ANSWERS to In-Class Exam #3 Review Sheet, Covers 11.5-11.10

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1. Converges absolutely. Geometric series with |r|=9/64 < 1.
2. Diverges. Ratio Test.
3. Converges absolutely. Ratio Test.
4. Converges conditionally. Alternating Series Test (converges) but $\sum |a_n|$ diverges by Comparison Test $\frac{1}{\sqrt{2n-3}} > \frac{1}{\sqrt{2n}}$ (p-series with p = ½ < 1)
5. Converges absolutely. Comparison Test $\frac{\cos^2 n}{4+4^n} \le \frac{1}{4^n}$ (geometric series with r = ¼ < 1)
6. Diverges. Ratio Test
7. Converges Absolutely. Comparison Test. $\frac{\sin\sqrt{n}}{n^2} \le \frac{1}{n^2}$ (p-series with p = 2 > 1)
8. Diverges. Ratio Test.
9. Converges absolutely. Ratio Test.
10. A) Series converges using the Alternating Series Test. (check decreasing by taking derivative, limit approaching 0)

    B) Since $|a_7| = 0.000427 < .001$, the partial sum of the first 6 terms is enough.

    C) $sum \approx s_6 \approx -0.1596679688 \approx -0.160$
11. R = 5 I: [-5,5]
12. R = ∞ I: (–∞,∞)
13. R=0 I: {-1} $x = -1$ makes the series and x + 1= 0
14. $f(x) = \frac{5}{1-4x^2} = 5\sum_{n=0}^{\infty} 4^n x^{2n} = 5 + 20x^2 + 80x^4 + 320x^6 + \ldots$ R = 1/2
15. $f(x) = \frac{x^3}{(2+x)^2} = x^3 \frac{1}{(2+x)^2} = x^3 \frac{d}{dx}\left[\frac{-1}{2+x}\right] = x^3 \frac{d}{dx}\left[\frac{-1}{2}\cdot\frac{1}{\left(1-\left(\frac{-x}{2}\right)\right)}\right] = x^3 \frac{d}{dx}\left[\frac{-1}{2}\sum_{n=0}^{\infty}\left(\frac{-x}{2}\right)^n\right]$

    $= x^3 \frac{d}{dx}\left[\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^n}{2^{n+1}}\right] = x^3 \cdot \sum_{n=0}^{\infty}\frac{(-1)^{n+1}nx^{n-1}}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}nx^{n+2}}{2^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}nx^{n+2}}{2^{n+1}}$

    $= \frac{x^3}{4} - \frac{2x^4}{8} - \frac{3x^5}{16} - \frac{4x^6}{32} + \ldots$ R = 2

    $= \frac{x^3}{4} - \frac{x^4}{4} - \frac{3x^5}{16} - \frac{x^6}{8} + \ldots$
16. $f(x) = \ln(x+5) = \int \frac{1}{x+5}dx = \int\left(\frac{1}{5}\cdot\frac{1}{1-\left(\frac{-x}{5}\right)}\right)dx = \frac{1}{5}\int\left(\sum_{n=0}^{\infty}\left(\frac{-x}{5}\right)\right) = \frac{1}{5}\int\left(\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{5^n}\right)$

    $= C + \frac{1}{5}\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{5^n(n+1)} = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{(n+1)5^{n+1}} = C + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n5^n} = \ln 5 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n5^n}$ R = 5

17. [R=∞], $f(x) = 2xe^{3x} = 2x\left(\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}\right) = 2\sum_{n=0}^{\infty}\frac{3^n x^{n+1}}{n!} = 2x + 6x^2 + 9x^3 + 9x^4 + \frac{27}{4}x^5 + \frac{81}{20}x^6 + \ldots$

18. [R=∞], $f(x) = \sin(x^4) = \sum_{n=0}^{\infty}(-1)^n\frac{(x^4)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{8n+4}}{(2n+1)!} = x^4 - \frac{1}{6}x^{12} + \frac{1}{120}x^{20} - \frac{1}{5040}x^{28}\ldots$

19. [R=∞], $f(x) = x\cos(2x^2) = x\sum_{n=0}^{\infty}(-1)^n\frac{(2x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{2^{2n}x^{4n+1}}{(2n)!}$

20. [R=∞], $f(x) = 3^x = \sum_{n=0}^{\infty}\frac{(\ln 3)^n x^n}{n!} = 1 + (\ln 3)x + \frac{(\ln 3)^2 x^2}{2!} + \frac{(\ln 3)^3 x^3}{3!} + \frac{(\ln 3)^4 x^4}{4!} + \ldots$

21. [R=1], $f(x) = \sqrt{1+x^4} = (1+x^4)^{1/2} = \sum_{n=0}^{\infty}\binom{1/2}{n}(x^4)^n = 1 + \frac{1}{2}x^4 + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}(1\cdot 3\cdot 5\cdots(2n-3))}{2^n n!}x^{4n}$

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22. $\int\sqrt{1+x^4}\,dx = \int\left(1 + \frac{1}{2}x^4 + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}(1\cdot 3\cdot 5\cdots(2n-3))}{2^n n!}x^{4n}\right)$

$$= x + \frac{1}{10}x^5 + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}(1\cdot 3\cdot 5\cdots(2n-3))}{2^n n!(4n+1)}x^{4n+1} = x + \frac{1}{10}x^5 - \frac{1}{72}x^9 + \frac{1}{208}x^{13} - \frac{5}{2176}x^{17} + \ldots$$

Since this series alternates after the first term, we can approximate the value of this function using the first 3 terms, since the 4th term has coefficient 1/208 = .00481 < .01

So, $\int_0^1\sqrt{1+x^4}\,dx \approx \left[x + \frac{1}{10}x^5 - \frac{1}{72}x^9\right]_0^1 \approx 1.09$

23. $\sum_{n=0}^{\infty}\frac{-(x+3)^n}{3^{n+1}} = -\sum_{n=0}^{\infty}\frac{(x+3)^n}{3^{n+1}}$ R = 3