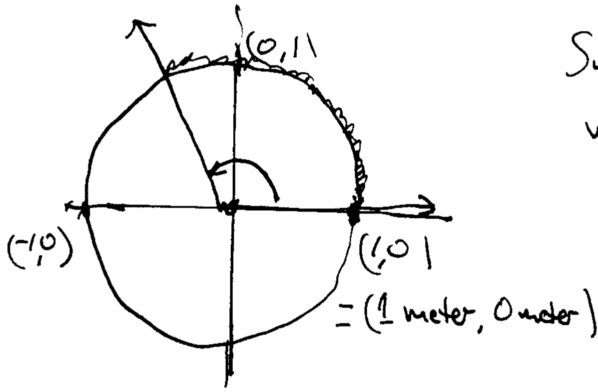
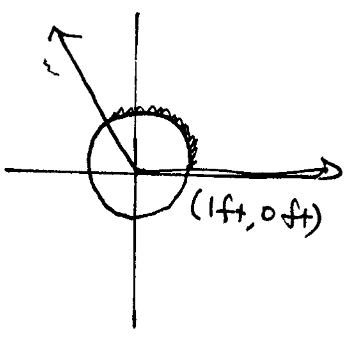
1.2 Radian measure



Suppose the length of a string wrapped around the unit (meter) circle is s meters.

Then the radian measure of the angle is s radians.

Remark: Radian measure is unitless. Reason: Had we used feet, rather than meters,



the circle is smaller, and the string is shorter as well.

$$\text{radian measure} = \alpha = \frac{1.9 \cancel{\text{meters}}}{1 \cancel{\text{meter}}} = \frac{1.9 \cancel{\text{ft}}}{1 \cancel{\text{ft}}} = 1.9$$

That is $\boxed{\alpha = \frac{s}{r}}$

where r = radius of the circle (meters or whatever)
s = arc length (meters or whatever)
α = radian measure (unitless)

In other words

$\boxed{s = \alpha r}$ provided α = radian measure of an angle
r = radius of circle
s = arc length

Remark: This is why we want to use radian measure for angles.

Converting between degrees and radians

Use: $\boxed{180 \text{ degrees} = \pi \text{ radians}}$

ex: Convert

$$72 \text{ degrees} = 72 \text{ degrees} \cdot 1$$
$$= 72\ \cancel{\text{degrees}} \cdot \frac{\pi \text{ radians}}{180\ \cancel{\text{degrees}}}$$
$$= \frac{72\pi}{180} \text{ radians} = \frac{2\pi}{5} \text{ radians}$$

Note: $72 \div 36 = 2$ and $180 \div 36 = 5$.

ex: Convert 1 radian to degrees:

$$1 \text{ radian} = 1\ \cancel{\text{radian}} \cdot \frac{180 \text{ degrees}}{\pi\ \cancel{\text{radian}}}$$
$$= \frac{180}{\pi} \text{ degrees} \approx 57.3 \text{ degrees}$$

Notation: If you write "$\alpha = 2.76$ is the measure of an angle" you are saying that $\alpha = 2.76$ radians.

If you write "$\alpha = 2.76°$" you mean $\alpha = 2.76$ degrees.

Calculation of arc length

$$\boxed{s = \alpha r}$$

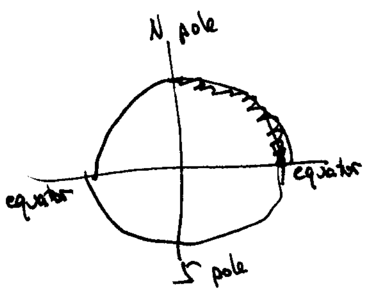
We must use radians to apply this formula

Remark: If you know two of the three of s, α or r, you can calculate the third.

#88) Given $\alpha = \frac{\pi}{8}$, $r = 30$ yd, Find s.

$$s = \frac{\pi}{8}\cdot(30\text{ yd}) = \frac{30\pi}{8}\text{ yd} \approx \frac{15\pi}{4}\text{ yd} \approx \cancel{5.89\text{ yd}}\ 11.78\text{ yd}$$

ex: Given: The distance from the equator to the north pole is 10,000 km, what is the radius of the earth?



$s = 10{,}000$ km $\quad \alpha = 90° = \frac{\pi}{2}$ radians $\quad r = ?$

$$s = \alpha r$$

$$10{,}000 = \frac{\pi}{2}\cdot r \Rightarrow r = \frac{20{,}000}{\pi}\text{ km}$$

$$\approx \boxed{6{,}366\text{ km}}$$

algebra:

$$\frac{2}{\pi}\cdot 10{,}000 = \cancel{\frac{2}{\pi}}\cdot\cancel{\frac{\pi}{2}}\cdot r$$

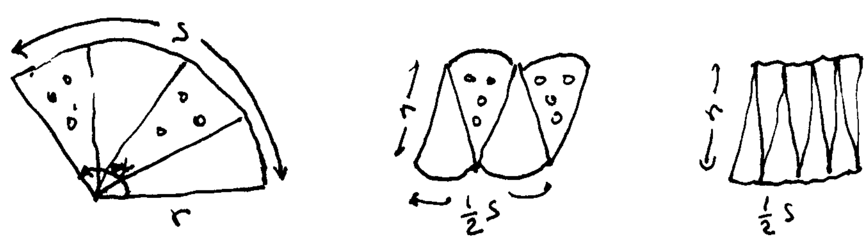
$$\frac{20{,}000}{\pi} = r$$

Area of a sector

$$\boxed{A = \frac{1}{2}\alpha r^2}$$

where α = radian measure of the angle forming the sector
r = radius of the circle

Derivation



Area = (base)(ht)
$= (\frac{1}{2}s)(r) = \frac{1}{2}sr$

Now substitute for s using $s = \alpha r$ so that

Area $= \frac{1}{2}\cdot\alpha r\cdot r = \frac{1}{2}\alpha r^2$

Remarks: (1) In the special case that $\alpha = 360° = 2\pi$, we get $A = \frac{1}{2}\cdot 2\pi\cdot r^2 = \pi r^2$ = area of a circle

(2) Since α is unitless and r has units of length, $A = \frac{1}{2}\alpha r^2$ will have units of $(\text{length})^2$.