1.5 Solving right triangles

examples

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$ means $\sin\alpha = \frac{1}{2}$ [that is $\alpha = 30°$]

$\alpha = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ means $\cos\alpha = \frac{\sqrt{2}}{2}$ [that is $\alpha = 45°$]

$\alpha = \tan^{-1}(\sqrt{3})$ means $\tan\alpha = \sqrt{3}$ [that is $\alpha = 60°$]

$= \arctan(\sqrt{3})$

Be able to "solve" a right triangle.

That is, given a right triangle and...

(1) one side and one more angle

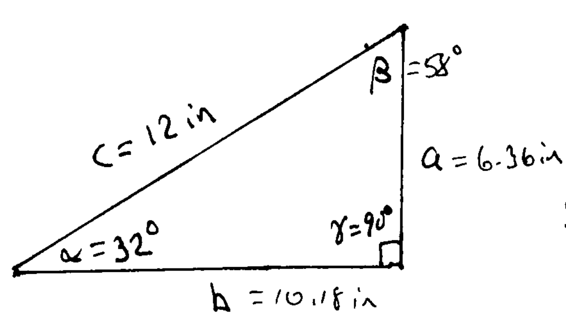
OR (2) two sides,

we can calculate the remaining sides and angles.

What we need:
- Pythagorean theorem
- Sum of angles in a triangle is 180°
- the definitions of sin, cos, and tan
- the definitions of arcsin, arccos, and arctan (and a calculator)

ex [Case (1)a: given the hypotenuse and one more angle]

Given hypotenuse = 12 inches, and $\alpha = 32°$.



easy: $\beta = 180° - 90° - 32°$
$= 90° - 32° = \boxed{58°}$

$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 32° = \frac{a}{12\text{ in}}$

$\Rightarrow a = 12\sin 32° = 12(.5299) = \boxed{6.36\text{ in}}$

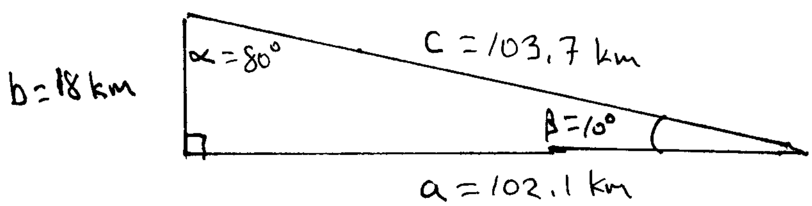
Also $\cos\alpha = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 32° = \frac{b}{12\text{ in}}$

$\Rightarrow b = 12\cos 32° = 12(.8480)$
$= \boxed{10.18\text{ in}}$

ex: [case (1)b: given one leg and another angle]

Given $\beta = 10^\circ$ and $b = 18$ km. Solve the right triangle.

[Implied by labeling conventions that b is a leg and it is opposite and β.]



$\alpha = 90^\circ - 10^\circ = \boxed{80^\circ}$

$\tan\beta = \frac{opp}{adj}$

$\tan 10^\circ = \frac{18 \text{ km}}{a}$

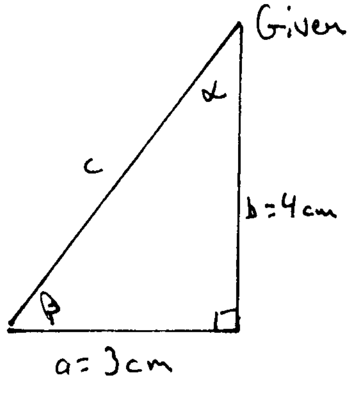
$\Rightarrow a \tan 10^\circ = 18$ km

$\Rightarrow a = \frac{18 \text{ km}}{\tan 10^\circ} = \frac{18 \text{ km}}{.1763} = \boxed{102.1 \text{ km}}$

Also $\sin\beta = \frac{opp}{hyp} \Rightarrow \sin 10^\circ = \frac{18 \text{ km}}{c}$

$\Rightarrow c \sin 10^\circ = 18\text{km} \Rightarrow c = \frac{18 \text{ km}}{\sin 10^\circ} = \frac{18 \text{ km}}{.1736} = \boxed{103.7 \text{ km}}$

ex [Case (2)a: Given two legs in a right triangle]



Given: In a right triangle the legs are $a = 3$ cm, $b = 4$ cm.

