Exam#4 Review: 4.9 & Chapter 5.1-5.4
Math 180, Vanden Eynden

1. Evaluate the Reimann sum for $f(x) = x^2 - x$, $0 \le x \le 2$ using five approximating rectangles and

   a. using right endpoints. Sketch the function and the five rectangles.

   b. using left endpoints. Sketch the function and the five rectangles.

   c. Consider $\int_0^2 \left(x^2 - x\right) dx$. Draw a diagram to explain the geometric meaning of this integral.

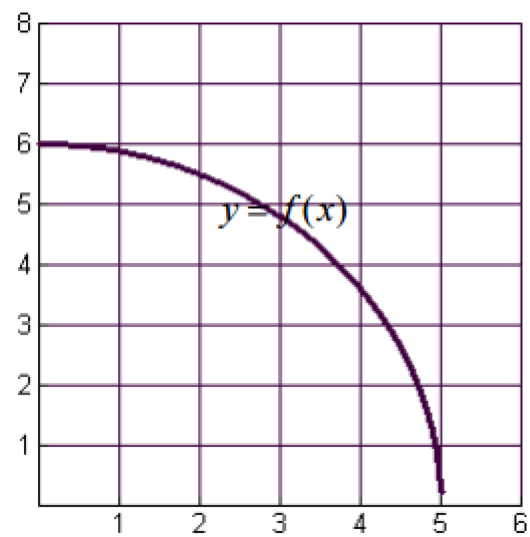
   d. Use the Fundamental Theorem of Calculus to find $\int_0^2 \left(x^2 - x\right) dx$ exactly.

2. Use 5 rectangles to approximate the area under the given graph of *f* from $x = 0$ to $x = 5$

   a. Use left endpoint approximations.

   b. Use right endpoint approximations.

   c. Use midpoint approximations.



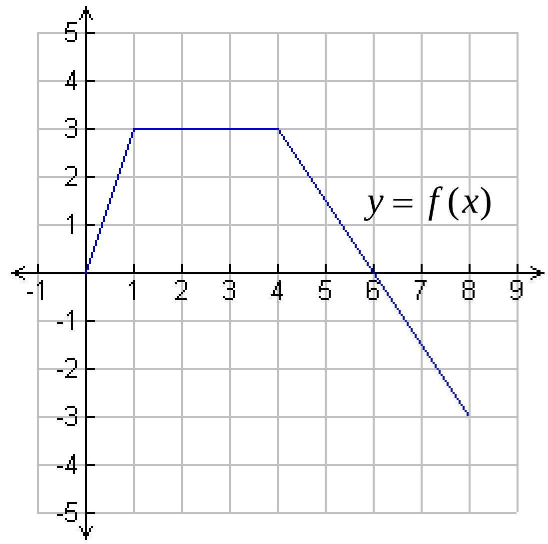
3. Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2x_i^3 - 4\right) \Delta x, \quad [1,7]$$

4. What is the difference between the types of answers obtained by computing $\int f(x)dx$ and $\int_a^b f(x)dx$?

5. If *f* is a differentiable function and 0 < a < b, then what does $\frac{d}{dx}\left[\int_a^b f(x)dx\right]$ equal?

6. Use the graph to evaluate the integrals



a. $\int_0^1 f(x)dx$

b. $\int_6^1 f(x)dx$

c. $\int_6^8 f(x)dx$

d. $\int_0^8 f(x)dx$

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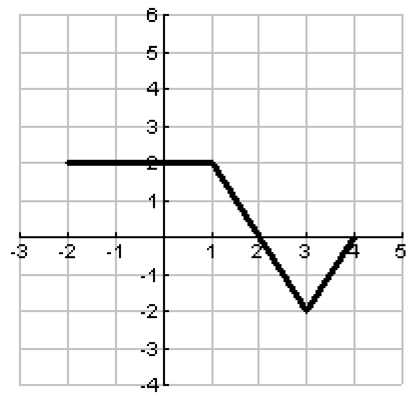
7. The graph of the function *f*, consisting of three line segments, is given.



Let $g(x) = \int_{-2}^{x} f(t)dt$

a. Compute g(–2), g(0), g(1) and g(4).

b. Sketch the graph of g(x) on the same grid to the right →

c. Find the instantaneous rate of change of *g*, with respect to *x*, at *x* = 3.
In other words: $g'(3)$

d. Find the absolute maximum value of *g* on the closed interval [–2, 4]. Justify your answer.

