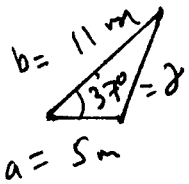
5.3 Area of a Triangle

ex. [SAS] Given $a = 5$ meters, $b = 11$ meters, $\gamma = 37°$

Area of the triangle?



$$\text{Area} = \frac{1}{2} ab \sin\gamma = \frac{1}{2}(5\text{m})(11\text{m})\sin 37°$$
$$= 16.54\ \text{m}^2$$

ex [SSS] Given a triangle has sides 7 ft, 5 ft and 8 ft.
What is the area? Use Heron's formula

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

where S = semiperimeter $= \frac{a+b+c}{2}$

perimeter $= 7 + 5 + 8 = 20$ ft

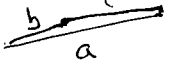
$S = 10$ ft

$S - a = 10 - 7 = 3$ ft

$S - b = 10 - 5 = 5$ ft

$S - c = 10 - 8 = 2$ ft

"Degenerate" triangle: (zero area)



$b + c = a$

$$\text{Area} = \sqrt{(10\text{ft})(3\text{ft})(5\text{ft})(2\text{ft})}$$
$$= \sqrt{300\ \text{ft}^4} = 10\sqrt{3}\ \text{ft}^2 \simeq 17.3\ \text{ft}^2$$

Remark: (1) Notice the units: $\sqrt{\text{ft}^4} = \text{ft}^2$

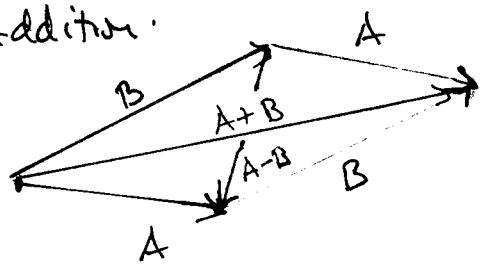
(2) $S - a = \frac{1}{2}(a+b+c) - a = \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c - a$
$= \frac{1}{2}b + \frac{1}{2} - \frac{1}{2}a = \frac{1}{2}(b+c-a)$

5.4 Vectors...

are represented as directed line segments in the plane. They have a direction (angle $= \theta$), and a length $|\vec{v}|$.

Operations:

(1) Addition.

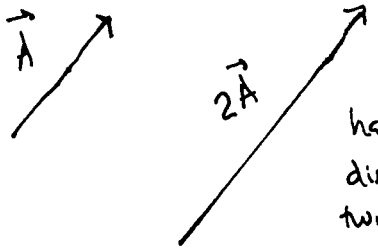


Observe:

(1) $\vec{A}+\vec{B} = \vec{B}+\vec{A}$

(2) $\vec{0}$ = zero vector = vector of zero length has the property that $\vec{0}+\vec{B} = \vec{B}$.

(2) Scalar multiplication ["scalar" means a real number in the context of vectors]



has same direction, twice the length.

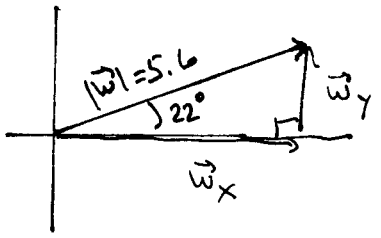
$-\frac{1}{2}\vec{A}$

has opposite direction, half the length.

Remark: $\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$ is the other diagonal of the parallelogram.

NOTE: $(\vec{A}-\vec{B})+\vec{B} = \vec{A}$

ex. A vector $\vec{w}$ has magnitude 5.6 Newtons and a direction angle $\theta = 22°$. Find $|\vec{w}_x|$ and $|\vec{w}_y|$.



$$\cos 22° = \frac{\text{adj}}{\text{hyp}} = \frac{|\vec{w}_x|}{|\vec{w}|}$$

So $|\vec{w}_x| = |\vec{w}| \cos 22°$

$= 5.6 \cos 22° = 5.2$ newtons

And $|\vec{w}_y| = |\vec{w}| \sin\theta$

$= 5.6 \sin 22° = 2.1$ newtons

What we did: Given r, θ → we Got x, y.

Next. Given x, y → find r, θ.

Look for a QUIZ on line Friday.