Section 5.5, Chapter 6 (no 6.4) & Section 8.1 Review **Name**____________________________
Math 180: Vanden Eynden

1. Evaluate the following definite and indefinite integrals (if they exist).

a. $\int_0^1 (3-x)^5\, dx$

b. $\int_0^1 \frac{dx}{(3x-1)^2}$

c. $\int \frac{\sin x}{\cos x}\, dx$

d. $\int \cos x \cdot e^{\sin x}\, dx$

e. $\int \frac{4}{2x-1}\, dx$

f. $\int_0^{\sqrt{\frac{\pi}{4}}} x\sin(x^2)\, dx$

g. $\int \left(x^3+1\right)\sqrt{x^4+4x}\, dx$

h. $\int x^4 \sec^2\left(x^5\right) dx$

i. $\int_1^e \frac{(\ln x)^3}{x}\, dx$

2. Find the **area** of the region bounded by the curves. Draw a picture and shade the appropriate region.
   a. $y = 20 - x^2$ and $y = x^2 - 12$

   b. $y = e^x - 1, \quad y = x^2 - x, \quad x = 2$

3. Consider the region bounded by the curves $y = x^3, \quad y = 8, \quad x = 1$, $x = 0$

   a. Find the **area** of this region. Draw a picture and shade the appropriate region.

   b. Find the **volume** of the solid obtained by rotating this same region about the y-axis. Draw a picture. **Indicate** which method you are using. _______________

4. Let R be the region in the first quadrant bounded by the graphs of $y = 8 - x^{\frac{3}{2}}$, $x = 0$ and $y = 0$.

   a. Find the volume of the solid generated when R is revolved about the x-axis.

   b. The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

5. Find the **volume** of the solid obtained by rotating the region bounded by the given curves about the specified axis. Use the specified method. Your set-up must include a graph of the region and at least one example of a disk/washer or cylindrical shell (depending on the method used).

a. $y = x^3, \quad y = 0, \quad x = 1$ about the x-axis. **<u>Disks/Washers</u>**

b. $y = \sqrt{x-1}, \quad x = 5, \quad y = 0$ about the y-axis **<u>Disks/Washers</u>**

c. $y = \dfrac{4}{x}, \quad x = 1, \quad x = 4, \quad y = 0$ about y-axis **<u>Cylindrical Shells</u>**

d. $y = 3x - x^2, \quad y = 0$ about the vertical line $x = -2$ **<u>Cylindrical Shells</u>**

e. $x = 4 - y^2, \quad x = 0$ about the horizontal line $y = 3$ **<u>Method: You Decide</u>**

6. Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$. Sketch the graph of *f* and a rectangle whose area is the same as the area under the graph of *f* on the interval.

7. The value, *V*, of a Tiffany lamp, worth \$225 in 1965, increases at 8% per year. This means that its value in dollars *t* years after 1965 is given by the function: $V(t) = 225(1.08)^t$. Find the average value of the lamp over the time period 1965 to 2015. Round your answer to the nearest dollar.

8. Find the arc length of $y = e^x$ on $[0, 2]$. Use your calculator and round to 4 decimal places.

9. Find the length of the curve $3x = 2(y-1)^{\frac{3}{2}}, \quad 2 \le y \le 5$

10. Use integration to prove that the circumference (length) of a semi-circle with radius 1 is π. Use the semi-circle $y = \sqrt{1-x^2}$ from $(-1,0)$ to $(1,0)$. [Hint: recall that $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$]