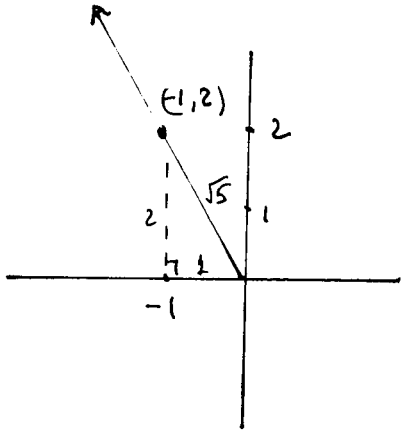
Warmup - No calculators - "Ratio Definition" of the trig functions

(1) For an angle α in standard position, the terminal ray passes through $(x,y) = (-1,2)$. Find the values of all six trig functions of α.



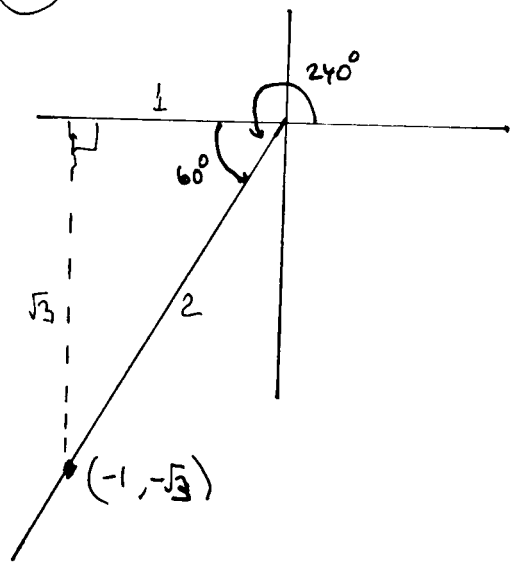
$$r = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

$$\sin\alpha = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad \csc\alpha = \frac{\sqrt{5}}{2}$$

$$\cos\alpha = \frac{x}{r} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5} \qquad \sec\alpha = -\sqrt{5}$$

$$\tan\alpha = \frac{y}{x} = \frac{-2}{1} = -2 \qquad \cot\alpha = -\frac{1}{2}$$

(2) NOTE: 240° is a famous angle.



$$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\sin 240^\circ = \frac{y}{r} = \frac{-\sqrt{3}}{2} \qquad \csc 240^\circ = -\frac{2}{\sqrt{3}}$$

$$\cos 240^\circ = \frac{x}{r} = \frac{-1}{2} \qquad \sec 240^\circ = -2$$

$$\tan 240^\circ = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3} \qquad \cot 240^\circ = \frac{1}{\sqrt{3}}$$

Remark: 60° = reference angle = acute angle the terminal ray makes with the closest half of the x-axis.

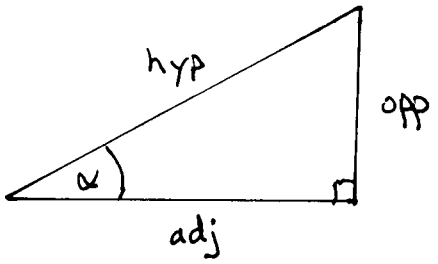
Behold:
$$\sin 240^\circ = -\sin 60^\circ$$
$$\cos 240^\circ = -\cos 60^\circ$$
$$\tan 240^\circ = +\tan 60^\circ$$

Recall:

S	A
T	C

1.5 Right triangle definition of trig functions

Defn: Given an acute angle of measure α, manufacture a right triangle with α as one of its angles. Then define:



$$\sin\alpha = \frac{opp}{hyp} \qquad \csc\alpha = \frac{hyp}{opp}$$

$$\cos\alpha = \frac{adj}{hyp} \qquad \sec\alpha = \frac{hyp}{adj}$$

$$\tan\alpha = \frac{opp}{adj} \qquad \cot\alpha = \frac{adj}{opp}$$

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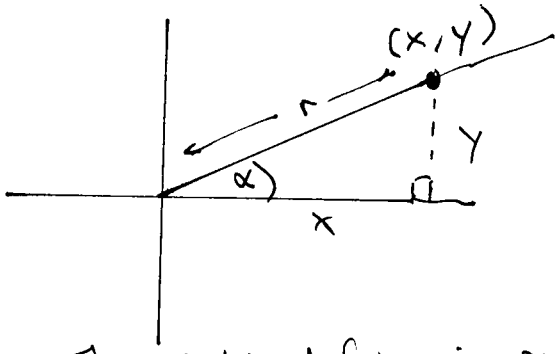
Remark: The size of the triangle doesn't matter. In any similar triangles, these ratios will still be the same.

