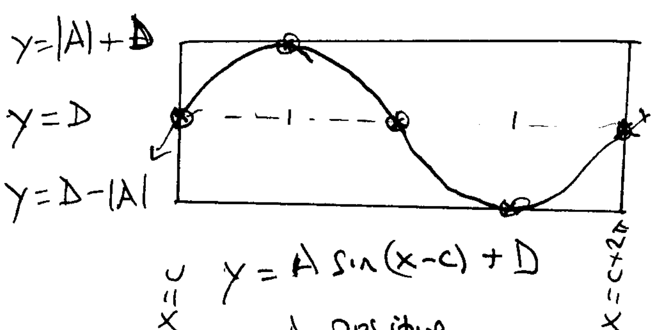
Summary of §2.1

$$y = A\sin(x - C) + D$$

or $$y = A\cos(x - C) + D$$

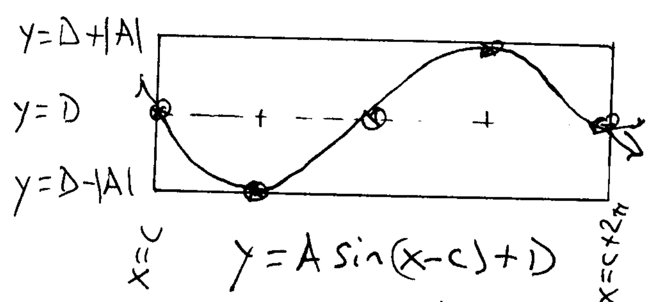
each are "waves" with amplitude $= |A|$

period $= 2\pi$, phase shift $= C$, vertical shift $= D$



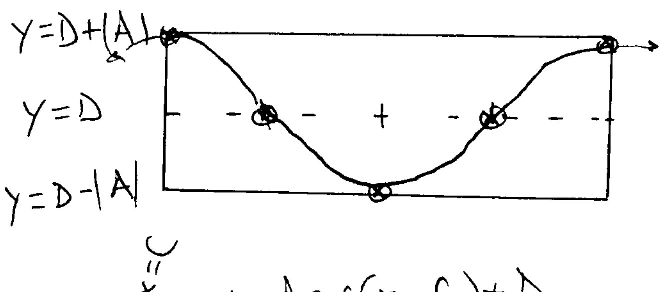
$y = A\sin(x-c) + D$

A positive



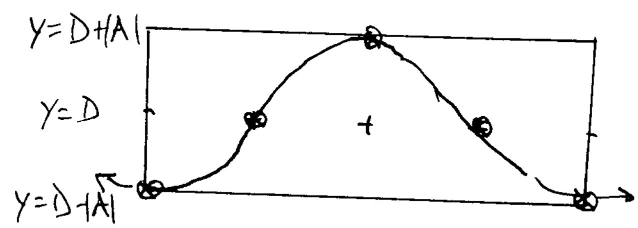
$y = A\sin(x-c) + D$

A negative



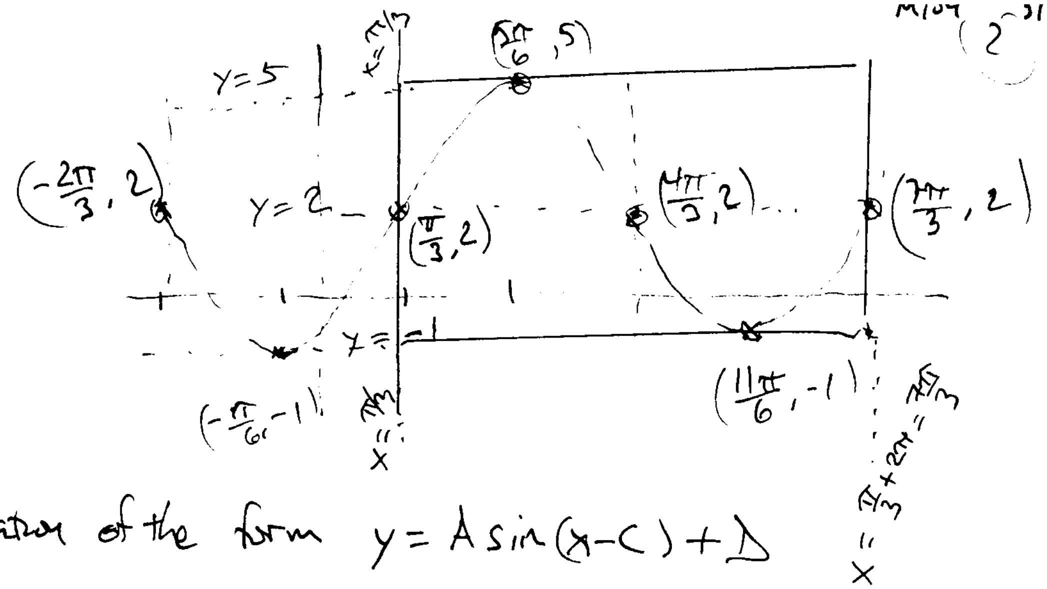
$y = A\cos(x-C) + D$

A positive



$y = A\cos(x-C) + D$

A negative



Find an equation of the form $y = A\sin(x-C)+D$

So $D = 2$ = ~~median~~ mean value, $A = 5-2 = 3$ or $A = \frac{5-(-1)}{2} = \frac{\text{max}-\text{min}}{2} = 3$ $\quad C = \frac{\pi}{3}$ = phase shift

Note: amplitude $= |A|$ is always positive, but A may be positive or negative.

Answer: $y = 3\sin\left(x - \frac{\pi}{3}\right) + 2$

Alternative answer: $D = 2$, $A = -3$, $C = -\frac{2\pi}{3}$

$$y = -3\sin\left(x + \frac{2\pi}{3}\right) + 2$$

§2.2 Period other than 2π

ex: What is the period of $y = \sin 7x$? (instead of $y = \sin x$)

