Practice Exam: Chapters 6 - 7 **NAME** Answer Key
Math 160
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100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

THERE ARE 4 PAGES IN THIS EXAM

Write down all formulas or calculator commands used to receive full credit!!!!!!!!

1. (8 pts) A survey of 300 union members in California reveals that 237 favor the Prop 30. Find the margin of error for the 90% confidence interval used to estimate the true population proportion of all California union members who favor the Prop 30. Round to the nearest thousandths.

$\hat{p} = \frac{237}{300} = 0.79$ $\quad E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\frac{(0.79)(0.21)}{300}}$

$\hat{q} = \frac{63}{300} = 0.21$ $\quad = 0.03868$

$z_{\alpha/2} = 1.645$ $\quad = \boxed{0.039}$

2. (8 pts) How many integrated circuits must be randomly selected and tested for time to failure in order to estimate the mean time to failure? We want 96% confidence that the sample mean is within 2 hr of the population mean, and the population standard deviation is known to be 18.6 hours.

$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left[\frac{(2.05)(18.6)}{2}\right]^2 = (19.065)^2$

$= 363.474$

$= \boxed{364 \text{ CIRCUITS}}$

3. (10 pts) Three percent of hair dryers produced in a particular plant are defective. Estimate the probability that out of 10,000 randomly selected hair dryers, between 340 and 360 are found defective. Use the normal approximation to the binomial distribution. Round to 4 decimal places.

$p = 0.03 \quad n = 10{,}000$

$\mu = np = 300$

$\sigma = \sqrt{npq} = 17.0587$

$X = 339.5$

$z = \frac{x - \mu}{\sigma} = \frac{339.5 - 300}{17.0587}$

$= 2.32$

$X = 360.5$

$z = \frac{360.5 - 300}{17.0587} = 3.55$

$P(340 \le x \le 360) = P(339.5 < x < 360.5)$

$= P(2.32 < z < 3.55)$

$= \text{normalcdf}(2.32, 3.55)$

$= \boxed{0.0100}$

$= P(z < 3.55) - P(z < 2.32)$

$= 0.9999 - 0.9898$

$= \boxed{0.0101}$

4. (6 pts) Use the given confidence interval (0.478, 0.510) to find the point estimate $\hat{p}$ and the margin of error E.

$\hat{p} = \frac{MAX + MIN}{2}$ $E = \frac{MAX - MIN}{2}$

Point Estimate: 0.494 Margin of Error: 0.016

5. (16 pts total) A random sample of 15 parking meters in a resort community showed the following incomes for a day:

$6.50 $10.25 $7.10 $9.35 $12.00

$8.20 $10.80 $9.55 $7.60 $5.85 ⇒ L_1

$11.30 $8.90 $4.95 $6.90 $7.45

1 VARSTATS L_1

$\bar{x} = \$8.45$

$s = \$2.07936$

$df = 14$

$n = 15$

A. Find a 98% confidence interval for the population mean μ. Assume the population has a standard normal distribution. $\alpha = 0.02$

$t_{\alpha/2} = 2.624$

$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = \frac{2.624(2.07936)}{\sqrt{15}}$

$= 1.4088$

$\bar{x} - E < \mu < \bar{x} + E$

$8.45 - 1.4088 < \mu < 8.45 + 1.4088$

$7.0412 < \mu < 9.8588$

$\boxed{\$7.04 < \mu < \$9.86}$

B. Find a 98% confidence interval for the population standard deviation σ.

$\chi^2_L = 4.660$

$\chi^2_R = 29.141$

$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$

$\sqrt{\frac{14(2.07936)^2}{29.141}} < \sigma < \sqrt{\frac{14(2.07936)^2}{4.660}}$

$\boxed{\$1.44 < \sigma < \$3.60}$

6. (8 pts) The probability of flu symptoms for a person not receiving any treatment is 0.019. In a clinical trial of Lipitor, a drug commonly used to lower cholesterol, 863 patients were given a treatment of 10-mg tablets, and 19 of those patients experienced flu symptoms. Assuming that these tablets have no effect on flu symptoms, estimate the probability that at least 19 of 863 people experience flu symptoms. Use the normal approximation to the binomial distribution. Round to the nearest ten thousandths.

