Math 180: Chapter 4 & 3.10 (Sections 3.10, 4.1-4.5, 4.7-4.8) Review Sheet
Vanden Eynden

1. Consider the function $f(x) = x^3 + \frac{3}{2}x^2 - 36x + 7$
   a. Make a sign diagram for $f'(x)$. Find the intervals on which *f* is increasing and decreasing.
   b. Find the local maximum and minimum values of *f*.
   c. Make a sign diagram for $f''(x)$. Find the intervals of concavity and any inflection points.
   d. Use Newton's Method to find the root of *f* on the interval (0, 1) accurate to 8 decimal places. Show your work by writing down any formulas used in your calculations. Make sure to write down your initial guess x_1 and each successive *x* value needed to get to the root.
   e. Sketch the graph of the function using your findings above.

2. Find the critical numbers of the function and classify them as local maximums, minimums or neither.
   $f(x) = x^3\sqrt{2-x}$

3. Find the absolute maximum and absolute minimum value of the function on the given interval.
   a. $g(x) = \dfrac{x}{x^2+4}$ $\quad [0, 3]$ b. $k(\theta) = \theta + \sin\theta$ $\quad [0, 2\pi]$

4. Sketch a graph of the following function by hand. Use both the 1st and 2nd derivative sign diagrams to help you (actually use all techniques taught in graphing)
   $f(x) = x^{4/3} - 8x^{1/3}$

5. Find the equation of the tangent line to the curve $y = x^3 - 6x^2$ at its point of inflection.

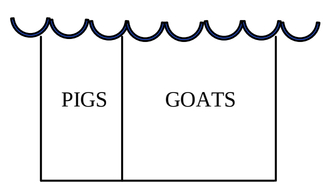
6. Use the linear approximation to $f(x) = \ln x$ at $a = 1$ to estimate $\ln(0.98)$.

7. Suppose that the function *f* is differentiable on the interval [–1, 2] and that *f*(–1) = –1 and *f*(2) = 5. Prove that there is a point on the graph of *f* at which the tangent line is parallel to the line *y* = 2*x*.

8. Each page of a book will contain 30 in^2 of print, and each page must have 2 in. margins at top and bottom and a 1 in. margin at each side. What is the minimum possible area of such a page?

9. Use L'Hospital's Rule to help you evaluate the following limits:
   a. $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt[3]{x}}$ b. $\lim\limits_{x\to 0} \dfrac{\cos(2x) - \cos(5x)}{x^2}$ c. $\lim\limits_{x\to 0} (\cos x)^{5/x}$

10. Patrick and DJ decide to raise pigs and goats. They have 1500 ft of fencing to build a rectangular enclosure. One side of their enclosure is going to be the babbling brook on their property. They will also need to build a fence to separate their pigs and goats. What dimensions will produce an enclosure with largest possible area?



11. Sketch the graph of a function that satisfies all of the given conditions:

$$f'(0) = f'(2) = 0 \quad f(2) = 0 \quad f(0) = 5$$

$$f''(x) > 0 \text{ if } x > 1 \text{ or } x < -1$$

$$f''(x) < 0 \text{ if } -1 < x < 1$$

$$f'(x) > 0 \text{ for } x < 0 \text{ or } x > 2$$

12. Let f be a function that is everywhere differentiable. The value of $f'(x)$, is given for several values of x in the table below.

x	–10	–5	0	5	10
$f'(x)$	2	1	0	1	2

If $f(x)$ is always increasing, which statement about $f(x)$ must be true? **Explain your reasoning.**

A) $f(x)$ has a local minimum at $x = 0$
B) $f(x)$ is concave downward for all x
C) $f(x)$ has a point of inflection at $(0, f(0))$
D) $f(x)$ passes through the origin
E) $f(x)$ is odd function

13. Use Newton's method to approximate $\sqrt[3]{2}$ to four decimal places.

14. a. Explain why Newton's method doesn't work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial guess is chosen to be $x_1 = -1$.

b. Choose another initial guess x_1 and use Newton's Method to find the root accurate to 6 decimal places. Show your work by writing down any formulas used in your calculations. Make sure to write down your initial guess x_1 and each successive x value needed to get to the root.

15. Consider the function $y = \sqrt{x}$.

a. Find the differential, dy

b. Evaluate dy when $x = 25$ and $dx = .03$

c. Use part (b) to estimate $\sqrt{25.03}$

16. Consider the function $f(x) = \dfrac{x}{x+2}$ on the interval [–1, 2].

a. Check that this function satisfies the conditions of the Mean Value Theorem (MVT).

b. Find the value(s) of c that are guaranteed to exist by the conclusion of the MVT.

c. Sketch a graph of $f(x)$ and the line that passes through the end points on the interval [–1, 2]. Then mark the point(s) at which the slope of the tangent line to the function equals the slope of the secant line.