4.2 (Continued) Basic sin, cos, tan equations

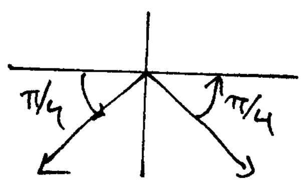
ex: Find all solutions of $\sin x = -\frac{\sqrt{2}}{2}$ (in radians)

Step 1 Ignore the minus sign and find the reference angle $x'$

Solve: $\sin x' = +\frac{\sqrt{2}}{2}$ in $[0, \pi/2]$.

$$x' = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \approx .785 = \frac{\pi}{4}$$

Step 2 where is sine negative? Q III or Q IV.
In those quadrants, which angles have a reference angle of $\frac{\pi}{4}$?




$x = \frac{5\pi}{4}$

or

$x = \frac{7\pi}{4}$

Step 3 What angles are coterminal with those angles?

$$\boxed{\begin{array}{l} x = \frac{5\pi}{4} + 2k\pi \\ \text{or } x = \frac{7\pi}{4} + 2k\pi \end{array}}$$

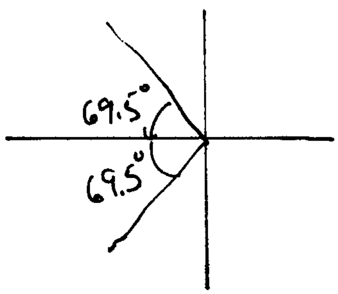
, (because sine has period $2\pi$)

ex: Solve $\cos \theta = -.35$ to three decimal places, (in degrees)

Step 1 Reference angle? Solve $\cos \theta' = +.35$

$$\theta' = \arccos(.35) = 69.51^\circ$$

Step 2 Solutions in QII and QIII, where cos is negative, with reference angle $\theta' = 69.51^\circ$




$$\theta = 180^\circ - 69.51^\circ = 110.49^\circ$$

or

$$\theta = 180^\circ + 69.51^\circ = 249.51^\circ$$

Step 3 All solutions

$$\boxed{\begin{array}{l} \theta = 110.49^\circ + 360^\circ k \\ \text{or} \\ \theta = 249.51^\circ + 360^\circ k \end{array}}$$

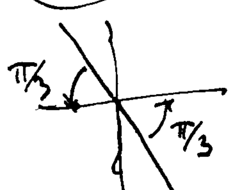
because cos has period 360°.

4.2 26) Solve $\tan x + \sqrt{3} = 0$ in radians.

$$\tan x = -\sqrt{3}$$

Step 1 Solve $\tan x' = +\sqrt{3}$ $\Rightarrow$ $x' = \frac{\pi}{3}$ $[= 60^\circ]$

Step 2 Because tan is <u>negative</u> in QII or QIV




$$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

or

$$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

Step 3 All solutions? $\boxed{x = \frac{2\pi}{3} + k\pi}$

because tan has period $\pi$, not $2\pi$.

4.3 Multiple angle equations

ex: a) Find all solutions in $(-\infty, \infty)$ of

$$\cos(3x) = -\frac{1}{2}.$$

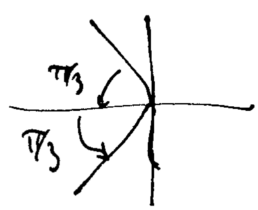
b) Find the solutions in $[0, 2\pi)$.

Remark: Up until now, the angle (i.e. the argument of the cosine function) and the variable have been the same thing. No longer: In this problem, the angle is $3x$, but the variable is $x$.
Method: First solve for the angle (step 1–3), then solve for $x$.

a) Step 1 Solve $\cos\theta' = +\frac{1}{2}$ in $[0, \pi/2)$

$$\theta' = \pi/3$$

Step 2 Solve in $[0, 2\pi)$ $\cos\theta = -\frac{1}{2}$, where $\theta = 3x$.




$$\theta = \frac{2\pi}{3}$$

or

$$\theta = \frac{4\pi}{3}$$

Step 3 All solutions

$$\theta = 3x = \frac{2\pi}{3} + 2k\pi$$

OR

$$\theta = 3x = \frac{4\pi}{3} + 2k\pi$$

Step 4 Solve for $x$:

$$x = \frac{1}{3}(3x) = \frac{1}{3}\left(\frac{2\pi}{3} + 2k\pi\right) = \boxed{\frac{2\pi}{9} + \frac{2k\pi}{3}}$$

OR

$$x = \frac{1}{3}(3x) = \frac{1}{3}\left(\frac{4\pi}{3} + 2k\pi\right) = \boxed{\frac{4\pi}{9} + \frac{2k\pi}{3}}$$

$$= \frac{2\pi + 6k\pi}{9} = \frac{2\pi(1+3k)}{9}$$

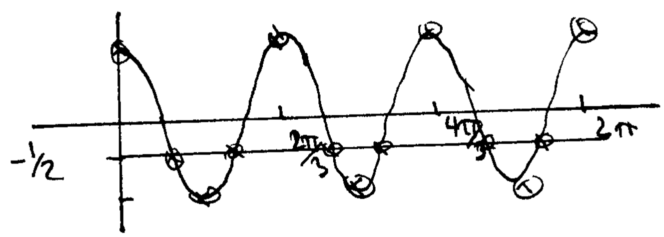
or

$$= \frac{4\pi + 6k\pi}{9} = \frac{2\pi(2+3k)}{9}$$

b) In $[0, 2\pi)$ the solutions are

$$x = \boxed{\underbrace{\frac{2\pi}{9}, \frac{4\pi}{9}}_{k=0}, \underbrace{\frac{8\pi}{9}, \frac{10\pi}{9}}_{k=1}, \underbrace{\frac{14\pi}{9}, \frac{16\pi}{9}}_{k=2}} \quad \cancel{\frac{20\pi}{9}} \leftarrow \text{Bigger than } 2\pi$$

What's going on graphically? $y = \cos 3x$ has 3 times the standard frequency ($\frac{1}{3}$ the standard period)




← Six solutions in $[0, 2\pi)$.

37) $\csc(4\alpha) = \sqrt{2} \iff \dfrac{1}{\sin(4\alpha)} = \sqrt{2}$

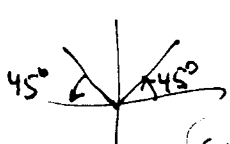
Equivalent: $\sin(4\alpha) = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ Solve $[0, 360°)$

Step 1 Reference angle $= 45°$ because $\sin 45° = \frac{\sqrt{2}}{2}$.

Step 2 angle $= 4\alpha = 45°$

or

$4\alpha = 180° - 45° = 135°$




Step 3 angle $= 4\alpha = 45° + 360°k$

or

$4\alpha = 135° + 360°k$

Step 4 $\alpha = \frac{1}{4}(45° + 360°k) = 11.25° + 90°k$

or $33.75° + 90°k$

In $[0, 360°)$

$$\boxed{\alpha = 11.25°, 33.75°, 101.25°, 123.75°, 191.25°, 213.75°, 281.25°, 303.75°}$$

Remark: Next time §4.4 Trig equations of Quadratic Type will be covered and will be on Monday's test.