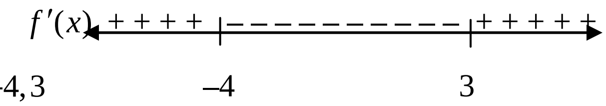
**Math 180: Chapter 4 Review Sheet ---SOLUTIONS**
**Vanden Eynden**

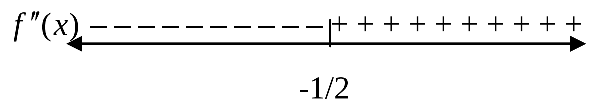



1. a. $f'(x)=3x^2+3x-36=0$, Critical Numbers: $x=-4,3$

Increasing: $(-\infty,-4)\cup(3,\infty)$

Decreasing: $(-4,3)$

b. Local Max = 111 @ x = –4, Local Min = –60.5 @ x = 3




c. $f'(x)=6x+3=0$, Critical value: $x=-\dfrac{1}{2}$

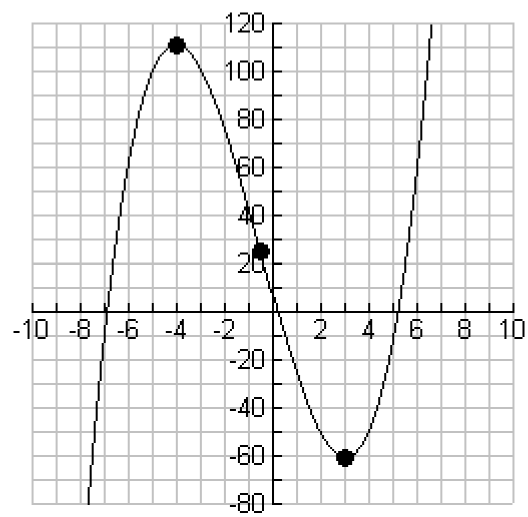
Concave up: $(-1/2,\infty)$

Inflection point: $(-1/2,25.25)$

Concave down: $(-\infty,-1/2)$

d. $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$

e.

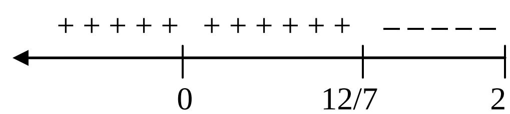



$x_1=0$
$x_2=0.19444444$
$x_3=0.19625914$
$x_4=0.19625933$
$x_5=0.19625933$
The root is 0.19625933.

2. $f(x)=x^3\sqrt{2-x}$ Domain: $(-\infty,2]$

$f'(x)=\dfrac{12x^2-7x^3}{2\sqrt{2-x}}=\dfrac{x^2(12-7x)}{2\sqrt{2-x}}$

C.N.'s: $x=0,\dfrac{12}{7},2$




Local MAX is approximately 2.69 @ $x=\dfrac{12}{7}$

$x=0$, $x=2$ are **neither** maximums nor minimums

3. a. Abs max value of ¼ at x = 2, Abs min value of 0 at x = 0.

b. Abs max value of $2\pi$ at θ= $2\pi$, Abs min value of 0 at θ = 0.

4. $f(x) = x^{4/3} - 8x^{1/3}$ Domain: $\mathbb{R}$

Intercepts: (0,0) and (8,0)

No Vertical or Horizontal asymptotes

$$f'(x) = \frac{4}{3}x^{1/3} - \frac{8}{3}x^{-2/3} = \frac{4(x-2)}{3x^{2/3}}$$ CNs: $x = 2, 0$

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0 2

Local min = -7.56 @ $x = 2$

$$f''(x) = \frac{4}{9}x^{-2/3} - \frac{16}{9}x^{-5/3} = \frac{4(x+4)}{9x^{5/3}}$$ PIPs: $x = -4, 0$