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8. Evaluate the definite integrals:

a. $\int_1^4 \frac{x-2}{\sqrt{x}}dx$

b. $\int_0^3 \left(2\sin x - e^x\right)dx$

c. $\int_1^3 \left(x^2 + 3x\right)dx$

d. $\int_1^4 \sqrt{x}\left(x-2\right)dx$

e. $\int_0^1 (3-x)^2 \, dx$

g. $\int_1^e \frac{x^2+x+1}{x} \, dx$

f. $\int_{-4}^{-2} \frac{3}{5x} dx$

h. $\int_0^1 \frac{1}{1+x^2} \, dx$

9. Evaluate the indefinite integral

a. $\int \frac{1}{4x^2} dx$

c. $\int \sqrt[3]{\frac{8}{x}} \, dx$

b. $\int (\sec^2 x + \sec x \tan x) \, dx$

d. $\int \frac{2}{\sqrt{1-x^2}} dx$

10. Find the exact velocity function and the exact position function given the following information:

$$a(t) = t - 3, \quad v(0) = 4, \quad s(0) = -2$$

11. The velocity of a particle is $v(t) = t^3 - 10t^2 + 24t$ ft/sec. Compute the

a. Displacement over the interval $0 \le t \le 6$

b. Total distance traveled over the interval $0 \le t \le 6$

12. A bottle of wine at room temperature (68° F) is placed in a refrigerator at 4pm. Its temperature after t hours is changing at a rate of $-18e^{-0.6t}$ degrees Fahrenheit/hour. Explain in context what the following definite integral represents: $\int_0^3 -18e^{-0.6t} dt$ and then compute the definite integral using your calculator.

13. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour? You will need to use your calculator to compute the integral.

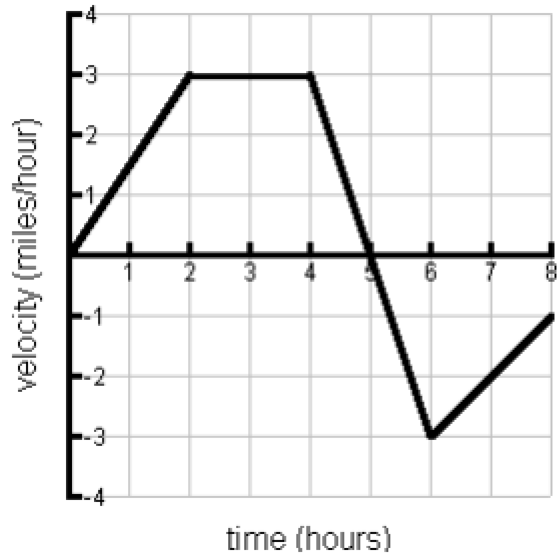
14. A car is moving along a straight road from A to B, starting at A at time $t = 0$. Below is a graph of the cars velocity $v(t)$, measured in miles per hour.

a. How many miles away from A is the car at time $t = 6$

b. At what time does the car change direction?

c. What does $\int_0^8 v(t)\,dt$ represent in this problem?

d. What was the TOTAL distance the car traveled in the 8 hours?



15. Let the function F be defined by $F(x) = \int_0^x \frac{t^2 - 2t}{e^t}\,dt$

a. Find $F'(x)$ and $F''(x)$.

b. If they exist, determine the critical numbers for F.

c. Discuss the concavity of the graph of F.

