

4.1 Inverse Trig (cont'd)

$y = \cos x$

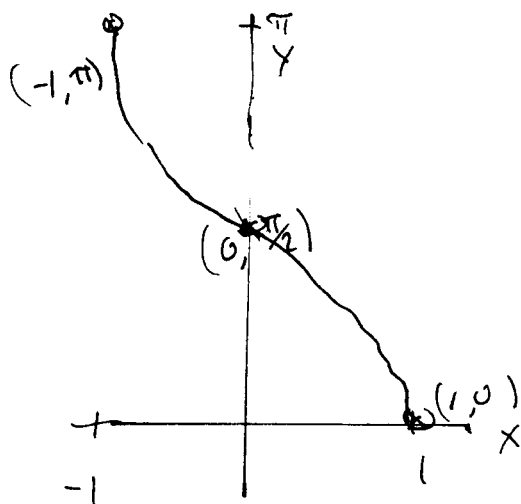
Restrict the domain to

domain = $[0, \pi]$

range = $[-1, 1]$



x	cos x
0	1
$\pi/2$	0
π	-1



$y = \arccos x$

domain = $[-1, 1]$

range = $[0, \pi]$

← memorize
i.e. in Q I or Q II

x	arccos x
1	0
0	$\pi/2$
-1	π

$y = \arccos x$ means (1) $\cos y = x$
 $= \cos^{-1} x$ and (2) $0 \leq y \leq \pi$

ex: What is $\arccos(-1/2)$? Let $y = \arccos(-1/2)$.

This says (1) $\cos y = -1/2$

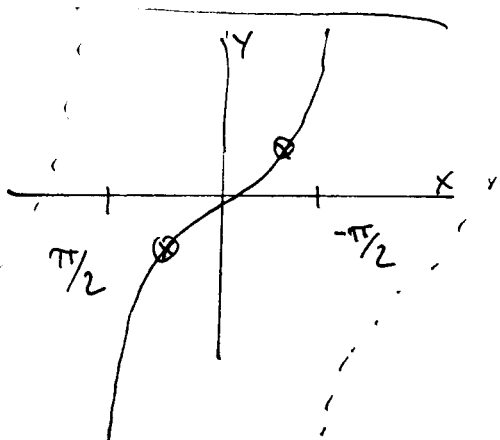
and (2) $0 \leq y \leq \pi$

possibilities satisfying (1): $\frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{2\pi}{3}$ (infinitely many others)

But (2) says we want $y = \frac{2\pi}{3}$

The arctangent function (most important in calculus)

x	tan x
$-\pi/4$	-1
0	0
$\pi/4$	1



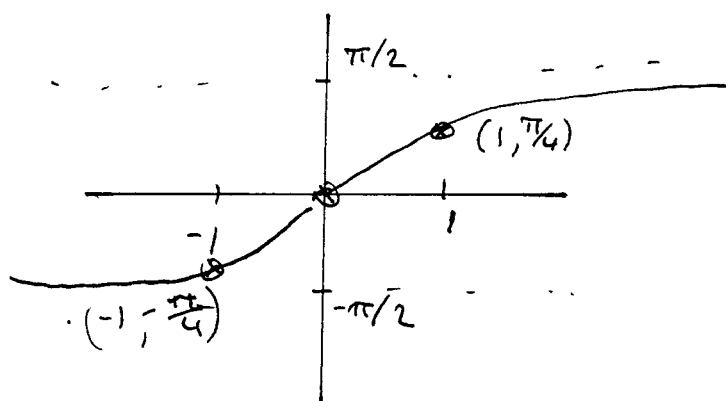
$y = \tan x$

Restrict domain to $(-\pi/2, \pi/2)$

domain = $(-\pi/2, \pi/2)$

range = $(-\infty, \infty)$

x	arctan x
-1	$-\pi/4$
0	0
1	$\pi/4$



$y = \arctan x$

domain = $(-\infty, \infty)$

range = $(-\pi/2, \pi/2)$

↑ memorize
i.e. QI or QIV

$y = \arctan x$
 $= \tan^{-1} x$

means

(1) $\tan y = x$

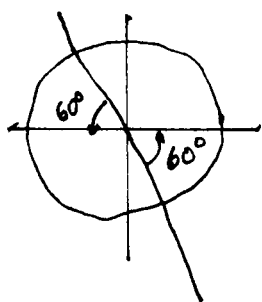
(2) $-\pi/2 < y < \pi/2$

ex: Find (without a calculator) $\arctan(-\sqrt{3})$

Let $y = \arctan(-\sqrt{3})$. Then (1) $\tan y = -\sqrt{3}$

and (2) $-\pi/2 < y < \pi/2$

What angles satisfy (1)?



