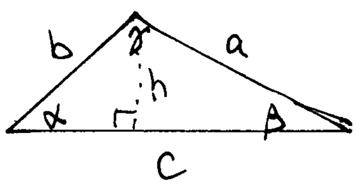
5.1 Law of Sines



$$\boxed{\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}}$$

Look at algebra of $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$. If you know three of the four of $a, b, \sin\alpha, \sin\beta$, you can calculate the ~~fourth~~.

How to derive this law: [surprisingly easy!]

Because ~~sin~~ $\sin\alpha = \frac{\text{opp}}{\text{hyp}}$ $\sin\alpha = \frac{h}{b}$. Also $\sin\beta = \frac{h}{a}$

Solve both ~~for~~ for h: $h = b\sin\alpha = a\sin\beta$

Divide by ab! $\frac{b\sin\alpha}{ab} = \frac{a\sin\beta}{ab} \Rightarrow \frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$

ex: $\boxed{AAS}$ $\beta = 94.7°$, $\alpha = 30.6°$, $b = 3.9$ in

5.1 10)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{3.9\sin 30.6°}{\sin 94.7°} = 1.992 \approx \boxed{2.0 \text{ in}}$$

$$\gamma = 180° - \alpha - \beta = 180° - 30.6° - 94.7° = \boxed{54.7°}$$

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta} \Rightarrow c = \frac{b\sin\gamma}{\sin\beta} = \frac{3.9\sin 54.7°}{\sin 94.7°} = 3.194 = \boxed{3.2 \text{ in}}$$

5.1

12) $\cancel{\beta = 28.6^\circ, \quad a = 40.7 \text{ cm}}$ Try not to think. Be laz

$\boxed{ASA}$

$\alpha = 39.7^\circ \qquad \gamma = 91.6^\circ \qquad b = 16.4 \text{ cm}$

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 39.7^\circ - 91.6^\circ = \boxed{48.7^\circ}$$

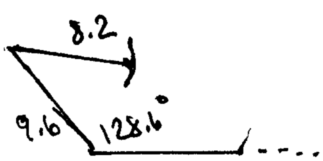
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{16.4 \sin 39.7^\circ}{\sin 48.7^\circ} = 13.944 \approx \boxed{13.9 \text{ cm}}$$

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta} \Rightarrow c = \frac{b\sin\gamma}{\sin\beta} = \frac{16.4 \sin 91.6^\circ}{\sin 48.7^\circ} = 21.821 \approx \boxed{21.8 \text{ cm}}$$

$\boxed{SSA}$ 18) $\gamma = 128.6^\circ \qquad a = 9.6 \text{ miles} \qquad c = 8.2 \text{ miles}$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} \Rightarrow \sin\alpha = \frac{a\sin\gamma}{c} = \frac{9.6 \sin 128.6^\circ}{8.2}$$

$\sin\alpha = 0.91495$ ← Could have two solutions.



$\alpha_1 = \sin^{-1} 0.91495 = 66.2^\circ$

$\beta_1 = 180^\circ - \alpha_1 - \gamma = 180^\circ - 66.2^\circ - 128.6^\circ = -14.8^\circ$ STOP

$\alpha_2 = 180^\circ - 66.2^\circ = 113.8^\circ$ STOP you can't have two obtuse angles

$\beta_2 = 180^\circ - \alpha_2 - \gamma = 180^\circ - 113.8^\circ - 128.6^\circ = -62.4^\circ$ STOP

$\boxed{\text{No solution}}$

16) $\alpha = 41.2^\circ$ $a = 8.1$ m $b = 10.6$ m

$\boxed{SSA}$

So use Law of sines

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \Rightarrow \sin\beta = \frac{b\sin\alpha}{a} = \frac{10.6\sin 41.2^\circ}{8.1} = .861989$$

This sine equation has two solutions, one in Q I, one in Q II.

$$\beta_1 = \sin^{-1} .861989 = \boxed{59.5^\circ}$$

$$\beta_2 = 180^\circ - 59.5^\circ = \boxed{120.5^\circ}$$

$$\gamma_1 = 180^\circ - \alpha - \beta_1 = 180^\circ - 41.2^\circ - 59.5^\circ = \boxed{79.3^\circ}$$

$$\gamma_2 = 180^\circ - \alpha - \beta_2 = 180^\circ - 41.2^\circ - 120.5^\circ = \boxed{18.3^\circ}$$

$$\frac{c_1}{\sin\gamma_1} = \frac{a}{\sin\alpha} \Rightarrow c_1 = \frac{a\sin\gamma_1}{\sin\alpha} = \frac{8.1\sin 79.3^\circ}{\sin 41.2^\circ} = 12.0833 \approx \boxed{12.1 \text{ m}}$$

$$c_2 = \frac{a\sin\gamma_2}{\sin\alpha} = \frac{8.1\sin 18.3^\circ}{\sin 41.2^\circ} = 3.861 \approx \boxed{3.8 \text{ m}}$$