)}{[2]^2} = \frac{1}{2}$

Using graph:

So $P'(-2) = \frac{1}{2}$

10. Use law of cosines. Angle between them is 45º = $\frac{\pi}{4}$ radians



$c^2 = a^2 + b^2 - 2ab\cos(\frac{\pi}{4})$

30 minutes = ½ hour

a = 2, b = 3, c =2.125, $\frac{da}{dt} = 4$, $\frac{db}{dt} = 6$

$c^2 = a^2 + b^2 - 2ab\frac{\sqrt{2}}{2}$

$c^2 = a^2 + b^2 - \sqrt{2}ab$ Now differentiate implicitly with respect to time, *t*

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} - \sqrt{2}\left(a\frac{db}{dt} + b\frac{da}{dt}\right)$ Plug and chug and solve for $\frac{dc}{dt}$ = 4.25 mph

11. a. $v(t) = s'(t) = 3t^2 - 12$

$a(t) = s''(t) = 6t$

b. The particle moving *upward* when $v(t) = 3t^2 - 12 = 3(t-2)(t+2) > 0$.

That happens when t > 2 or t < –2. In this problem, only **t > 2** makes sense.

The particle moving *downward* when $v(t) = 3t^2 - 12 < 0$.

That happens when –2 < t < 2. Only **0< t< 2** makes sense in this problem.

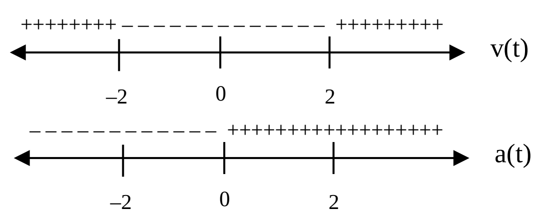
c. When is the particle speeding up and when is it slowing down?

The particle is *speeding up* when the sign of v(t) is the same as the sign of a(t).

That happens when –2 < t< 0 or t >2. Only **t >2** makes sense in this problem.

The particle is *slowing down* when the signs are different.

That happens when t <–2 or 0< t< 2 . Only **0< t< 2** makes sense in this problem.



12. f = c, the zero of “c” <u>does not</u> correspond to a max or min (0 slope) in the other 2 functions.

f' = a, the zero of “a” corresponds the minimum (0 slope) of “c”.

f'' = b, the zero of “b” corresponds to the maximum (0 slope) of “a”.

13. a. t = 6 sec, when v(t) switches from positive to negative, the bug will stop crawling UP the wire, and start crawling DOWN the wire.

b. 0 < t < 6 sec

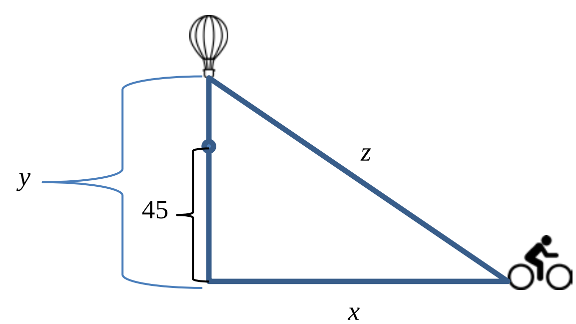
c. 6 < t < 9 sec

d. speed = |v(t)| , is greatest when $2 \le t \le 5$ and when $7 \le t \le 8$. The bug is crawling up (or down) the wire at 6 mm/sec during those times.

e. t = 0 sec, 6 sec, 9 sec. This is when v(t) = 0.

f. No. The bug is crawling up the wire for 6 seconds, and only crawling down the wire for 4 seconds. The bug will not return to the bottom within the first 9 seconds.

14. Find $\frac{dz}{dt}$ after 3 seconds.



We know: $\frac{dx}{dt} = 15$, $\frac{dy}{dt} = 5$

<u>Moment in time: 3 seconds has passed</u>

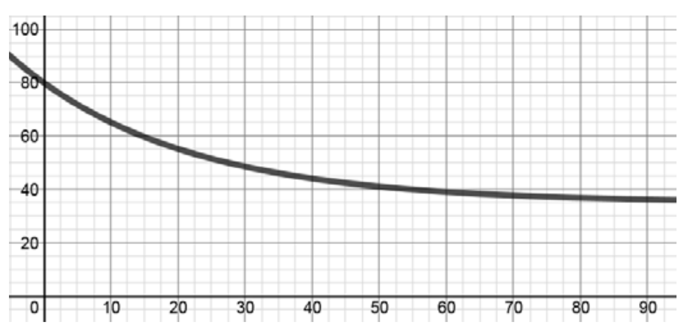
$$x = 15 \cdot 3 = 45\,\text{ft}$$
$$y = 45 + 5 \cdot 3 = 60\,\text{ft}$$
$$z = \sqrt{5625} = 75\,\text{ft}$$

Pythagorean Thrm: $z^2 = x^2 + y^2$

Implicit Diff: $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

Plug and chug and solve for $\frac{dz}{dt}$ = 13 ft/s

15. a. See graph



b. $\frac{dT}{dx}$ is the rate of change of the temperature of the can of soda with respect to time.

c. degrees Fahrenheit per minute, ^{o} F/min

d. $\frac{dT}{dx}$ is negative, since the soda can is cooling and it's temperature is decreasing.