Idea: $\sin x$ goes through one cycle when

$$0 \le x \le 2\pi$$

So $\sin 7x$ goes through one cycle when

$$0 \le 7x \le 2\pi \qquad \text{So}$$

$$\frac{0}{7} \le \frac{7x}{7} \le \frac{2\pi}{7} \Rightarrow 0 \le x \le \frac{2\pi}{7}$$

$\therefore$ $\boxed{\text{period} = \frac{2\pi}{7}}$

ex: what is the period of

$$y = 2\sin 3\left(x - \frac{\pi}{4}\right) \quad ?$$

Remark: The amplitude $= |2| = 2$ doesn't matter for this question.

Idea: $y = 2\sin 3(x - \pi/4)$ goes through one cycle

when

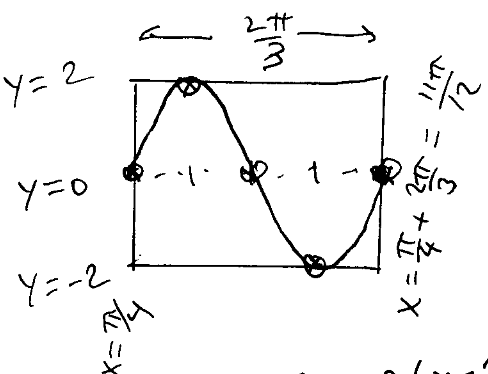
$$0 \le 3\left(x - \pi/4\right) \le 2\pi$$

$$\frac{0}{3} \le \frac{3\left(x - \frac{\pi}{4}\right)}{3} \le \frac{2\pi}{3}$$

$$0 \le x - \frac{\pi}{4} \le \frac{2\pi}{3}$$

$$+\frac{\pi}{4} \qquad +\frac{\pi}{4} \qquad +\frac{\pi}{4}$$

$$\frac{\pi}{4} \le x \le \frac{2\pi}{3} + \frac{\pi}{4}$$



$$\text{period} = \left(\frac{2\pi}{3} + \frac{\pi}{4}\right) - \frac{\pi}{4} = \boxed{\frac{2\pi}{3}}$$

$$\frac{1}{4}\cdot\text{period} = \frac{1}{4}\cdot\frac{2\pi}{3} = \frac{\pi}{6}$$

