Lots of Practice Problems for Math 180
Chapters 2 - 6, 8.1 and 8.2 ☺
All problems can be done without a graphing calculator.

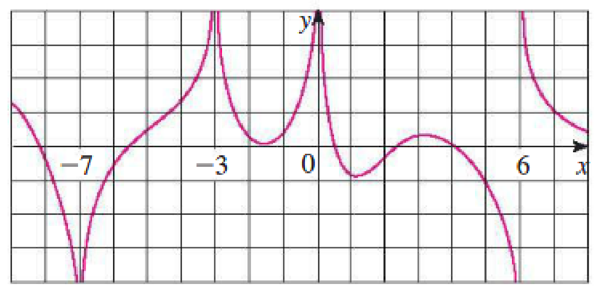
1. The position of a car is given by the values in the table.

t (seconds)	0	1	2	3	4	5
s (feet)	0	10	32	70	119	178

   a) Find the average velocity for the time period beginning when t = 2 and lasting
      i) 3 s   ii) 2 s   iii) 1 s

   b) Use the graph of s as a function of t to estimate the instantaneous velocity when t = 2.

2. For the function g whose graph is shown, state the following.



   a) $\lim_{x\to -7} g(x)$
   b) $\lim_{x\to 6^-} g(x)$
   c) $\lim_{x\to 6^+} g(x)$
   d) $\lim_{x\to 4} g(x)$
   e) The equations of the vertical asymptotes.

3. Evaluate the limit if it exists

   a) $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-3x+2}$

   b) $\lim_{t\to 0} \dfrac{\sqrt{2-t}-\sqrt{2}}{t}$

4. Find the following limits of the greatest integer function.
   a) $\lim_{x\to 1^+} [\![x]\!]$
   b) $\lim_{x\to -2^-} [\![x]\!]$

5. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval: $x^2 = \sqrt{x+1}$, (1, 2)

6. Find each limit

   a) $\lim_{x\to \infty} \dfrac{2x^2+x-2}{3x^2-3x+2}$

   b) $\lim_{x\to -\infty} \dfrac{\sqrt{4x+x^2}}{4x+1}$

7. Find an equation of the tangent line to the curve at the given point.

   $y = \frac{1}{\sqrt{x}}, \quad (1, 1)$

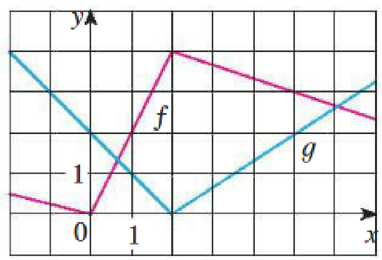
8. If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $s(t) = 40t - 16t^2$. Find the velocity when t = 2.

9. Find the derivative of the given function using the limit definition of the derivative.
   a) $f(x) = 5 - 3x + x^2$
   b) $g(x) = \sqrt{1 + 2x}$

10. Find an equation of the tangent line to the curve at the given point

    $y = 2xe^x, \quad (0,0)$

11. If f and g are the functions whose graphs are shown, let *u(x)=f(x)g(x)* and *v(x)=f(x)/g(x)*.
    a) Find $u'(1)$
    b) Find $v'(5)$



12. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is

    $s = 80t - 16t^2$
    a) What is the maximum height reached by the ball?
    b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

13. Find the equation of the tangent line to the curve $y = \sec x - 2\cos x$ at the point $(\pi/3, 1)$

14. Find the limit: $\lim_{t \to 0} \frac{\sin 3t}{\sin 5t}$

15. Differentiate:
    a. $f(x) = e^{-5x} \cos 3x$
    b. $g(x) = (x^2 + 1)\sqrt[3]{x^2 + 2}$

16. Find $\frac{dy}{dx}$ by implicit differentiation

    $x^4 + y^2 x^2 + xy^4 = x + 1$

17. Find the derivative

$y = \tan^{-1}(e^x)$

18. Differentiate (logarithmic differentiation may help here)

$y = x^x$

19. If a snowball melts so that its surface area decreases at a rate of 1 sq cm/min, find the rate at which the diameter decreases when the diameter is 10 cm. (Note: $S = 4\pi r^2$)

20. At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

21. Find the differential dy and evaluate dy for the given values of x and dx.

$y = (x^2 + 5)^3, \quad x = 1, \quad dx = 0.05$

22. Find all values of c (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval [0, 1]

23. Graph the following functions. Find the intervals where it is increasing and decreasing. Find the local maximum and minimum values. Find the intervals of concavity and the inflection points.

    a. $f(x) = \frac{x}{(x-1)^2}$

    b. $f(x) = 3x^5 - 5x^3 + 3$

24. Use L'Hospital's Rule to find the following limits:

    a. $\lim_{x \to 0} \frac{x + \tan x}{\sin x}$

    b. $\lim_{x \to \infty} \frac{\ln(\ln x)}{x}$

    c. $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$

25. A box with a square base and open top must have a volume of 32,000 cubic cm. Find the dimensions of the box that minimize the amount of material used.

26. Estimate the area under the graph of $f(x) = \frac{1}{x}$ from x = 1 to x = 5 using four rectangles and right endpoints. Sketch the graph and the rectangles. Repeat using left endpoints.