Find the hypotenuse and the two non-right angles.

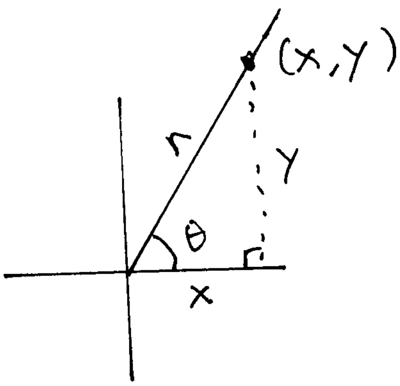
$a^2 + b^2 = c^2 \Rightarrow c^2 = 3^2 + 4^2 = 25 \Rightarrow c = \boxed{5 \text{ cm}}$

$\tan\beta = \frac{opp}{adj} \Rightarrow \tan\beta = \frac{4}{3} \Rightarrow \beta = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1^\circ}$

$\alpha = 90^\circ - 53.1^\circ = \boxed{36.9^\circ}$

exercise: Given $c = 13$ ft, $b = 12$ ft, and $\gamma = 90^\circ$, find a, α, and β.

1.6 Reference angles and the pythagorean identities



$x^2 + y^2 = r^2$ Divide by r^2:

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

ex: $\sin^2(30^\circ) + \cos^2(30^\circ) = (\sin 30^\circ)^2 + (\cos 30^\circ)^2$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

Added after class ended

1.6 p102 6) Find $\sin(\alpha)$, given that $\cos(\alpha) = -\frac{4}{5}$ and α is in quadrant III.

Method 1 [Using the Pythagorean identity]

$\sin^2\alpha + \cos^2\alpha = 1$ is true for all values of α. Now, if α is an angle such that $\cos\alpha = -\frac{4}{5}$, then $\sin^2\alpha + \left(-\frac{4}{5}\right)^2 = 1 \Rightarrow (\sin\alpha)^2 + \frac{16}{25} = 1$

$\Rightarrow (\sin\alpha)^2 = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25} \Rightarrow \sin\alpha = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$.

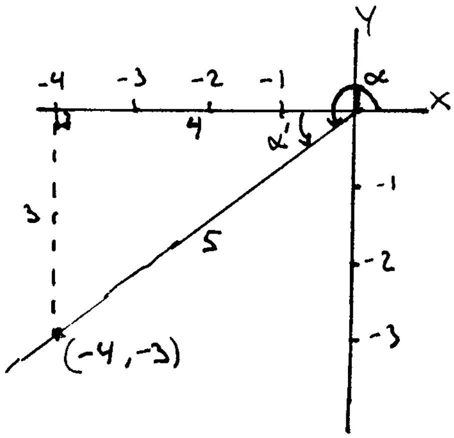
So which is it? Is it $\sin\alpha = +\frac{3}{5}$ or $\sin\alpha = -\frac{3}{5}$?

In quadrant III, sine is negative so $\boxed{\sin\alpha = -\frac{3}{5}}$

1.6 6) <u>Method 2</u> [Using geometry]

Because α is in quadrant III and $\cos\alpha = \frac{-4}{5}$, draw an angle in Q III so that for a point on the ray,

$x = -4$ and $r = 5$.



Because $x^2 + y^2 = r^2$ (or by drawing a right triangle and using the pythagorean theorem), we get

$$(-4)^2 + y^2 = 5^2$$
$$16 + y^2 = 25$$
$$y^2 = 25 - 16 = 9$$
$$y = \pm\sqrt{3} = \pm 3$$

But since (x,y) is in quadrant III, y is negative, so $y = -3$. Hence

$$\sin\alpha = \frac{y}{r} = \boxed{\frac{-3}{5}}$$

↑ Yet another way to look at this:

$$\sin\alpha' = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

where α' = reference angle

but since α is in Q III,

$$\sin\alpha = -\sin\alpha' = -\frac{3}{5}$$