5.4 Vectors (cont'd)

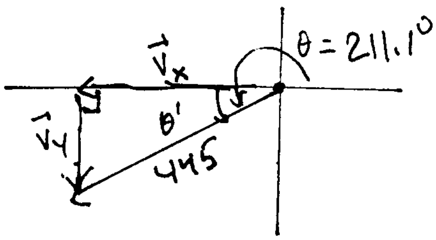
Components $\longleftrightarrow$ Direction and magnitude

x, y $\qquad$ r, θ

ex: #24) Given $|\vec{v}| = 445 = r$ and $\theta = 211.1^\circ$

Find $\vec{v}_x$ and $\vec{v}_y$.

Note: $\theta' = 211.1^\circ - 180^\circ = 31.1^\circ$



$|\vec{v}_x| = r\cos\theta'$ because $\frac{adj}{hyp} = \cos\theta'$

So $adj = hyp \cos\theta'$

$|\vec{v}_x| = 445\cos 31.1^\circ = \boxed{381.0}$

Likewise, because $\sin\theta' = \frac{opp}{hyp}$

then $opp = (hyp)\sin\theta'$

$|\vec{v}_y| = 445 \sin 31.1^\circ = \boxed{229.9}$

"Component form" of the vector $\vec{v}$ $\qquad$ $\vec{v} = \langle -381.0, -229.9 \rangle$

Essential idea:

Because $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$, it follows that:

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

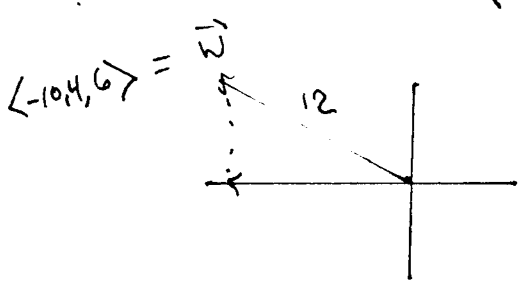
We're using these equations with $r = |\vec{v}|$ and $\theta =$ direction.

ex: For some vector $\vec{w}$, $|\vec{w}| = 12$ lbs and $\theta = 150°$. Find the component form of $\vec{w}$.

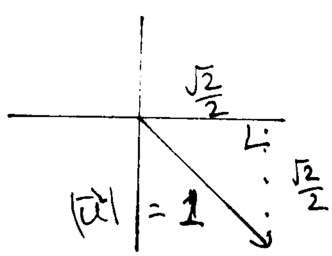
$$x = r\cos\theta = 12\cos 150° = 12\left(-\frac{\sqrt{3}}{2}\right) = -6\sqrt{3} \approx -10.4$$

$$y = r\sin\theta = 12\sin 150° = 12\left(\frac{1}{2}\right) = 6$$

$$\vec{w} = \langle -6\sqrt{3}, 6\rangle \approx \langle -10.4, 6\rangle$$



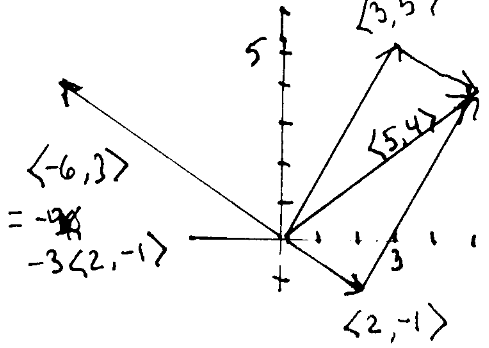
ex: $\vec{u}$ has $|\vec{u}| = 1$ and direction $\theta = 315°$. Find the component form of $\vec{u}$.



$$\vec{u} = \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle = 1\langle \cos 315°, \sin 315°\rangle$$

Addition and scalar multiplication using component form:

ex: $\vec{v} = \langle 3, 5\rangle$ $\quad\vec{w} = \langle 2, -1\rangle$



$$\vec{v} + \vec{w} = \langle 3+2, 5+(-1)\rangle = \langle 5, 4\rangle$$

ex: $\vec{w} = \langle 2, -1 \rangle$ Find $-3\vec{w} = -3\langle 2, -1 \rangle$

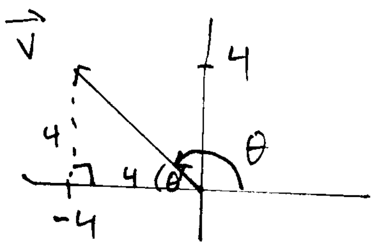
$$= \langle -3\cdot 2, -3(-1) \rangle$$
$$= \langle -6, 3 \rangle$$

ex: For $\vec{v} = \langle 3, 5 \rangle$ and $\vec{w} = \langle 2, -1 \rangle$

what is $5\vec{v} - 2\vec{w}$?

$$5\vec{v} - 2\vec{w} = 5\langle 3, 5 \rangle - 2\langle 2, -1 \rangle$$
$$= \langle 15, 25 \rangle + \langle -4, 2 \rangle$$
$$= \langle 11, 27 \rangle$$

ex: Suppose $\vec{v} = \langle -4, 4 \rangle$. Find the magnitude and direction of $\vec{v}$.



$$|\vec{v}| = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$\tan\theta' = \frac{4}{4} = 1 \Rightarrow \theta' = \tan^{-1} 1 = 45^\circ$$

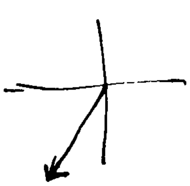
So $\theta = 180^\circ - 45^\circ = 135^\circ$

Defn: If $\vec{v} = \langle a, b \rangle$ then $|\vec{v}| = \sqrt{a^2 + b^2}$.

Remark: To find θ = direction of $\vec{v}$

use $\tan\theta = \frac{b}{a}$ and choose θ to be in the correct quadrant.

32) $\vec{v} = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle$ Find $|\vec{v}|$ and θ.



$$|\vec{v}| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

$$\tan\theta = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$$ Take $\theta = 60^\circ + 180^\circ = 240^\circ$

Defn: If $\vec{A} = \langle a_1, a_2 \rangle$ and $\vec{B} = \langle b_1, b_2 \rangle$

the dot product of $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = a_1 b_1 + a_2 b_2$$

ex: $\vec{A} = \langle 3, 5 \rangle$ $B = \langle 2, -1 \rangle$

$$\vec{A} \cdot \vec{B} = (3)(2) + (5)(-1)$$
$$= 6 - 1 = \boxed{1}$$

ex: $\vec{u} = \langle 4, -3 \rangle$ $\vec{w} = \langle 3, 4 \rangle$

Then $\vec{u} \cdot \vec{w} = (4)(3) + (-3)(4)$
$$= 12 - 12 = \boxed{0}$$

ex: $\vec{v} = \langle 5, 0 \rangle$ $\vec{w} = \langle -3, -3 \rangle$

$$\vec{v} \cdot \vec{w} = (5)(-3) + (0)(-3)$$
$$= \boxed{-15}$$

Fact (Geometric interpretation of the Dot Product)

If $\vec{A}$ and $\vec{B}$ are vectors, then

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos \alpha}$$

where α = angle between $\vec{A}$ and $\vec{B}$.

Solve $\cos\alpha$: If $|\vec{A}| \neq 0$ and $|\vec{B}| = 0$, then

$$\boxed{\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}}$$