**Section 5.5, Chapter 6 (no 6.4) & Section 8.1 Review ANSWERS**
**Math 180: Vanden Eynden**

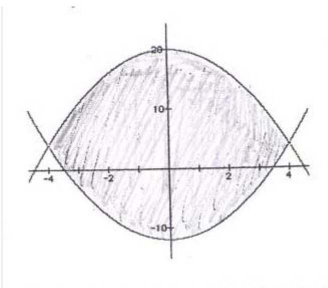
1. a. $\dfrac{665}{6}$

   b. We can't compute this since we can't use the FTC. $\dfrac{1}{(3x-1)^2}$ is not continuous at $x = 1/3$

   c. $-\ln|\cos x| + C$    d. $e^{\sin x} + C$    e. $2\ln|2x-1| + C$    f. $\dfrac{1}{2} - \dfrac{\sqrt{2}}{4}$

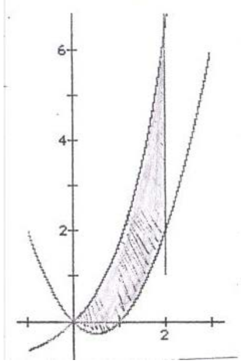
   g. $\dfrac{1}{6}\left(x^4+4x\right)^{3/2} + C = \dfrac{1}{6}\left(x^4+4x\right)\sqrt{x^4+4x} + C$    h. $\dfrac{1}{5}\tan(x^5) + C$    i. $\dfrac{1}{4}$

2. a.



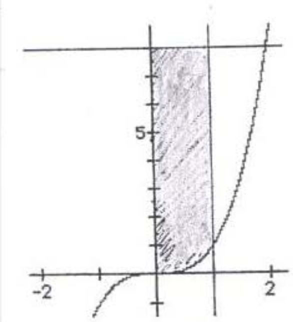
$$\int_{-4}^{4}\left[20-x^2-(x^2-12)\right]dx = \frac{512}{3}$$

b.



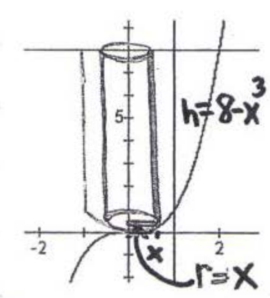
$$\int_0^2 \left[e^x-1-(x^2-x)\right]dx = e^2 - \frac{11}{3}$$

3. a)



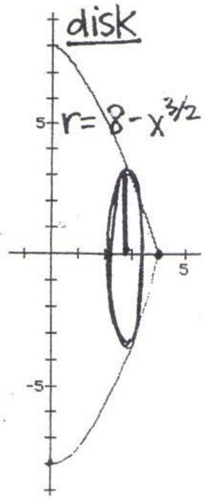
$$\int_0^1 (8-x^3)\,dx = \frac{31}{4}$$

b) shells.




$$\int_0^1 2\pi x(8-x^3)\,dx = \frac{38\pi}{5}$$

4.

a) 


$$\int_0^4 \pi\left(8-x^{3/2}\right)^2 dx = \frac{576\pi}{5}$$

b) need to find $k \in [0,4]$ such that $\int_0^k \pi(8-x^{3/2})^2 dx$ is $\frac{1}{2}$ of $\frac{576\pi}{5} = 361.91$

$$\int_0^k \pi(8-x^{3/2})^2 dx = \pi\int_0^k (64-16x^{3/2}+x^3)\,dx$$

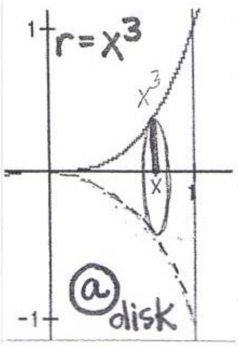
$$= \pi\left(64x - \frac{32}{5}x^{5/2} + \frac{x^4}{4}\right)\Big|_0^k = \pi\left(64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4}\right)$$

So set = to $\frac{288\pi}{5}$, and get all terms to one side of equation.

$$\pi\left(64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4}\right) - \frac{288\pi}{5} = 0$$

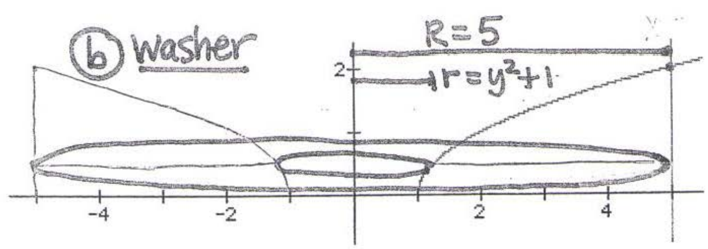
Use calculator to find zero on $[0,4]$: $K = .994904$

5.