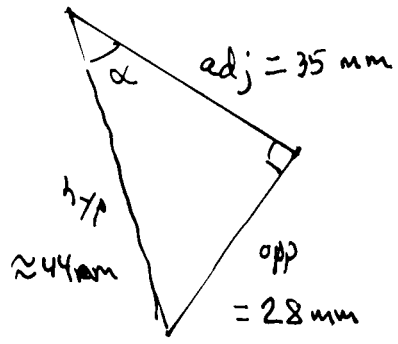
Remark: What's the connection between the ratio definition and the right triangle.



$y = opp$
$x = adj$
$r = hyp$

The ratio definition is restricted to acute angles (drawback) but it's more portable:

ex:



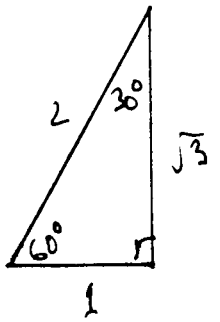
Find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$ to the nearest 1000th.

$$\sin\alpha = \frac{opp}{hyp} \approx \frac{28}{44} \approx .636$$

$$\cos\alpha = \frac{adj}{hyp} = \frac{35}{44} = .795$$

$$\tan\alpha = \frac{opp}{adj} = \frac{28}{35} = .800$$

Famous angles (again)



$\sin 30° = \frac{opp(30°)}{hyp} = \frac{1}{2}$

$\cos 30° = \frac{adj(30°)}{hyp} = \frac{\sqrt{3}}{2}$

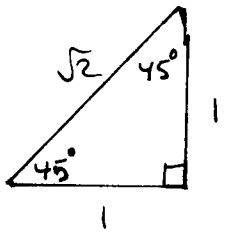
$\tan 30° = \frac{opp(30°)}{adj(30°)} = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{opp(60°)}{hyp} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{adj(60°)}{hyp} = \frac{1}{2}$

$\tan 60° = \frac{opp(60°)}{adj(60°)} = \frac{\sqrt{3}}{1}$

$[= \cot 30°]$



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{opp}{hyp} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{adj}{hyp} = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{1}{1} = \frac{opp}{adj}$

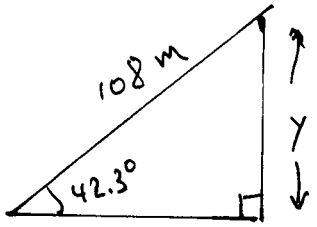
How memorize trig values of the famous acute angles

	sine	cosine	tangent $= \frac{\text{sine}}{\text{cosine}}$
$0 = 0°$	$\frac{\sqrt{0}}{2} = 0$	1	0
$\frac{\pi}{6} = 30°$	$\frac{\sqrt{1}}{2} = \frac{1}{2} = .5$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2} \approx .707$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3} = 60°$	$\frac{\sqrt{3}}{2} \approx .866$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
$\frac{\pi}{2} = 90°$	$\frac{\sqrt{4}}{2} = 1$	0	undefined

Calculator (1) angle α $\longrightarrow$ $\begin{cases} \sin\alpha \\ \cos\alpha \\ \tan\alpha \end{cases}$

(2) $\left.\begin{matrix} \sin\alpha \text{ or} \\ \cos\alpha \text{ or} \\ \tan\alpha \end{matrix}\right\} \longrightarrow$ angle α

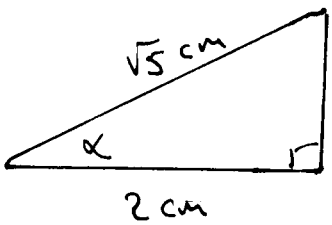
Example (1.86) A guy wire for an antenna has length 108 m. makes an angle of elevation of 42.3°. What is the height of the antenna?



$$\sin 42.3^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{Y}{108}$$

$$\Rightarrow Y = (108 \text{ meters}) \sin 42.3^\circ$$
$$= (108)(.6730125)$$
$$= 72.7 \text{ meters}$$

ex. In a triangle with sides 1 cm, 2 cm, and $\sqrt{5}$ cm, what is α?



$$\tan\alpha = \frac{1}{2} = .5$$

α = angle whose tan is .5 = $\tan^{-1}(.5)$ (called the "arctangent of .5")

$$= 26.57^\circ$$