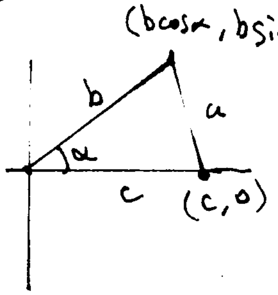
5.2 Law of Cosines



$$\boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

Good for the SAS case.

<u>proof</u>: Recall that

$$\cos\alpha = \frac{x}{r} = \frac{x}{b} \Rightarrow x = b\cos\alpha$$

$$\sin\alpha = \frac{y}{r} = \frac{y}{b} \Rightarrow y = b\sin\alpha$$

Use (Distance formula)^2 to write a^2 two ways:

$$a^2 = (b\cos\alpha - c)^2 + (b\sin\alpha - 0)^2$$

$$a^2 = b^2\cos^2\alpha - 2bc\cos\alpha + c^2 + b^2\sin^2\alpha$$

$$a^2 = b^2\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1} + c^2 - 2bc\cos\alpha$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

QED.
Quid Erat Demonstrandum.

<u>Remark</u>: To get the other two, just permute the letters

$a \to b \quad b \to c \quad c \to a \quad \alpha \to \beta \quad \beta \to \gamma, \quad \gamma \to \alpha$

<u>Remark</u>. $a^2 = b^2 + c^2 - 2bc\cos\alpha$ If we solve for $\cos\alpha$:

$$\frac{2bc\cos\alpha}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\boxed{\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}}$$

Good for the SSS case.

5.2 10) $a = 1.3$ ft $\quad b = 14.9$ ft $\quad \gamma = 9.8^\circ$

$\boxed{SAS}$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$= 1.3^2 + 14.9^2 - 2(1.3)(14.9)\cos 9.8^\circ$$
$$= 185.525 \Rightarrow c = \sqrt{185.525} = \boxed{13.6 \text{ ft}}$$

Strategy: We can now use the Law of Sines to find the next angle. A consideration: it's a pain in the rear to have to worry about a sine equation with two solutions.

Idea: A triangle can have at most one obtuse angle ($> 90^\circ$). ~~The second, hence~~, So, the second (or third) largest angle is guaranteed to be acute so we would only need the QI solution.

Note: β will be the largest angle, so let's look for α.

Law of Sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\gamma}{c} \Rightarrow \sin\alpha = \dfrac{a\sin\gamma}{c} = \dfrac{1.3\sin 9.8^\circ}{13.6}$

$$= 0.016245$$

$$\alpha = \sin^{-1} 0.016245 = 0.9308^\circ \approx \boxed{0.93^\circ} \approx 0.9^\circ$$

$$\beta = 180^\circ - \gamma - \alpha = 180^\circ - 9.8^\circ - 0.9^\circ = \boxed{169.3^\circ}$$

5.2 16) $a = 4.1$ cm $\quad b = 9.8$ cm $\quad c = 6.2$ cm

$\boxed{SSS}$

Strategy: Since there can only be one (or zero) obtuse angles, let's find it first. Then when we later use the Law of Sines, we won't have to be concerned with Q II solutions of a sine equation.

$$b^2 = a^2 + c^2 - 2ac\cos\beta \Rightarrow \cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$\cancel{= 4.1^2 + 9.8^2 -}$$

$$= \frac{4.1^2 + 6.2^2 - 9.8^2}{2(4.1)(6.2)}$$

$$= -0.8023$$

$$\beta = \cos^{-1}(-0.8023) = 143.352 \approx \boxed{143.4^\circ}$$

Now use the Law of Sines with no worries about two solutions to sine equations.

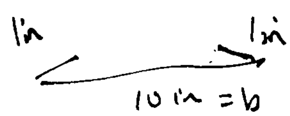
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \Rightarrow \sin\alpha = \frac{a\sin\beta}{b} = \frac{4.1\sin 143.4^\circ}{9.8}$$

$$= 0.2497$$

$$\alpha = \sin^{-1}(0.2497) = 14.46^\circ \approx \boxed{14.5^\circ}$$

$$\gamma = 180^\circ - 143.4^\circ - 14.5^\circ = \boxed{22.1^\circ}$$

Remark: SSS may have no solution e.g. $a = 1$ in, $b = 1$ in, $c = 10$ in.



$$\cos\beta = \frac{1^2+1^2-10^2}{2\cdot1\cdot1} = \frac{-98}{2} = -49$$

No solution