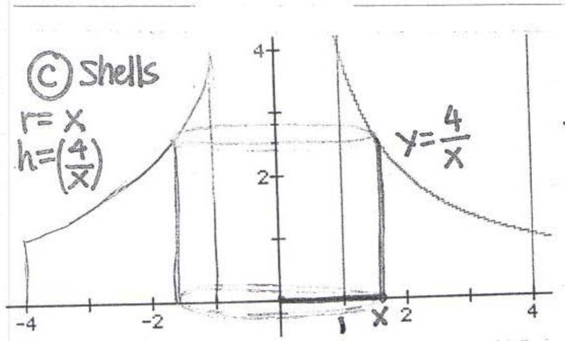
a) 


disks: $r = x^3$

$$\int_0^1 \pi(x^3)^2 dx = \frac{\pi}{7}$$

b) 

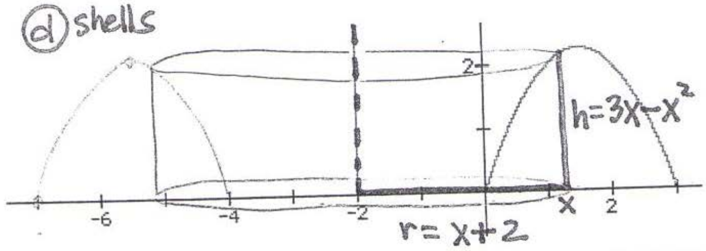

$$\int_0^2 \pi\left(5^2 - (y^2+1)^2\right) dy = \frac{544\pi}{15}$$

c) 


$$\int_1^4 2\pi x\left(\frac{4}{x}\right) dx = 24\pi$$

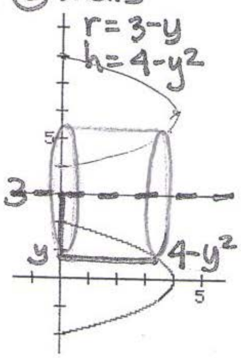
5.

d) (d) shells



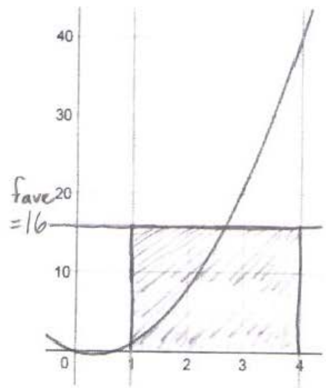

$$\int_0^3 2\pi(x+2)(3x-x^2)\,dx = \frac{63\pi}{2}$$

e) (e) shells




$$\int_{-2}^{2} 2\pi(3-y)(4-y^2)\,dy = 64\pi$$

6.




$$f_{ave} = \frac{1}{4-1}\int_1^4 (3x^2-2x)\,dx = \boxed{16}$$

7. \$2684

8. 6.7887

9. $\dfrac{10\sqrt{5}}{3} - \dfrac{4\sqrt{2}}{3} \approx 5.5679$

10. Let $y = \sqrt{1-x^2}$. So $\dfrac{dy}{dx} = \dfrac{1}{2}\left(1-x^2\right)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{1-x^2}}$. Now calculate arc length, *L*.

$$L = \int_{-1}^{1}\sqrt{1+\left(\frac{-x}{\sqrt{1-x^2}}\right)^2}\,dx \;=\; 2\int_0^1\sqrt{1+\frac{x^2}{1-x^2}}\,dx \;=\; 2\int_0^1\sqrt{\frac{1}{1-x^2}}\,dx \;=\; 2\int_0^1\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \; 2\left(\sin^{-1}x\right)\Big|_0^1 \;=\; 2\left(\sin^{-1}1-\sin^{-1}0\right) \;=\; 2\left(\frac{\pi}{2}-0\right) \;=\; \pi$$