$p = 0.019$

$\mu = np = 863(0.019)$

$= \underline{16.397}$

$\sigma = \sqrt{npq}$

$= \sqrt{863(0.019)(0.981)}$

$= \underline{4.0107}$

$P(\text{AT LEAST } 19) = P(X \geq 19) = P(X > 18.5)$

$z = \frac{x - \mu}{\sigma} = \frac{18.5 - 16.397}{4.0107} = 0.5243$

$= \underline{0.52}$

$P(\text{AT LEAST } 19) = P(X > 18.5) = P(z > 0.52)$

$= \text{NORMALCDF}(0.52, 1000)$

$= \boxed{0.3015}$

7. (8 pts) Of 288 employees selected randomly from one company, 14.58% of them commute by carpooling. Find a 99% confidence interval for the true proportion of all employees who carpool. (Round your answers to three decimal places.)

$n = 288$

$\hat{p} = 0.1458$

$\hat{q} = 0.8542$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$z_{\alpha/2} = 2.575$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.575\sqrt{\frac{(0.1458)(0.8542)}{288}}$

$= 0.05355$

$\hat{p} - E < p < \hat{p} + E$

$0.1458 - 0.05355 < p < 0.1458 + 0.05355$

$0.09225 < p < 0.19935$

$\boxed{0.092 < p < 0.199}$

8. (8 pts) Find the margin of error for the following: 90% confidence interval, $n = 91$, $\bar{x} = 53$, $s = 17.2$.

$\alpha = 0.10$

$t_{\alpha/2,90} = 1.662$

$E = t_{\alpha/2}\frac{s}{\sqrt{n}}$

$= \frac{1.662(17.2)}{\sqrt{91}} = 2.9967 = \boxed{3.0}$

9. (8 pts) Suppose you are interested in estimating the percentage of all California high school students who passed the high school exit exam on the first try. If the goal is to estimate the percentage with 98% confidence and a margin of error of 6%, how many current California high school students' records should be sampled?

$n = \frac{(z_{\alpha/2})^2\hat{p}\hat{q}}{E^2} = \frac{(2.33)^2(0.5)(0.5)}{(0.06)^2} = (19.4167)^2 = 377.0069$

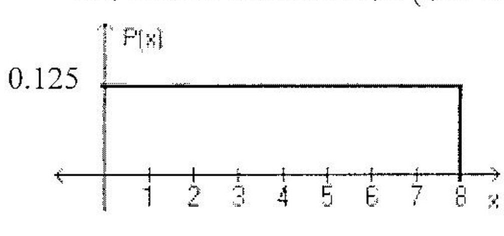
$= \boxed{378 \text{ STUDENTS}}$

10. (4 pts) Do one of the following as appropriate: a) Find the critical value $z_{\alpha/2}$, b) find the critical value $t_{\alpha/2}$, c) state neither the normal nor the t distribution apply (state why).

90%; $n = 17$; σ is unknown; population appears to be normally distributed.

$\boxed{t_{\alpha/2,16} = 1.746}$

11. (8 pts) Using the following uniform density curve, what is the probability that the random variable has a value less than 3.9? (Use three decimal places).



$P(X < 3.9)$ = AREA BELOW $X = 3.9$

$= 3.9(0.125)$

$= \boxed{0.488}$

12. (8 pts) A final exam in Math 160 has a mean of 73 with a standard deviation of 8.5. If 44 students are randomly selected, find the probability that their mean of their test scores is less than 70. Round to three decimal places.

$P(x<70) =$

$$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{70-73}{8.5/\sqrt{44}} = -2.34$$

$P(x<70) = P(z<-2.34) =$ NORMALcdf $(-1000, -2.34)$

$= 0.0096 = \boxed{0.010}$