ANSWERS to In-Class Exam #4 Review Sheet, Covers 11.10, 11.11, 10.1-10.4

Math 280, Vanden Eynden

1. $f(x) = x^2 e^{-3x} = x^2\left(\sum_{n=0}^{\infty}\frac{(-3x)^n}{n!}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n 3^n x^{n+2}}{n!} = x^2 - 3x^3 + \frac{9x^4}{2} - \frac{9x^5}{2} + \frac{27x^6}{8} - \frac{81x^7}{40} + \ldots$

2. $f(x) = \frac{x - \sin x}{x^3} = \frac{1}{x^3}\left(x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots\right)\right) = \frac{1}{x^3}\left(\frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!} + \ldots\right)$

$$= \frac{1}{3!} - \frac{x^2}{5!} + \frac{x^4}{7!} - \frac{x^6}{9!} + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+3)!}$$

3. $f(x) = 2^x = \sum_{n=0}^{\infty}\frac{(\ln 2)^n x^n}{n!} = 1 + (\ln 2)x + \frac{(\ln 2)^2 x^2}{2!} + \frac{(\ln 2)^3 x^3}{3!} + \frac{(\ln 2)^4 x^4}{4!} + \ldots$

4. $f(x) = \sqrt[3]{1-x} = (1+(-x))^{1/3} = \sum_{n=0}^{\infty}\binom{1/3}{n}(-x)^n = 1 - \frac{1}{3}x - \sum_{n=2}^{\infty}\frac{(2\cdot 5\cdot 8\cdots(3n-4))}{3^n n!}x^n$

$$= 1 - \frac{x}{3} - \frac{x^2}{9} - \frac{5x^3}{81} - \frac{10x^4}{243} - \ldots$$

5. $f(x) = (1+x)^{-3} = \sum_{n=0}^{\infty}\binom{-3}{n}x^n = \sum_{n=0}^{\infty}\frac{(-1)^n(3\cdot 4\cdot 5\cdots(n+2))}{n!}x^n = \sum_{n=0}^{\infty}\frac{(-1)^n(n+2)!}{2(n!)}x^n$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n(n+2)(n+1)}{2}x^n = \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n(n+2)(n+1)\,x^n$$

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6. $\int\left(\frac{x-\sin x}{x^3}\right)dx = \int\left(\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+3)!}\right)dx = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+3)!} = \frac{x}{6} - \frac{x^3}{360} + \frac{x^5}{25200} - \frac{x^7}{2,540,160} + \ldots$

Since this is an alternating series, we can approximate the value of this function using the first 2 terms, since the 3rd term has coefficient 1/25,200 = .0000397 < .001.

So, $\int_{0.1}^{0.4}\frac{x-\sin x}{x^3}\,dx \approx \left.\frac{x}{6} - \frac{x^3}{360}\right|_{0.1}^{0.4} = 0.049825$

7. a. $T_3(x) = \frac{1}{e^2} + \frac{2(x+1)}{e^2} + \frac{2(x+1)^2}{e^2} + \frac{4(x+1)^3}{3e^2}$

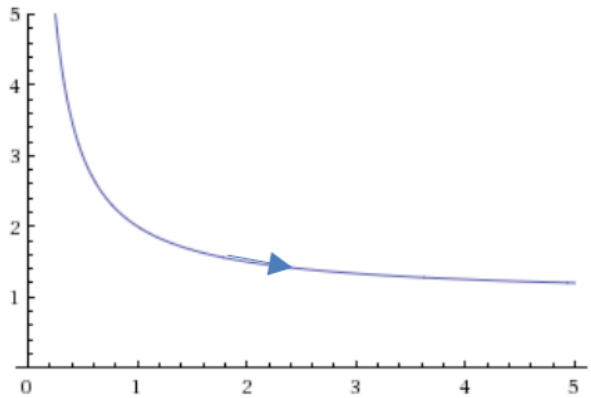
b. $e^{-2.2} = e^{2(-1.1)} \approx T_3(-1.1) \approx 0.1083$

8. a. $T_4(x) = \ln 4 + (x-2) - \frac{(x-2)^2}{4} + \frac{(x-2)^3}{12} - \frac{(x-2)^4}{32}$

b. $\ln(1.8^2) \approx T_4(1.8) \approx 1.1756$

9. a.

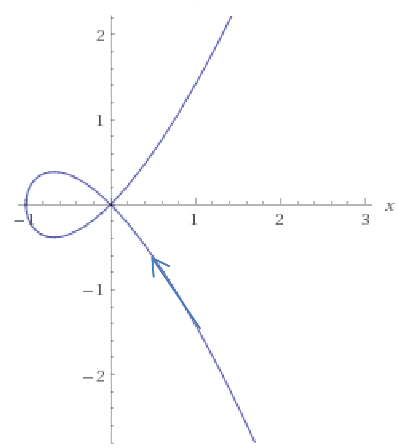
t	x	y
1.5	.5	3
2	1	2
2.5	1.5	5/3
3	2	1.5
3.5	2.5	1.4
4	3	4/3
4.5	3.5	9/7
5	4	1.25



b. $y = \frac{x+1}{x}$

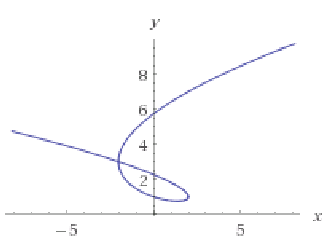
10. $r = 3$

11. See loop-dee-loop graph →



12. $y = \frac{4}{9}x - \frac{1}{3}$

13.



