

## Answers to Practice Problems from Math 280 Final Exam Study Guide

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1.

a.  $\frac{3}{2\sqrt{e}} - \frac{2}{e}$

b.  $2048/15$

c. (partial fraction decomp)  $-\frac{1}{4x} - \frac{1}{16} \ln|2-x| + \frac{1}{16} \ln|x+2| + C$

d.  $1$

e.  $x \ln|x+2| + 2 \ln|x+2| - x + C$

f. diverges

g. (partial fraction decomp)  $8 \ln 2 - 8 \ln 4 + 6 \ln 3 = \ln\left(\frac{729}{256}\right)$

h.  $\frac{1}{6} x^6 \ln x - \frac{1}{36} x^6 + C$

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2.

a.  $R = 2$ , Interval:  $(-4, 0]$

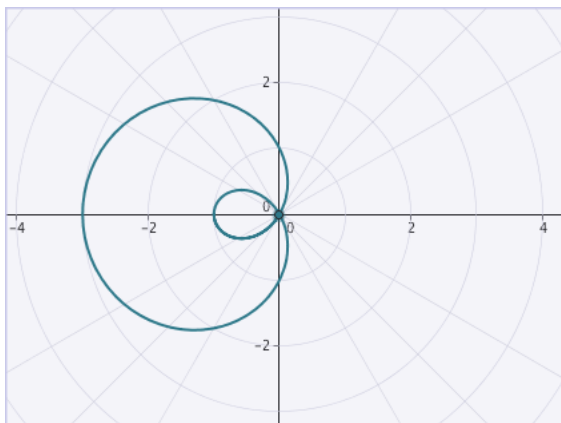
b.  $-5.015625$

c.  $-0.682$

d.  $f(-1)$ , since  $x = -1$  is within the radius of convergence. (This means  $f(1)$  diverges)

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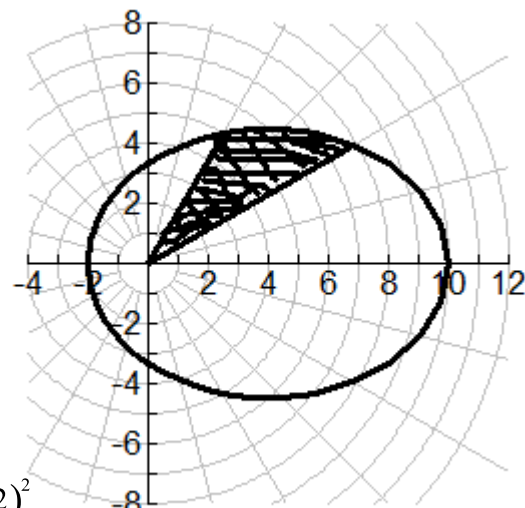
3.a.



b.  $\frac{dy}{dx} = -2$

c.  $\pi + 3\sqrt{3}$

4.
  - a. Converges (Ratio Test)
  - b. Converges (Alternating Series Test); Converges conditionally (limit comp. test)
  - c. Diverges (Divergence Test)
  - d. Converges (Alternating Series Test) ; Converges absolutely (integral test)
  - e. Converges (Comparison test)
  - f. Converges (Limit comp. test)
  - g. Diverges (Geometric Series with  $r > 1$ )
  - h. Converges (Ratio Test)



5. Converges by integral test, since  $\int_1^{\infty} \frac{1}{(4x+1)^5} dx = \frac{1}{10000}$

6.  $\sum_{n=0}^{\infty} 4^n x^n$ , for  $|x| < \frac{1}{4}$ . Interval:  $\left(-\frac{1}{4}, \frac{1}{4}\right)$

7. This problem is super messy!

a.  $\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{6}} = \frac{4}{3} - \sqrt{3}$       b. My Area estimate  $\approx \frac{\pi}{6} \cdot \frac{(6.2)^2}{2} \approx 10.1$

WolframAlpha integral estimate = 10.7538

8.  $xe^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$        $R = \infty$

