3.3 Sum + Diff for cosine [continued]

$$\boxed{\begin{aligned} \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \end{aligned}}$$

Sum for cosine

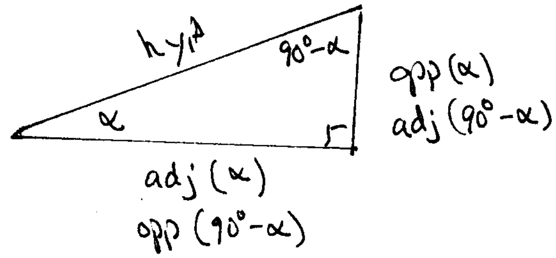
Difference

ex: In the Diff. formula take $\alpha=\beta$.

$$\cos(\alpha-\alpha) = \cos\alpha\cos\alpha + \sin\alpha\sin\alpha$$

$$1 = \cos(0) = \cos^2\alpha + \sin^2\alpha$$

Cofunction identities



$$\sin\alpha = \frac{\text{opp}(\alpha)}{\text{hyp}} = \frac{\text{adj}(90^\circ-\alpha)}{\text{hyp}} = \cos(90^\circ-\alpha)$$

$$\cos\alpha = \sin(90^\circ-\alpha)$$

$$\tan\alpha = \frac{\text{opp}(\alpha)}{\text{adj}(\alpha)} = \frac{\text{adj}(90^\circ-\alpha)}{\text{opp}(90^\circ-\alpha)} = \cot(90^\circ-\alpha)$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{\cos(90^\circ-\alpha)} = \sec(90^\circ-\alpha)$$

$\tan\alpha = \cot(90^\circ-\alpha)$ Let $\beta = 90^\circ-\alpha$ so $\alpha = 90^\circ-\beta$

$\tan(90^\circ-\beta) = \cot\beta$

Then, since α and β are arbitrary letters

$\tan(90^\circ-\alpha) = \cot\alpha$

In $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$,

let $\alpha = \frac{\pi}{2}$ and $\beta = u$.

$$\cos\left(\frac{\pi}{2}-u\right) = \cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u$$
$$= 0\cdot\cos u + 1\cdot\sin u$$

$$\therefore \boxed{\cos\left(\frac{\pi}{2}-u\right) = \sin u}$$

Now substitute $\frac{\pi}{2}-u$ for u

Actually $w = \frac{\pi}{2}-u$

so $u = \frac{\pi}{2}-w$

$$\cos w = \sin\left(\frac{\pi}{2}-w\right)$$

But w is just a letter, so u works just as well:

$$\boxed{\cos u = \sin\left(\frac{\pi}{2}-u\right)}$$

$$\tan u = \frac{\sin u}{\cos u} = \frac{\cos\left(\frac{\pi}{2}-u\right)}{\sin\left(\frac{\pi}{2}-u\right)} = \cot\left(\frac{\pi}{2}-u\right)$$

$$\boxed{\tan u = \cot\left(\frac{\pi}{2}-u\right)}$$

$$\cot u = \frac{\cos u}{\sin u} = \frac{\sin\left(\frac{\pi}{2}-u\right)}{\cos\left(\frac{\pi}{2}-u\right)} = \tan\left(\frac{\pi}{2}-u\right)$$ etc

The rest: $$\boxed{\cot u = \tan\left(\frac{\pi}{2}-u\right)}$$

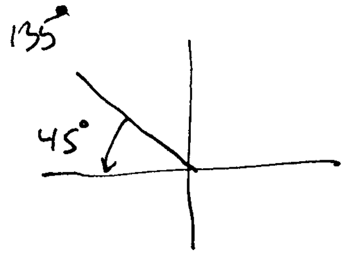
$$\sec u = \frac{1}{\cos u} = \frac{1}{\sin\left(\frac{\pi}{2}-u\right)} = \csc\left(\frac{\pi}{2}-u\right)$$

$$\csc u = \qquad = \sec\left(\frac{\pi}{2}-u\right)$$

3.3 24) Find the exact value of $\cos(165^\circ)$.

Hint: $165^\circ = 90^\circ + 75^\circ = 135^\circ + 30^\circ = 120^\circ + 45^\circ$

would also work



$\sin 135^\circ = +\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

$\sin 30^\circ = \frac{1}{2}$

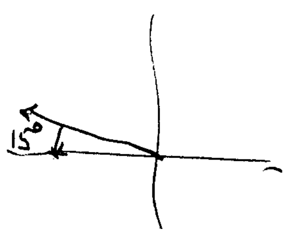
$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\cos 165^\circ = \cos(135^\circ + 30^\circ)$$

$$= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$$



Simplify

34) $\cos(34^\circ)\cos(13^\circ) + \sin(34^\circ)\sin(13^\circ)$

$$= \cos(34^\circ - 13^\circ) = \cos(21^\circ)$$

39) $\cos\left(-\frac{\pi}{2}\right)\cos\left(\frac{\pi}{5}\right) + \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{5}\right)$

$$= \cos\left(\frac{\pi}{2}\right)\cos\frac{\pi}{5} + \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{5}\right)$$

Because $\cos(-x) = \cos(x)$, i.e. cosine is even.

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$$

$$= \cos\left(\frac{5\pi}{10} - \frac{2\pi}{10}\right) = \boxed{\cos\left(\frac{3\pi}{10}\right)}$$

OR $\boxed{\sin\frac{\pi}{5}}$ by a cofunction identity

Two hard problems

62) $\sin(85^\circ)\sin(40^\circ) + \sin(-5^\circ)\sin(-50^\circ)$

$= \sin(85^\circ)\sin(40^\circ) + (-\sin 5^\circ)\cdot(-\sin 50^\circ)$ — Sine is odd

$= \sin(85^\circ)\sin(40^\circ) + \sin(5^\circ)\cdot\sin(50^\circ)$ — $(-1)\cdot(-1) = 1$

$= \sin(90^\circ - 5^\circ)\cdot\sin(90^\circ - 50^\circ) + \sin(5^\circ)\sin(50^\circ)$ — 85° and 5° are complementary, 40° and 50° "

$= \cos(5^\circ)\cdot\cos(50^\circ) + \sin(5^\circ)\cdot\sin(50^\circ)$

$= \cos(5^\circ - 50^\circ)$

$= \cos(-45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$

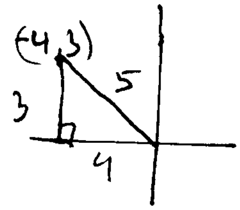
67) Find the exact value of $\cos(\alpha+\beta)$
if $\sin\alpha = \frac{3}{5}$ with α in Q II and
$\sin\beta = \frac{5}{13}$ with β in Q I.

Easy part: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= (\cos\alpha)(\cos\beta) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

to be figured out — given

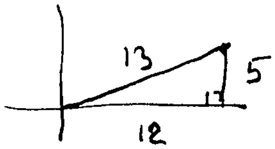
What is $\cos\alpha$? $\sin\alpha = \frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$



adj $= \sqrt{5^2 - 3^2} = 4$

$\cos\alpha = \frac{-4}{5}$

What is $\cos\beta$? $\sin\beta = \frac{5}{13}$



adj $= \sqrt{13^2 - 5^2} = 12$

$\cos\beta = +\frac{12}{13}$

$\cos(\alpha+\beta) = \left(\frac{-4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{-48-15}{65} = \boxed{\frac{-63}{65}}$