Math 280 FINAL EXAM Formula/Theorem Sheet

Vanden Eynden **(You will be provided a fresh copy on exam day)**

Derivatives of Inverse Trig Functions:

$$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}x=\frac{-1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$$

Established Integration Formulas

$$\int \tan x\,dx=\ln|\sec x|+C$$

$$\int \cot x\,dx=\ln|\sin x|+C$$

$$\int \sec x\,dx=\ln|\sec x+\tan x|+C$$

$$\int \csc x\,dx=\ln|\csc x-\cot x|+C$$

Trigonometric Substitution

Expression in the integrand	*Substitution*
$\sqrt{a^2-x^2}$	$x=a\sin\theta$
$\sqrt{x^2+a^2}$	$x=a\tan\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$

Half-Angle Formulas

$$\sin^2 x=\frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x=\frac{1}{2}(1+\cos 2x)$$

$$\tan^2 x=\frac{1-\cos 2x}{1+\cos 2x}$$

Double-Angle Formulas

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Product Formulas

$$\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]$$

$$\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

(A) Definition Partial Sums:

Given a series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$, let s_n denote its nth partial sum,

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

If the sequence $\{s_n\}$, the sequence of partial sums $\{s_1, s_2, s_3, \ldots\}$ is convergent and its limit is a real number, $\lim_{n\to\infty} s_n = s$, then the series $\sum_{n=1}^{\infty} a_n$ is called convergent and we write

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = s \qquad \text{or} \qquad \sum_{n=1}^{\infty} a_n = s$$

The number s is called the sum of the series. Otherwise, the series is called divergent.

(B) Geometric Series: The series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots$ is convergent if $|r| < 1$ and its sum is $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$. If $|r| \geq 1$, the series is divergent.

(C) Test for Divergence: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(D) Integral Test: Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)dx$ is convergent. In other words,

(i) If $\int_1^{\infty} f(x)dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(ii) If $\int_1^{\infty} f(x)dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

(E) P-series: The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

(F) Remainder Estimate for the Integral Test: Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum a_n$ is convergent. If $R_n = S - S_n$, then $\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)dx$.

(G) Series Sum Estimate for the Integral Test: $$s_n + \int_{n+1}^{\infty} f(x)dx \;\leq\; s \;\leq\; s_n + \int_n^{\infty} f(x)dx$$

The midpoint of this interval is an estimate of *s*, with error < (half the interval's length).

(H) The Comparison Test: If $\sum a_n$ and $\sum b_n$ are series with positive terms and

(i) If $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum a_n$ is also divergent.

(I) The Limit Comparison test: Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number and $c > 0$, then either both series converge or both diverge.

(J) The Alternating Series Test:

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + ...$ where $b_n > 0$ satisfies

(i) $b_{n+1} \le b_n$

(ii) $\lim_{n\to\infty} b_n = 0$

then the series converges.

(K) Alternating Series Estimation Theorem:

If $s = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$ is the sum of an alternating series that satisfies

(i) $0 \le b_{n+1} \le b_n$ and (ii) $\lim_{n\to\infty} b_n = 0$

Then $|R_n| = |s - s_n| \le b_{n+1}$

(L) Absolute Convergence:

If $\sum |a_n|$ converges, then $\sum a_n$ converges (absolutely).

(M) The Ratio Test for Absolute Convergence:

Let $\sum a_n$ be a series with non-zero terms and suppose $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$;

i. If L < 1, the series $\sum a_n$ is absolutely convergent.

ii. If L > 1 or $L = \infty$, then the series $\sum a_n$ diverges.

iii. If L = 1 , the test is inconclusive. The series may be convergent or divergent. Use another test (not The Root Test)

(N) The Root Test for Absolute Convergence:

Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$

i. If L < 1, then the series $\sum a_n$ is absolutely convergent.

ii. If L > 1 or $L=\infty$, then the series $\sum a_n$ diverges .

iii. If L = 1 , the test is inconclusive. The series may be convergent or divergent. Use another test (not The Ratio Test)

Taylor series of the function *f* centered at *a*:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

First Derivative of parametric equations:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \text{if } \frac{dx}{dt}\neq 0$$

Second Derivative of parametric equations:

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \qquad \text{if } \frac{dx}{dt}\neq 0$$

Arc Length for parametric equations, for $\alpha < t < \beta$**:**

$$L=\int_{\alpha}^{\beta}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ dt$$

First Derivative of polar equations:

$$\frac{dy}{dx}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

Area of polar region:

$$A=\int_a^b \frac{1}{2}\left[f(\theta)\right]^2\ d\theta$$

Conic Sections in Polar Coordinates

$$r=\frac{ed}{1\pm e\cos\theta} \quad or \quad r=\frac{ed}{1\pm e\sin\theta}$$

The conic is:

a) an ellipse if $e<1$

b) a parabola if $e=1$

c) a hyperbola if $e>1$