Answers: Exam#4 Review: 4.9 & Chapter 5.1-5.4
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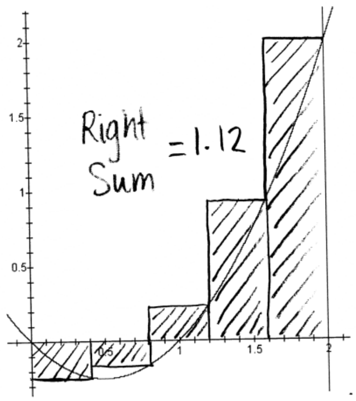
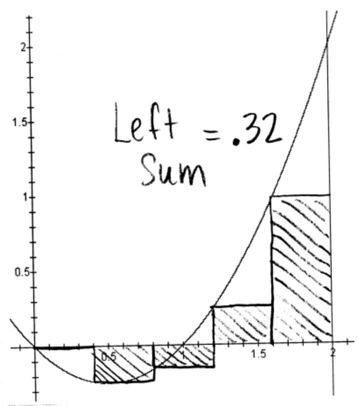
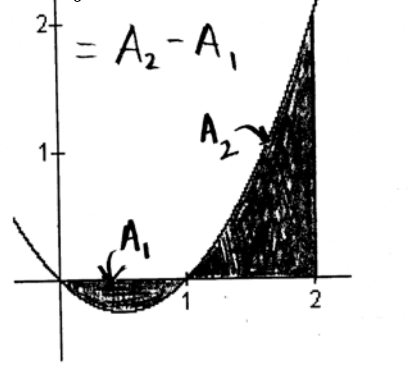
1. a. $R_5 = 1.12$ b $L_5 = 0.32$ c. $\int_0^2 \left(x^2 - x\right) dx$ d. $\frac{2}{3}$







2. These are approximate from the graph, your answers may differ slightly

   a. $L_5 \approx 25.6$ b. $R_5 \approx 19.6$ c. $M_5 \approx 23.8$

3. $\int_1^7 \left(2x - 4\right) dx$

4. $\int f(x)dx$ is an indefinite integral and gives a family of functions, the antiderivatives of $f(x)$

   $\int_a^b f(x)dx$ is a definite integral from *a* to *b* and gives a number, which can be interpreted as net area between the curve $y = f(x)$ and the *x*-axis.

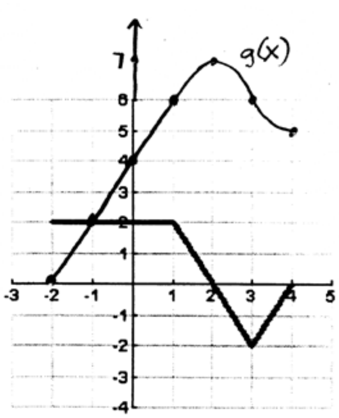
5. $\frac{d}{dx}\left[\int_a^b f(x)dx\right] = 0$, since $\int_a^b f(x)dx$ is a number, and the derivative of a constant is zero.

6. a. $\frac{3}{2}$ b. –12 c. -3 d. 10.5

7. a. Interpret the integral as net area under the curve from -2 to *x*, so

   g(–2) = 0 g(0) = 4 g(1) = 6 g(4) = 5

   b.



   c. $g'(3) = f(3) = -2$ by FTC- Part 1.

   d. Looking at graph of g(x) from part (b), The absolute maximum value is 7, and is found at *x* = 2.

8. a. $\frac{2}{3}$ b. $3-2\cos 3-e^{3}$ c. $\frac{62}{3}$ d. $\frac{46}{15}$ e. $\frac{19}{3}$

f. $-\frac{3}{5}\ln(2)$ g. $\frac{e^2}{2}+e-\frac{1}{2}$ h. $\frac{\pi}{4}$

9. a. $-\frac{1}{4x}+C$ b. $\tan x+\sec x+C$ c. $3\sqrt[3]{x^2}+C$ d. $2\sin^{-1}x+C$

10. $v(t)=\frac{1}{2}t^2-3t+4$, and then $s(t)=\frac{1}{6}t^3-\frac{3}{2}t^2+4t-2$

11. a. 36 feet b. $49\frac{1}{3}$ feet

12. The integral represents the net change in the temperature of the bottle of wine over the 3 hour time period from 4pm to 7pm in degrees Fahrenheit. That is, it is the number of °F the temperature of the bottle fell in 3 hours while being refrigerated. We expect this value to be negative, as the temperature is falling as the bottle is cooled. $\int_0^3 -18e^{-0.6t}\,dt=-25.04$ °F

13. $\int_0^{60} 100e^{-0.01t}\,dt=4511.9$ liters

14. a. 9 miles b. $t=5$ hours

c. Net distance or displacement from A after 8 hours. d. 16 miles

15. a. $F'(x)=\frac{x^2-2x}{e^x}$, and $F''(x)=\frac{-x^2+4x-2}{e^x}$

b. critical values are x = 0 and x = 2 (local max at x = 0, local min at x = 2)

c. Concave Up $\left(2-\sqrt{2},2+\sqrt{2}\right)$, Concave Down $\left(-\infty,2-\sqrt{2}\right)\cup\left(2+\sqrt{2},\infty\right)$