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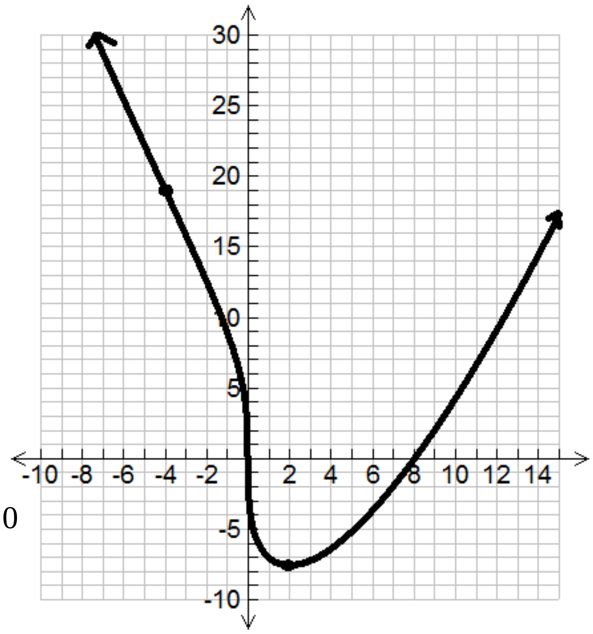
- 4 0

Inflection points x=$(0,0),(-4,19.05)$




5. Set y'' = 0, to find inflection point at (2, –16), y'(2) = –12 = slope of tangent line

y + 16 = –12 (x –2)

So line tangent to y at (2, –16) is **y = –12x +8**

6. $f'(x) = \frac{1}{x}$, so $f'(1) = 1$ and the linear approximation is $L(x) = x - 1$.

$\ln(0.98) \approx L(0.98) = 0.98 - 1 = -0.02$.

7. Since f(x) is differentiable on [–1, 2], it is also continuous on [–1, 2], thus f satisfies the conditions of the Mean Value Theorem. Therefore, there exists *c* in the interval (–1, 2), such that

$$f'(c) = \frac{f(2) - f(-1)}{2-(-1)} = \frac{5-(-1)}{2+1} = \frac{6}{3} = 2.$$

This means that at x = c, the slope of the line tangent is 2.

Which is the same slope of the line y=2x

8. See figure labeled with the unknown lengths →

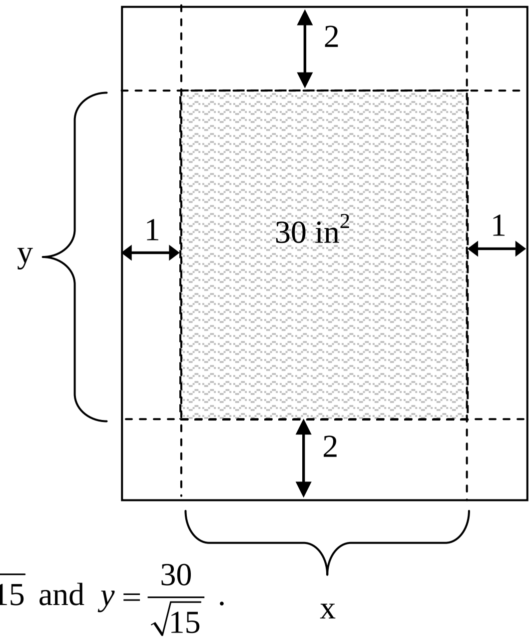
The constraint equation: $xy = 30$ which means $y = \frac{30}{x}$

The objective function to be minimize is:

$A = (x+2)(y+4)$ with the substitution: $y = \frac{30}{x}$.

$A = (x+2)\left(\frac{30}{x}+4\right) = 30 + 4x + \frac{60}{x} + 8$

$A = 38 + 4x + \frac{60}{x}$



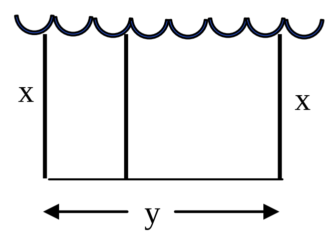

And the derivative: $A' = 4 - \frac{60}{x^2}$ =0 gives the solution, $x = \sqrt{15}$ and $y = \frac{30}{\sqrt{15}}$ .

