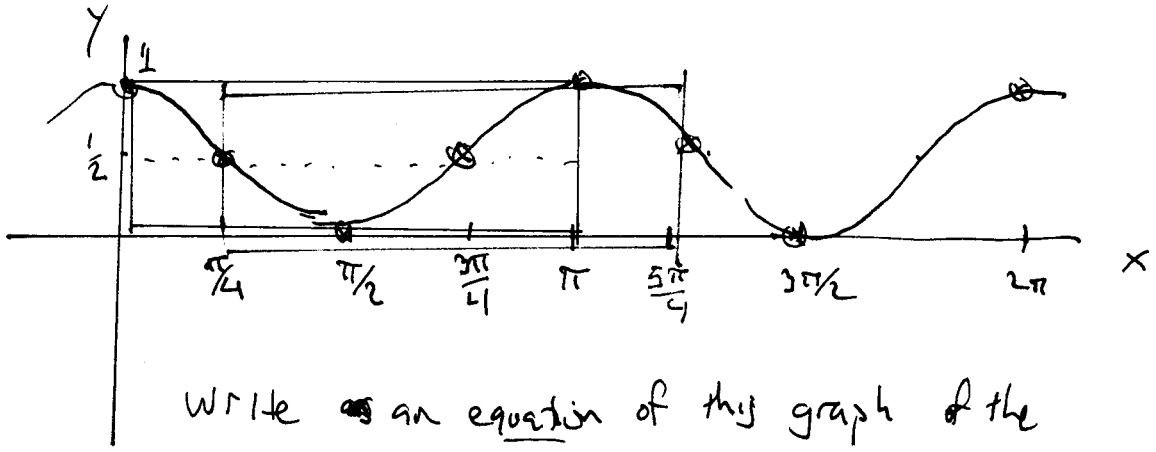
# 2.2 [Loose ends] Graph → Equation




Write an equation of this graph of the form $y = A \sin B(x-C) + D$ OR

$y = A \cos B(x-C) + D$

Observations about graph:

Range of this function = $[0, 1]$

amplitude = $\frac{1}{2} = \frac{1-0}{2}$ [= amount of deviation from the mean value]

mean value = $\frac{1}{2} = \frac{1+0}{2}$ = [halfway between max and min]

period = $\pi$

phase shift = ? many possible answers. It depends

$y = A \cos B(x-C) + D$

becomes

$$\boxed{y = \frac{1}{2}\cos 2x + \frac{1}{2}}$$

Now pick a "frame" which encloses exactly one cycle

Given this frame: this is a cosine graph, with phase shift = 0 = left x-value of the frame

Conclude:

$|A|$ = amplitude = $\frac{1}{2} \Rightarrow A = \boxed{\frac{1}{2}}$ (because we start at top of frame)

$\frac{2\pi}{B}$ = period $\underset{\text{(algebra)}}{\Rightarrow}$ $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{(\ )} = \boxed{2}$

$C$ = phase shift = $\boxed{0}$

$D$ = mean value = vertical shift = $\boxed{\frac{1}{2}}$

What if we chose a different frame (say to express this as a sine instead of cosine)

green ink = new frame

What changes? sine, not cosine
(reason: sine begins at mid frame)

C = phase shift = $\frac{\pi}{4}$ (= new left edge of frame)

$|A| = \frac{1}{2} \Rightarrow A = \pm\frac{1}{2}$ which?

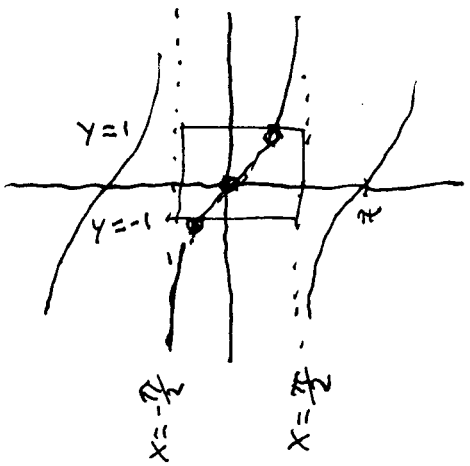
$A = -\frac{1}{2}$

$y = A \sin B(x-C) + D$ becomes

$$\boxed{y = -\frac{1}{2}\sin 2\left(x - \frac{\pi}{4}\right) + \frac{1}{2}}$$

Remark. There are infinitely many correct answers.