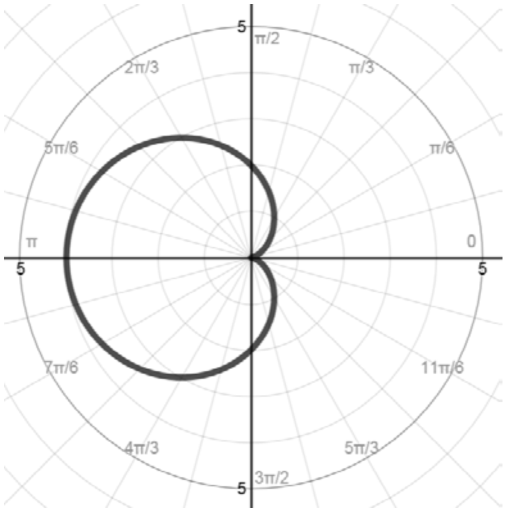
On calculator, minimum looks around (1.5, .75). For exact, set dy/dt = 0 and solve for t.
Minimum is at t = –0.5, at the point (11/8, ¾) or (1.375, 0.75)

14. Find $\frac{d^2y}{dx^2} = \frac{\cos t + \sin t + 1}{(1+\cos t)^3}$. At t = 0, $\frac{d^2y}{dx^2} = 0.25 > 0$, so curve is concave up at (0, -1).

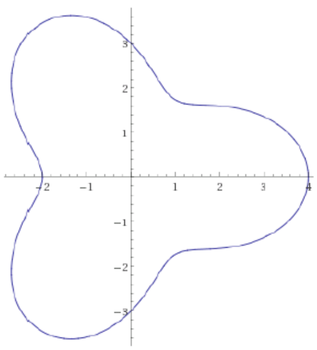
15. $\frac{d^2y}{dx^2} = \frac{-(3t^2+1)}{4t^3}$. So concave down for t > 0

16. $\int_0^2 \sqrt{36t^2 + 36t^4}\, dt = 10\sqrt{5} - 2.$

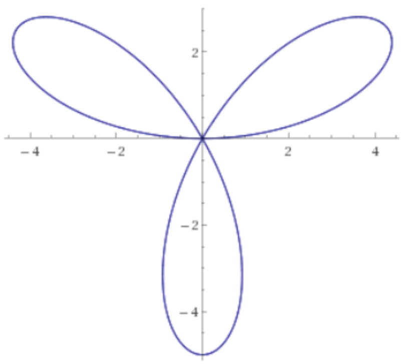
17.



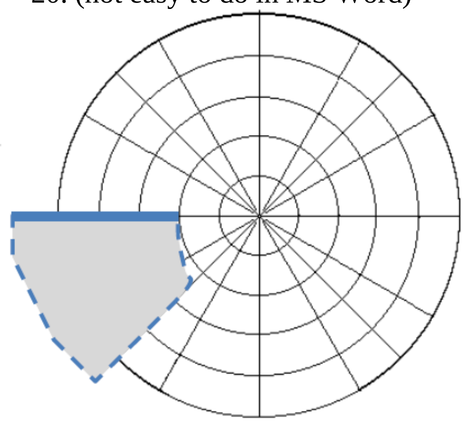
18.



19. 2-leaved rose



20. (not easy to do in MS Word)



21. polar: $(2,\ \pi/3)$, $(2,\ 5\pi/3)$; Cartesian: $(1,\sqrt{3})$, $(1,-\sqrt{3})$

22. $x + y = x^2 + y^2$. Complete the square in x and y: $1/2 = (x-1/2)^2 + (y-1/2)^2$
This is a circle centered at $(1/2,\ 1/2)$ with radius $\sqrt{1/2}$

23. $$2\int_{\pi/6}^{\pi/2} \frac{1}{2}(1-2\sin\theta)^2\ d\theta = \pi - \frac{3\sqrt{3}}{2}$$

24. $$\int_0^{\pi/4} \frac{1}{2}(2\sin\theta)^2\ d\theta\ +\ \int_{\pi/4}^{3\pi/4} \frac{1}{2}(\sin\theta+\cos\theta)^2\ d\theta = \pi/2 - 1/2$$

25. Find [area inside the big/outer loop] – [area inside small/inner loop]
$$A = \left[2\int_0^{2\pi/3} \frac{1}{2}(1+2\cos\theta)^2\ d\theta\right] - \left[2\int_{2\pi/3}^{\pi} \frac{1}{2}(1+2\cos\theta)^2\ d\theta\right] = \pi + 3\sqrt{3} \approx 8.3378$$