So the dimensions of the page are $\left(\sqrt{15}+2\right)$ *inches* *by* $\left(\frac{30}{\sqrt{15}}+4\right)$ *inches*,

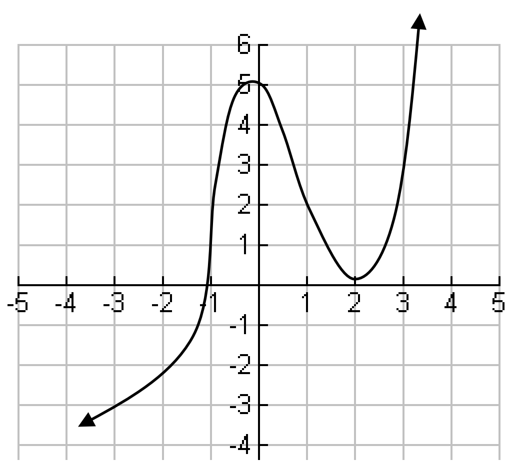
giving a minimum area of 69.98 square inches.

9. a. 0 b. 10.5 c. $e^0 = 1$

10. Constraint: P = 1500 = 3x +y → y = 1500 – 3x
Objective fctn: A = xy = x(1500 – 3x)
Set A'(x) = 0 and solve….. x = 250ft, y = 750ft




11.




12. A) No. $f'(x) > 0$ always

B) No. $f'(x)$ decreases until 0 and then increases, so $f(x)$ is concave down then concave up.

**C) YES.** $f(x)$ has inflection point at (0, $f$(0)) . $f''(x)$ is negative for x<0, and positive for x>0.

D) Not necessarily

E) Not necessarily

13. We are finding the root of $x^3-2=0$ , so $f(x)=x^3-2$

In calculator: $y1=x^3-2$, $y2=f'(x)=3x^2$

In calculator: $1\to x$ ENTER, $x-y_1/y_2\to x$ ENTER, ENTER, ….

$x_1$ = 1 | $x_4$ = 1.2599

$x_2$ = 1.3333 | $x_5$ = **1.2599** This is the value of $\sqrt[3]{2}$ correct to 4 decimals.

$x_3$ = 1.2639

14. a. $x_1$ = –1 is a "bad seed" in that the function happens to have a local minimum at x = –1. Thus Newton's method does not work since the horizontal tangent does not produce an x-intercept. In other words, since $f'(1)=0$, you can not compute $x_2$, since you can not divide by 0 in the calculation: $x_2=-1-\dfrac{f(-1)}{f'(-1)}$.

b. We are finding the root of $x^3-3x+6=0$ , so $f(x)=x^3-3x+6$

In calculator: $y1=x^3-3x+6$, $y2=f'(x)=3x^2-3$

In calculator: $-2\to x$ ENTER, $x-y_1/y_2\to x$ ENTER, ENTER, ….

$x_1$ = –2 | $x_4$ = –2.355309

$x_2$ = –2.444444 | $x_5$ = –2.355301 This is the root correct to 6 decimals.

$x_3$ = –2.359158

15. a. $dy=\dfrac{1}{2\sqrt{x}}dx$ b. $dy=\dfrac{1}{2\sqrt{25}}(.03)=.003$

c. $\sqrt{25.03}=\sqrt{25}+\Delta y\approx 5+dy=5+.003=5.003$, so $\sqrt{25.03}\approx 5.003$

16. a. $f(x)$ is continuous and differentiable on [–1, 2].

b. Set $f'(x)=\dfrac{f(2)-f(-1)}{2-(-1)}$ → $\dfrac{2}{(x+2)^2}=\dfrac{1}{2}$

Solve for x: x = 0, –4 c = 0 is in the interval [–1, 2].