## 2.4 Graph of Tangent function




domain = all reals except odd multiples of $\frac{\pi}{2}$

range = all real numbers = $(-\infty, \infty)$

tangent is an odd function (like sine)

$\tan(-x) = -\tan(x)$

period = $\pi$

$x$ = left edge of frame = $-\frac{\pi}{2}$ right edge = $\frac{\pi}{2} = x$

↑ ↑
vertical asymptotes of the graph

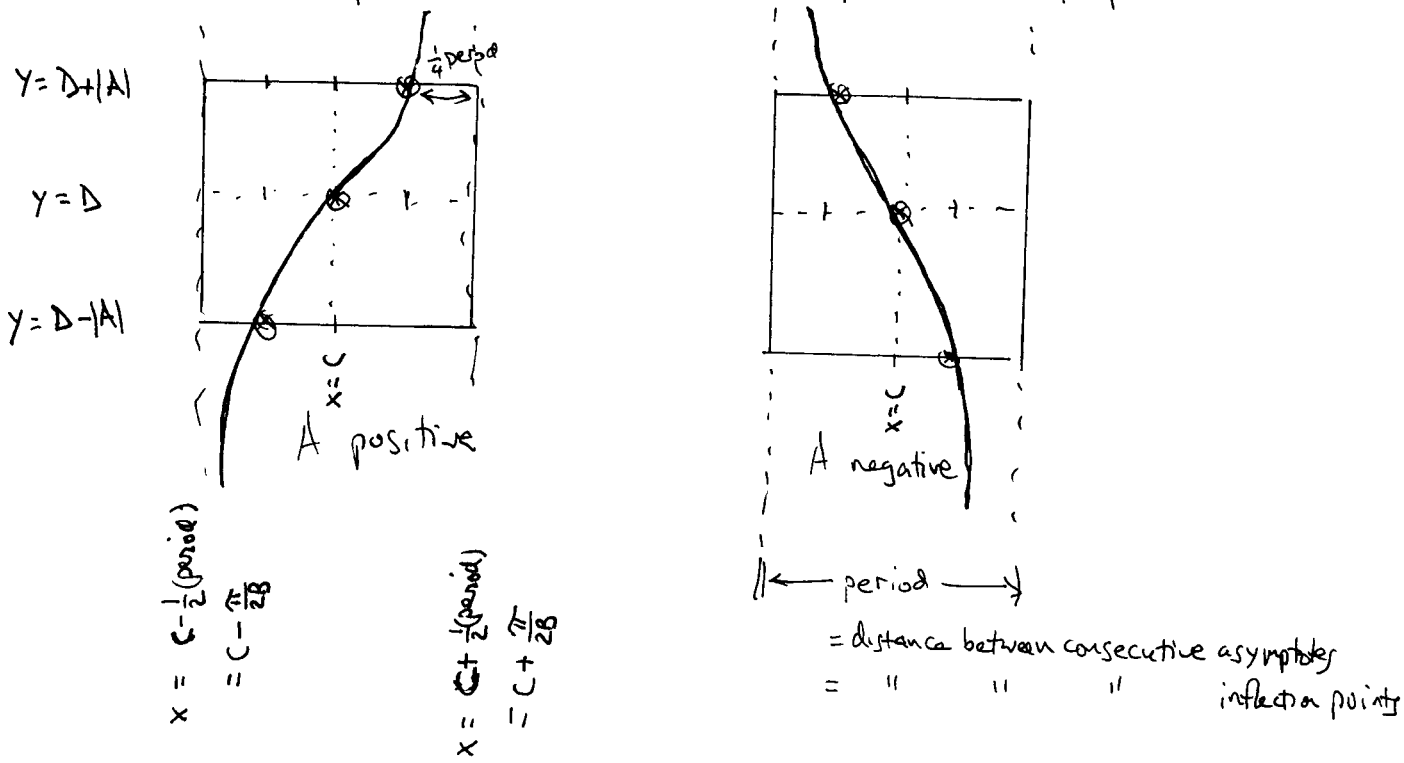
$$y = A \tan[B(x-C)] + D$$

D = vertical shift = y-coordinate of center (inflection point)

C = phase (horizontal) shift = x-coord of inflection pt.

$$\text{period} = \frac{\pi}{B} \qquad \text{so} \qquad B = \frac{\pi}{\text{period}}$$

top + bottom of frame = $D+|A|$ and $D-|A|$




= distance between consecutive asymptotes
= " " " inflection points