27. Use the definition of the definite integral to evaluate the following:

$$\int_0^1 \left(3x^2 + x^3\right) dx$$

28. Integrate:

a. $\int_1^9 \frac{1}{2x}\, dx$

b. $\int_{\pi}^{2\pi} \cos\theta \; d\theta$

c. $\int_1^2 \frac{3}{t^4}\, dt$

d. $\int x(1+2x^4)dx$

e. $\int \left(2-\sqrt{x}\right)^2 dx$

f. $\int (\sin x)^3 \cos x \; dx$

g. $\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\; dx$

29. Find the area under the curve $y = x^4 + 2$ from x = 1 to x = 3.

30. Find the area of the region enclosed by $y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2$

31. Find the area of the region enclosed by the curves $x = 1 - y^2, \quad x = y^2 - 1$

32. Find the volume of the solid obtained by rotating the region (in quadrant I only) bounded by $y = x^3$, $x = y^3$ about the x-axis.

33. Find the volume generated by rotating the region bounded by $y = 4x - x^2, \quad y = 8x - 2x^2$ about the line x = -2

34. A spring, whose natural length is 1 m, extends to a length of 3 m when a force of 3 N is applied. Find the work needed to extend the spring to a length of 2 m from its natural length.

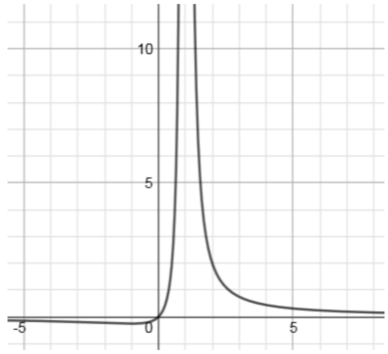
35. Find the arc length of the curve $y^2 = x^3$ from (0, 0) to $\left(\frac{1}{4}, \frac{1}{8}\right)$

36. Find the surface area obtained by rotating the curve $y = \frac{1}{2}x^2, \quad 0 \le x \le 1$ about the y-axis.

Solutions to Problems:

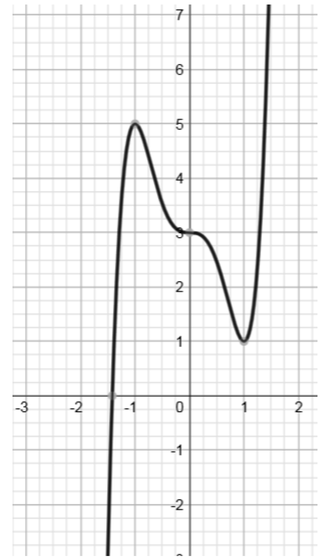
1. a. i. 48.7 ft/s
   ii. 43.5 ft/s
   iii. 38 ft/s
   b. approx. 28 ft/s   Draw a picture to help you get this answer
2. a. $-\infty$ b. $-\infty$ c. ∞ d. 0   e. x = -7, x =-3, x = 0, x = 6
3. a. –3
   c. $-\sqrt{2}/4$
4. a. 1
   b. –3
5. Do this one on your own
6. a. 2/3
   b. –1/4
7. $-\frac{1}{2}x+\frac{3}{2}$
8. –24 ft/s
9. a. $f'(x)=-3+2x$ Must get this the long way
   b. $g'(x)=\frac{1}{\sqrt{1+2x}}$
10. y = 2x
11. a. 0
    b. –2/3
12. a. 100 ft
    b. 16 ft/s; -16 ft/s
13. $y=3\sqrt{3}\,x+1-\pi\sqrt{3}$
14. 3/5
15. a. $f'(x)=-e^{-5x}(3\sin 3x+5\cos 3x)$
    b. $g'(x)=2x(x^2+2)^{1/3}\left[1+\frac{x^2+1}{3(x^2+2)}\right]$
16. $\frac{dy}{dx}=\frac{1-4x^3-2xy^2-y^4}{2x^2y+4xy^3}$
17. $y'=\frac{e^x}{1+e^{2x}}$
18. $y'=x^x(\ln x+1)$
19. decreases at a rate of $\frac{1}{20\pi}$ $cm/\min$
20. 55.4 km/h
21. a. $dy=6x(x^2+5)^2\,dx$
    b. dy = 10.8
22. $c=\sqrt{2}-1$

23. a. x = 1 vertical asymptote
(-1, -¼) local minimum
CU: (-2, 1) and (1, ∞)
Inflection point (-2, -2/9)



b. local maximum: (-1, 5) local minimum: (1, 1)

Inflection points: $\left(\pm\frac{1}{\sqrt{2}}, 3\pm\frac{7\sqrt{2}}{8}\right)$



24. a. 2
b. 0
c. $\frac{1}{2}(n^2 - m^2)$

25. 40 x 40 x 20

26. $R_4 = 77/60 \quad L_4 = 25/12$

27. 5/4 (Do this the long way)

28. a. ln 3 b. 0 c. 7/8 d. $\frac{x^2}{2}+\frac{x^6}{3}+C$ e. $4x-\frac{8}{3}x^{3/2}+\frac{1}{2}x^2+C$

f. $\frac{\sin^4 x}{4}+C$ g. $\frac{4\sqrt{2}}{3}-\frac{5\sqrt{5}}{12}$

29. 262/5

30. ln2 – ½

31. 8/3

32. $\frac{16\pi}{35}$

33. $\frac{256\pi}{3}$

34. 0.75 J

35. 61/216

36. $\frac{\left(4\sqrt{2}-2\right)\pi}{3}$