<u>Warmup</u> Solve, in $[0°, 360°)$ [See section 4.2.]

(1) $\sin\theta = -0.6$ (2) $\cos\theta = -0.8$ (3) $\tan\theta = -0.75$

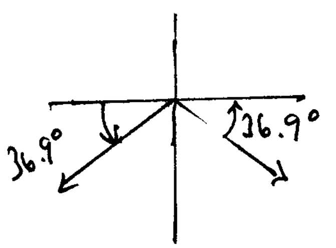
(1) $\sin\theta = -0.6$

Step 1 Solve $\sin\theta' = 0.6$ in $[0°, 90°)$

$\theta' = \sin^{-1} 0.6 = 36.9°$ = reference angle.

Step 2 Where is sine negative? QIII and QIV.

What angles θ in these quadrants have reference angle $\theta' = 36.9°$?



$\theta = 180° + 36.9° = \boxed{216.9°}$

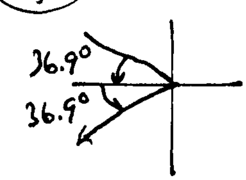
or

$\theta = 360° - 36.9° = \boxed{323.1°}$

(2) $\cos\theta = -0.8$

Step 1 $\cos\theta' = 0.8 \Rightarrow \theta' = \arccos(0.8) = 36.9°$

Step 2 cos is negative in QII and QIII.



$\theta = 180° - 36.9° = \boxed{143.1°}$

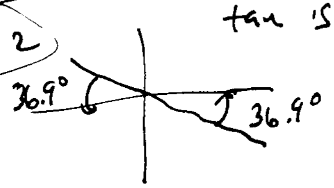
or

$\theta = 180° + 36.9° = \boxed{216.9°}$

(3) $\tan\theta = -0.75$

Step 1 Solve $\tan\theta' = 0.75 \Rightarrow \theta' = \arctan(0.75) = 36.9°$

Step 2 tan is negative in QII and QIV



$\theta = \boxed{143.1°} = 180° - 36.9°$

or

$\theta = \boxed{323.1°} = 360° - 36.9°$

§4.4 Trig equations of quadratic type

ex. [Flashback to intermediate algebra.]

$$x^{2/3} - 3x^{1/3} + 2 = 0$$

$$(x^{1/3})^2 - 3x^{1/3} + 2 = 0$$

Let $u = x^{1/3}$:

$$u^2 - 3u + 2 = 0$$

$$(u-2)(u-1) = 0$$

$u - 2 = 0$ or $u - 1 = 0$

$u = 2$ or $u = 1$

$\sqrt[3]{x} = x^{1/3} = 2$ or $x^{1/3} = 1$

$x = (x^{1/3})^3 = 2^3 = \boxed{8}$ or $x = (x^{1/3})^3 = 1^3 = \boxed{1}$

or

$$x^4 - 3x^2 + 2 = 0$$

$$(x^2)^2 - 3x^2 + 2 = 0$$

Let $u = x^2$:

$$u^2 - 3u + 2 = 0$$

$$(u-2)(u-1) = 0$$

$u = 2$ or $u = 1$

$x^2 = 2$ or $x^2 = 1$

$x = \boxed{\pm\sqrt{2}}$ or $x = \pm\sqrt{1} = \boxed{\pm 1}$

[End of flashback.]

ex: Solve $2\sin^2\theta - 3\sin\theta + 1 = 0$ in $[0°, 360°)$

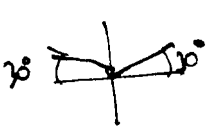
Let $u = \sin\theta$: $2u^2 - 3u + 1 = 0$

$$(2u - 1)(u - 1) = 0$$

$2u - 1 = 0$ or $u - 1 = 0$

$u = \frac{1}{2}$ OR $u = 1$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = 1$



$\theta = \boxed{30° \text{ or } 150°}$ OR $\theta = \boxed{90°}$

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sample test 13) Solve in $[0, 2\pi)$

$$4\sin^2 x - 3 = 0$$

optional:

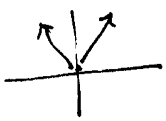
$$4u^2 - 3 = 0$$
$$4u^2 = 3$$
$$u^2 = \frac{3}{4}$$
$$u = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm\frac{\sqrt{3}}{2}$$

$\sin x = \frac{\sqrt{3}}{2}$ OR $\sin x = -\frac{\sqrt{3}}{2}$

NOTE:
$\sin \pi/3 = \frac{\sqrt{3}}{2}$



$x = \boxed{\frac{\pi}{3} \text{ or } \frac{2\pi}{3}}$

$x = \boxed{\frac{4\pi}{3} \text{ or } \frac{5\pi}{3}}$

sample test 14) Solve in $[0, 2\pi)$

$$\sin x = 1 - 2\sin^2 x$$

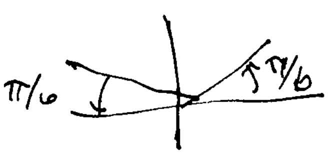
$$2\sin^2 x + \sin x - 1 = 0 \qquad \text{Let } u = \sin x$$
$$2u^2 + u - 1 = 0$$
$$(2u - 1)(u + 1) = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

