Math 280: 11.1 Sequences
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Determine whether the sequence converges or diverges. If it converges, find the limit.

1. $a_n = \dfrac{3^n}{5^n}$

$\lim_{n\to\infty} \frac{3^n}{5^n} = \lim_{n\to\infty} \left(\frac{3}{5}\right)^n = 0$ Since, $r = \frac{3}{5} < 1$

By Thrm 9, pg696

$\boxed{\text{Converges to } 0}$

2. $a_n = \dfrac{\sin n}{n!}$

$-1 \le \sin n \le 1$

$\frac{-1}{n!} \le \frac{\sin n}{n!} \le \frac{1}{n!}$

$\lim_{n\to\infty} \frac{-1}{n!} = 0 = \lim_{n\to\infty} \frac{1}{n!}$

So by Squeeze thrm,

$\lim_{n\to\infty} \frac{\sin n}{n!} = 0$

$\boxed{\text{Converges to } 0}$

3. $a_n = \dfrac{(-1)^{n-1} n}{n^2+1}$

test $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \left| \frac{(-1)^{n-1} n}{n^2+1} \right| = \lim_{n\to\infty} \frac{n}{n^2+1}$

$= \lim_{n\to\infty} \frac{\frac{1}{n}}{1 + \frac{1}{n^2}} = 0$. So by Thrm 6,

$\lim_{n\to\infty} a_n = 0$ also.

$\boxed{\text{Converges to } 0}$

4. $a_n = \dfrac{\ln n}{\ln 2n}$

$\lim_{x\to\infty} \frac{\ln x}{\ln 2x} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2}{2x}} = \lim_{x\to\infty} \frac{1}{x} \cdot \frac{x}{1} = 1$

So $\lim_{n\to\infty} \frac{\ln n}{\ln 2n} = 1$ by thrm 3

$\boxed{\text{converges to } 1}$

5. $a_n = \dfrac{(-1)^n n^3}{n^3+2n+1}$

$\lim_{n\to\infty} \left| \frac{(-1)^n n^3}{n^3+2n+1} \right| = \lim_{n\to\infty} \frac{n^3}{n^3+2n+1} = 1$ (not 0)

So the odd terms $a_1, a_3, a_5, a_7, \ldots$ are approaching -1 while the even terms $a_2, a_4, a_6, a_8, \ldots$ are approaching 1.

$\lim_{n\to\infty} a_n$ Does not exist. So $\boxed{a_n \text{ diverges}}$

6. $a_n = \cos\left(\frac{n}{2}\right)$ as $n \to \infty$, $\frac{n}{2} \to \infty$.

Since $\cos x$ ocillates between -1 and 1,

$\lim_{n\to\infty} \cos\left(\frac{n}{2}\right)$ does not exist. So $\boxed{a_n \text{ diverges}}$

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7. $a_n = \cos\left(\frac{2}{n}\right)$ $\lim_{n\to\infty} \cos\left(\frac{2}{n}\right) = \cos\left(\lim_{n\to\infty}\left(\frac{2}{n}\right)\right) = \cos 0 = 1$

by thrm 7, pg. 695

$\boxed{\text{converges to } 1}$

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8. $a_n = \ln(n+1) - \ln n = \ln\left(\frac{n+1}{n}\right)$

$\lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right) = \ln\left(\lim_{n\to\infty} \frac{n+1}{n}\right) = \ln(1) = 0$ by Thrm 7

$\boxed{\text{converges to } 0}$

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9. $a_n = n \cos n\pi = \{-1, 2, -3, 4, -5, 6, \ldots\} = (-1)^n n$

$\lim_{n\to\infty} n\cos(n\pi)$ does not exist.

$\boxed{\text{diverges}}$

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10. $a_n = \frac{(n-1)!}{(n+1)!} = \frac{(n-1)!}{(n+1)n(n-1)!} = \frac{1}{(n+1)n}$

$\lim_{n\to\infty} \frac{1}{n^2+n} = 0$

$\boxed{\text{converges to } 0}$