Warm up

Simplify

1

$(\sec x)(\cos x) = \frac{1}{\cos x} \cdot \frac{\cos x}{1} = \boxed{1}$

2 $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta$

$= \sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta - \cancel{2\sin\theta\cos\theta}$

$= \sin^2\theta + \cos^2\theta = \boxed{1}$

3

$(\cos x - 1)(\cos x + 1) = (\cos x)^2 - 1^2$

$= \cos^2 x - 1 = -(1 - \cos^2 x)$

$= \boxed{-\sin^2 x}$

4 $\tan x + \tan(-x) = \tan x + (-\tan x) = \boxed{0}$

example : Verify the identity, using the basic identities,

$(\cos x - 1)(\cos x + 1) = -\sin^2 x$

3.2 verifying Identities

16) Find the product

$(\tan\alpha + 2)(\tan\alpha - 2) = \tan^2\alpha - 2^2$

$= \tan^2\alpha - 4$ ← This says

order of operations
(0) Take α
(1) $\tan\alpha$
(2) $(\tan\alpha)^2 = \tan^2\alpha$
(3) $\tan^2\alpha - 4$

24) $(3\cos\theta - 2)^2$

$= (3\cos\theta)^2 - 2(3\cos\theta)(2) + 2^2$ ← using $(A-B)^2 = A^2 - 2AB + B^2$ with $A = 3\cos\theta$, $B = 2$

$= 9\cos^2\theta - 12\cos\theta + 4$

28) Factor the expression

$\cos^2\gamma - \cos\gamma - 6$

$= (\cos\gamma - 3)(\cos\gamma + 2)$

Hint: Consider factoring $u^2 - u - 6 = (u-3)(u+2)$

Now use this with $u = \cos\gamma$

38) Factor completely

$\cos^4 x - 2\cos^2 x + 1$

Try letting $u = \cos x$

$= u^4 - 2u^2 + 1$

$= (u^2)^2 - 2u^2 + 1$

$= (u^2 - 1)^2$

$= [(u-1)(u+1)]^2 = (u-1)^2 (u+1)^2 = (u-1)(u-1)(u+1)(u+1)$

$= (\cos x - 1)^2 (\cos x + 1)^2$

$= (\cos x - 1)(\cos x - 1)(\cos x + 1)(\cos x + 1)$

44) Verify the identity

$$1 - \csc(x)\sin^3(x) = \cos^2(x)$$

$$\text{L.H.S.} = 1 - c\ x \sin^3 x = 1 - (\csc x \cdot \sin x)\cdot \sin^2 x$$

$$= 1 - \left(\frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x}\right)\cdot \sin^2 x \quad \text{by Reciprocal}$$

$$= 1 - \sin^2 x = \cos^2 x = \text{R.H.S} \quad \text{by Pythagorean}$$