1.4 (cont'd) Quotient identities

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{Y}{r}}{\frac{X}{r}} \cdot \frac{\frac{r}{1}}{\frac{r}{1}} = \frac{\frac{Y}{1}}{\frac{X}{1}} = \frac{Y}{X} = \tan\theta$$

Similarly,

$$\frac{\cos\theta}{\sin\theta} = \frac{\frac{X}{r}}{\frac{Y}{r}} = \frac{X}{r} \div \frac{Y}{r} = \frac{X}{r} \cdot \frac{r}{Y} = \frac{X}{Y} = \cot\theta$$

Behold: Start with

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Now here's to derive the other two pythagorean identities.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Similarly $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

14) Given: $\sin\theta = -\frac{8}{43}$ for some angle θ.

Find $\csc\theta$. Answer: $\csc\theta = \frac{1}{\sin\theta} = -\frac{43}{8}$.

66) Given: $\cos\theta = \frac{4}{5}$ and θ is in quadrant IV

Find $\sin\theta$.

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{or}$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$(\sin\theta)^2 + \left(\frac{4}{5}\right)^2 = 1$$

$$(\sin\theta)^2 = 1 - \left(\frac{4}{5}\right)^2 = \frac{25}{25} - \frac{16}{25} = \frac{9}{25}$$

So $\sin\theta = \pm\sqrt{\frac{9}{25}} = \frac{3}{5}$ or $\frac{-3}{5}$

But θ is q. IV, so $\boxed{\sin\theta = -\frac{3}{5}}$

80) Given: $\cos\theta = \frac{\sqrt{5}}{8}$ and $\tan\theta < 0$.

Find all six trig functions.

$\sin\theta \overset{(2)}{=} -\frac{\sqrt{59}}{8}$ $\quad \cos\theta = \frac{\sqrt{5}}{8}$ $\quad \tan\theta \overset{(4)}{=} \frac{-\sqrt{59}/8}{\sqrt{5}/8} = \frac{-\sqrt{59}}{\sqrt{5}}$

$\csc\theta \overset{(3)}{=} -\frac{8}{\sqrt{59}}$ $\quad \sec\theta \overset{(1)}{=} \frac{8}{\sqrt{5}}$ $\quad \cot\theta \overset{(5)}{=} -\frac{\sqrt{5}}{\sqrt{59}}$

Scratch work

(2) $\sin^2\theta + \cos^2\theta = 1$

$(\sin\theta)^2 + \left(\frac{\sqrt{5}}{8}\right)^2 = 1$

$(\sin\theta)^2 = 1 - \left(\frac{\sqrt{5}}{8}\right)^2 = \frac{64}{64} - \frac{5}{64} = \frac{59}{64}$

$\sin\theta = \pm\sqrt{\frac{59}{64}} = \pm\frac{\sqrt{59}}{8}$ but choose '$-$' to make $\tan\theta < 0$.

MORAL: If you know one trig function (for a given θ) you know all six, provided you also know the quadrant.