$y = \frac{2\pi}{3}, \frac{5\pi}{3}, \dots$

What also satisfies (2)?

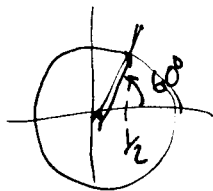
$\arctan(-\sqrt{3}) = -\pi/3$

without a calculator, find:

$$32) y = \arccos\left(\frac{1}{2}\right)$$

$$\textcircled{1} \cos y = \frac{1}{2}$$

$$\textcircled{2} 0 \leq y \leq \pi$$

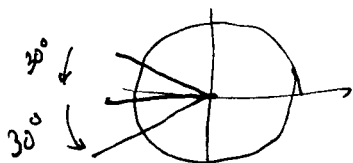


$$\text{Answer: } y = \frac{\pi}{3}$$

$$34) y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\textcircled{1} \cos y = -\frac{\sqrt{3}}{2}$$

$$\textcircled{2} 0 \leq y \leq \pi$$

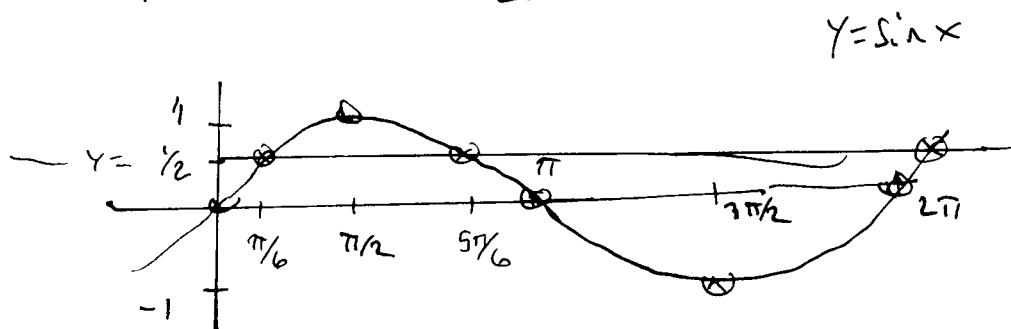


$$\text{Answer: } \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

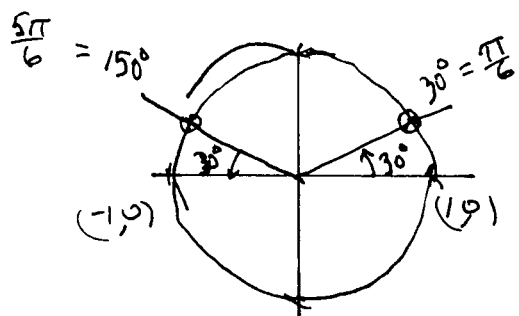
[Skip, for now, composition of trig functions with inverse trig functions.]

4.2 Basic Trig Equations

$$\text{Solve } \sin x = \frac{1}{2}$$



Two solutions would be $x = \frac{\pi}{6}$ because $\sin \frac{\pi}{6} = \frac{1}{2}$
and $x = \frac{5\pi}{6}$ " $\sin \frac{5\pi}{6} = \frac{1}{2}$

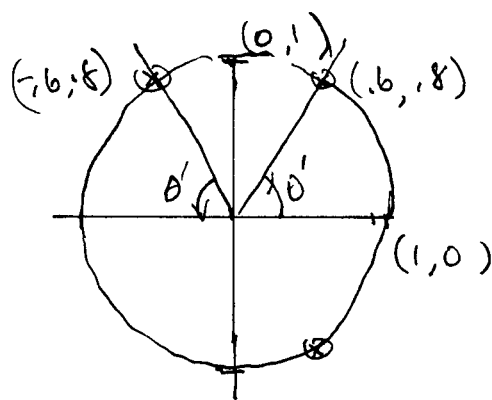


Answer:

$$x = 30^\circ + 360^\circ k = \frac{\pi}{6} + 2k\pi$$

or

$$x = 150^\circ + 360^\circ k = \frac{5\pi}{6} + 2k\pi$$

where $k = \dots, -2, -1, 0, 1, 2, 3, \dots$ ex: Find all solutions (in degrees) of

$$\sin \theta = 0.8$$

$$\text{reference angle} = \theta' = \sin^{-1} 0.8 = 53.13^\circ$$

$$\theta = 53.13^\circ = \sin^{-1}(0.8)$$

$$\theta = 180^\circ - 53.13^\circ = 126.87^\circ$$

by calculator

Answer:

$$\theta = 53.13^\circ + 360^\circ k$$

or

$$\theta = 126.87^\circ + 360^\circ k$$

To list every angle which is coterminal with these two angles.