x	$2\sin 3\left(x - \frac{\pi}{4}\right)$
$\pi/4$	$2\sin 0 = 0$
$\frac{\pi}{4} + \frac{\pi}{6} =$	$2\sin 3\left(\frac{\pi}{6}\right)$ $2\sin\frac{\pi}{2} = 2$
$\frac{\pi}{4} + \frac{2\pi}{6} =$	$2\sin 3\left(\frac{\pi}{3}\right) = 2\sin\pi = 0$
$\frac{\pi}{4} + \frac{3\pi}{6} =$	$2\sin 3\left(\frac{\pi}{2}\right) = 2\sin\frac{3\pi}{2} = -2$
$\frac{\pi}{4} + \frac{4\pi}{6} =$	$2\sin 3\left(\frac{2\pi}{3}\right) = 2\sin 2\pi = 0$

$$= \frac{\pi}{4} + \frac{2\pi}{3}$$

$$= \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$$

More generally : $\boxed{y = A \sin B(x-c) + D}$

$|A|$ = amplitude $\qquad$ c = phase shift

$\frac{2\pi}{B}$ = period $\qquad$ D = vertical shift or mean value

Likewise for $\boxed{y = A \cos B(x-c) + D}$

Remark: Frequency $= \frac{1}{\text{period}} = \frac{B}{2\pi}$

ex: Find the amplitude, period, phase shift of the following

example	amplitude	period	phase shift	vertical shift
$y = 3\sin 2x$	3	$\frac{2\pi}{2} = \pi$	0	0
$y = -5\cos \pi x$	5	$\frac{2\pi}{\pi} = 2$	0	0
$y = \frac{1}{2}\sin 2(x-\pi)$	$\frac{1}{2}$	$\frac{2\pi}{2} = \pi$	π	0
$y = \sin 3x + 5$	1	$\frac{2\pi}{3}$	0	5
$y = -\cos(3x+\pi)$ $= -\cos 3\left[x + \frac{\pi}{3}\right]$ $= -\cos 3\left[x - \left(-\frac{\pi}{3}\right)\right]$	1	$\frac{2\pi}{3}$	$-\frac{\pi}{3}$	0

$\uparrow$ why?

Solve $0 = 3x + \pi$

$3x = -\pi$

$x = -\pi/3$

ex [eqn → graph] $y = -\frac{1}{2}\sin\left[4\left(x+\frac{\pi}{4}\right)\right] + 1$

2.2 #28

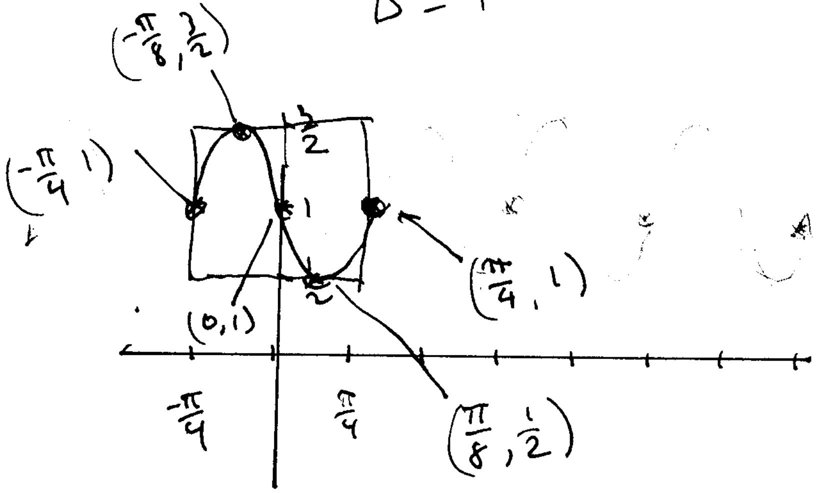
$A = -\frac{1}{2} \Rightarrow$ amplitude $= \left|-\frac{1}{2}\right| = \frac{1}{2}$

$B = 4 \Rightarrow$ period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$C = -\frac{\pi}{4} \Rightarrow$ phase shift $= -\pi/4$

$D = 1 \Rightarrow$ mean value $= 1$

Range $= \left[1-\frac{1}{2}, 1+\frac{1}{2}\right]$
$= \left[\frac{1}{2}, \frac{3}{2}\right]$



Left edge of frame $= -\frac{\pi}{4} = C$

right of frame $= -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4} = C + \text{period}$

top of frame $= 1 + \frac{1}{2} = \frac{3}{2} = D + \text{amplitude}$

bottom of frame $= 1 - \frac{1}{2} = \frac{1}{2} = D - \text{amplitude}$