3.3 Sum and Difference Identities for Cosine

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

Remark: This is true if α, β are numbers, variables, or expressions

ex: $\cos(30^\circ+45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

That is,

$$\cos 75^\circ = \frac{\sqrt3}{2}\cdot\frac{\sqrt2}{2} - \frac{1}{2}\cdot\frac{\sqrt2}{2} = \frac{\sqrt6-\sqrt2}{4}$$

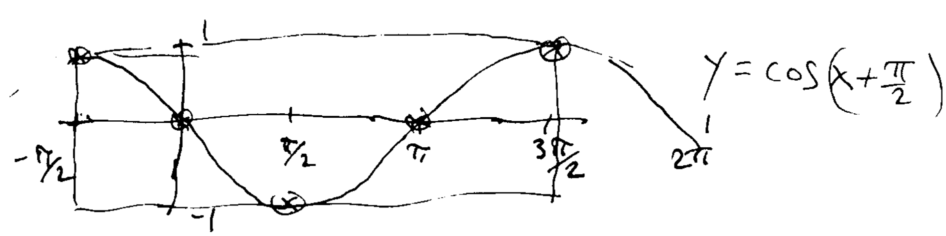
ex: Find another way to write:

$$\cos(90^\circ+x) = \underbrace{\cos 90^\circ}_{0}\cos x - \underbrace{\sin 90^\circ}_{1}\sin x$$

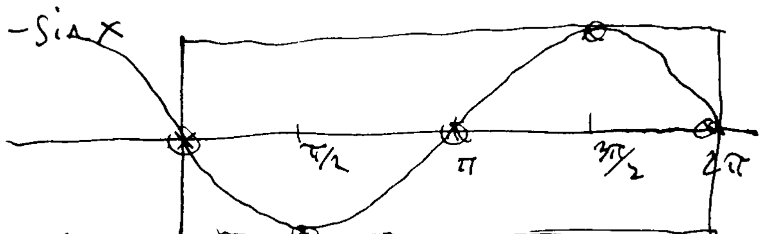
$$= -\sin x$$

OR $-\sin x = \cos\left(x+\frac{\pi}{2}\right)$

TRUE? Graph $y=\cos\left(x+\frac{\pi}{2}\right)$ Phase shift $= -\frac{\pi}{2}$



Graph $y=-\sin x$



YES! These two functions have the same graph, so they are the same function.

ex: In the sum identity

let $\alpha = x$ and $\beta = x$.

$$\cos(x+x) = \cos x \cos x - \sin x \sin x$$

$$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$$

Double-Angle Identity for Cosine

ex: In the sum identity,

let $\alpha = \alpha$ and $\beta = -\beta$.

$$\cos[\alpha + (-\beta)] = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta - (\sin\alpha)\cdot(-\sin\beta)$$

$$\boxed{\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta}$$

Difference Identity for cosine