9.
  - a. Absolutely convergent by Ratio Test.
  - b. Yes (alternating, decreasing (check derivative), limit goes to 0)  
Need 52 terms (n=0 up to and including n=51)

10. a.  $T_3(x) = \frac{1}{2} - \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{3}\right) - \frac{1}{4} \left(x - \frac{\pi}{3}\right)^2 + \frac{1}{4\sqrt{3}} \left(x - \frac{\pi}{3}\right)^3$

b.  $\cos(70^\circ) \approx T_3\left(\frac{7\pi}{18}\right) \approx 0.3420$

11. [3,5]

12.  $y = 2x + 1$

13. polar:  $(2, \pi/3), (2, 5\pi/3)$ ;      Cartesian:  $(1, \sqrt{3}), (1, -\sqrt{3})$

14.  $x + y = x^2 + y^2$ . Complete the square in x and y:  $1/2 = (x - 1/2)^2 + (y - 1/2)^2$

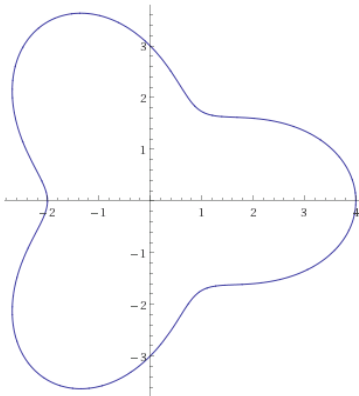
This is a circle centered at  $(1/2, 1/2)$  with radius  $\sqrt{1/2}$

15.  $2 \int_{\pi/6}^{\pi/2} \frac{1}{2} (1 - 2\sin\theta)^2 d\theta = \pi - \frac{3\sqrt{3}}{2}$

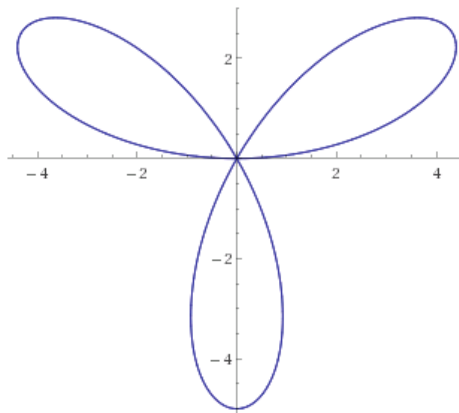
$$16. \int_0^{\pi/4} \frac{1}{2} (2 \sin \theta)^2 d\theta + \int_{\pi/4}^{3\pi/4} \frac{1}{2} (\sin \theta + \cos \theta)^2 d\theta = \pi/2 - 1/2$$


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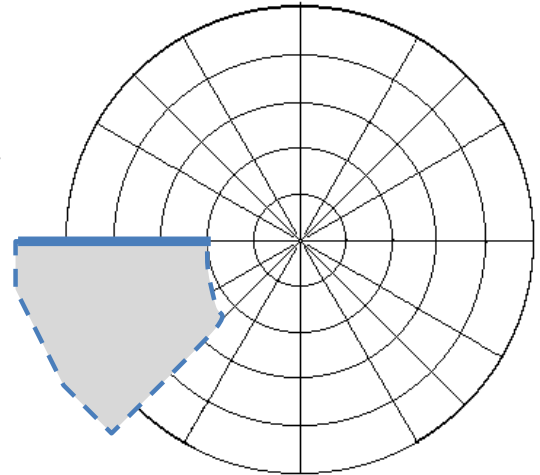
17.



18. 3-leaved rose



19. Not easy to do in MSWord



20. a. ellipse,  $e=1/3$ ,  $d=12$ , directrix:  $y = 12$   
 b. hyperbola,  $e=2$ ,  $d=1/2$ , directrix:  $y = -1/2$   
 c. parabola,  $e=1$ ,  $d=5/3$ , directrix:  $x = -5/3$

21. a.  $r = \frac{6}{1 - \cos \theta}$   
 b.  $r = \frac{3}{2 + \sin \theta}$