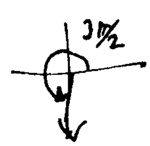
okay to go directly

$2\sin x - 1 = 0$ OR $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ OR





$x = \frac{3\pi}{2}$

solution set: $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

4.4
53) Solve in $[0, 2\pi)$. Approximate to nearest 10^{th} (of a radian)

$$5\sin^2 x - 2\sin x = \cos^2 x$$

<u>Use</u>: $\cos^2 x = 1 - \sin^2 x$.

$$5\sin^2 x - 2\sin x = 1 - \sin^2 x$$

$$+\sin^2 x \quad -1 \quad -1 + \sin^2 x$$

$$6\sin^2 x - 2\sin x - 1 = 0$$

Let $u = \sin x$

$$6u^2 - 2u - 1 = 0$$

$$u = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-1)}}{2(6)}$$

by the quadratic formula.

$$\sin x = \frac{2 \pm \sqrt{4 + 24}}{12} = \frac{2 \pm \sqrt{28}}{12} = \frac{2 \pm 2\sqrt{7}}{12} = \frac{\cancel{2}^{1}(1 \pm \sqrt{7})}{\cancel{12}_{6}}$$

$$\sin x = \frac{1 + \sqrt{7}}{6}$$

$$= 0.607625$$

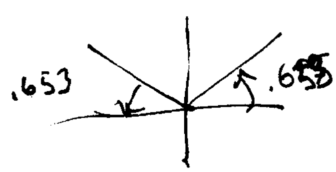
$\theta' =$ reference angle $= \sin^{-1}(0.607625) = .653$ radians

$$x = 0.653 \approx \boxed{0.7}$$

or

$$x = \pi - 0.653$$

$$= 2.489 \approx \boxed{2.5}$$



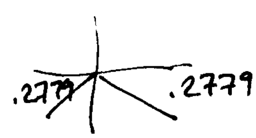
OR

$$\sin x = \frac{1 - \sqrt{7}}{6}$$

$$= -.2743$$

$\theta' = \sin^{-1}(+.2743) =$ ref. angle

$= .2779$ radians



$$x = \pi + .2779 = 3.419 \approx \boxed{3.4}$$

or

$$x = 2\pi - .2779$$

$$= \boxed{6.0}$$

Problems added after class

ex [Another flashback to Intermediate Algebra]
To solve radical equations, isolate the radical and square both sides,
But this may introduce extraneous solutions So you must check.

$$x - \sqrt{x} - 2 = 0 \Rightarrow x - 2 = \sqrt{x} \Rightarrow (x-2)^2 = \sqrt{x}^2$$

$$\Rightarrow x^2 - 4x + 4 = x \Rightarrow 0 = x^2 - 5x + 4 = (x-1)(x-4)$$

$$\Rightarrow \underbrace{\cancel{x=1}}_{\text{extraneous}} \text{ or } \boxed{x = 4}$$

[End of flashback.]

Sample Test

16) $\sin\theta + 1 = \sqrt{3}\cos\theta$

Motivated by the observation that $\cos\theta = \pm\sqrt{1-\sin^2\theta}$, we can isolate the radical and square, but watch for extraneous solutions;
[See also Example 5 in §4.4.]

$$\sin\theta + 1 = \sqrt{3}\cdot\sqrt{1-\sin^2\theta}$$

$$(\sin\theta + 1)^2 = \left(\sqrt{3}\sqrt{1-\sin^2\theta}\right)^2$$

$$\sin^2\theta + 2\sin\theta + 1 = 3(1-\sin^2\theta)$$

$$\sin^2\theta + 2\sin\theta + 1 = 3 - 3\sin^2\theta$$

$$+3\sin^2\theta \quad -3 \qquad -3 + 3\sin^2\theta$$

$$4\sin^2\theta + 2\sin\theta - 2 = 0 \Rightarrow$$

Divide by 2: $2\sin^2\theta + \sin\theta - 1 = 0$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\boxed{\theta = 30^\circ}$ or $\cancel{\theta = 150^\circ}$ or $\boxed{\theta = 270^\circ}$

Extraneous, for $\sin 150^\circ + 1 = \frac{1}{2} + 1 = \frac{3}{2} \neq -\frac{3}{2} = \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right) = \sqrt{3}\cos 150^\circ$